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DIFFERENCE BETWEEN STANDARD
AND QUAZI-CONFORMAL BFKL KERNELS

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Difference between standard and quasi-conformal BFKL kernels ^{*}

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Abstract

As it was recently shown, the colour singlet BFKL kernel, taken in Möbius representation in the space of impact parameters, can be written in quasi-conformal shape, which is unbelievably simple compared with the conventional form of the BFKL kernel in momentum space. It was also proved that the total kernel is completely defined by its Möbius representation. In this paper we calculated the difference between standard and quasi-conformal BFKL kernels in momentum space and discovered that it is rather simple. Therefore we come to the conclusion that the simplicity of the quasi-conformal kernel is caused mainly by using the impact parameter space.

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1 Introduction

The BFKL (Balitsky-Fadin-Kuraev-Lipatov) approach [1] was formulated in the momentum space. In this space the kernel of the BFKL equation was calculated in the next-to-leading order (NLO) long ago, at first for the forward scattering (i.e. for $t = 0$ and colour singlet in the t -channel) [2] and then for any fixed (not growing with energy) squared momentum transfer t and any possible two-gluon colour state in the t -channel [3]. Unfortunately, the NLO kernel is rather complicated. In particular, the colour singlet kernel for $t \neq 0$ is found in the NLO in the form of an intricate two-dimensional integral.

In the most interesting for phenomenological applications case of colourless particle scattering, the leading-order (LO) BFKL kernel has a remarkable property [4]: it can be taken in the Möbius representation (i.e. in the space of functions vanishing at coinciding transverse coordinates of Reggeons), where it turns out to be invariant in regard to conformal transformations of these coordinates. Moreover, in the coordinate space the Möbius representation (we will call it “Möbius form”) of the LO BFKL kernel coincides [5] with the kernel of the colour dipole model [6].

In the NLO the conformal invariance is violated in QCD by the running coupling. One could hope that the Möbius form of the colour singlet NLO kernel is quasi-conformal, i.e. conformal invariance is violated only by terms proportional to the β -function. However, the direct transformation of the colour singlet kernel found in Ref. [3] from momentum to coordinate space with the restriction of Möbius representation gives a kernel which is not quasi-conformal [7, 8, 9]. But in the NLO kernel there is an ambiguity [5, 10], analogous to the well known ambiguity of the NLO anomalous dimensions, because it is possible to redistribute radiative corrections between the kernel and the impact factors. The ambiguity, discussed in details in Ref. [11], permits to make transformations

$$\hat{\mathcal{K}} \rightarrow \hat{\mathcal{K}} - \alpha_s[\hat{\mathcal{K}}^{(B)}, \hat{U}] \quad (1)$$

conserving the LO kernel $\hat{\mathcal{K}}^{(B)}$ (which is fixed in our case by the requirement of conformal invariance of its Möbius form) and changing the NLO part of

the kernel. Note that this transformation must conserve the gauge invariance properties of the kernel, so that the operator \hat{U} must have in this respect the same properties as $\hat{\mathcal{K}}^{(B)}$.

The NLO kernel calculated in Ref. [3] is defined according to the prescriptions given in Ref. [12]. We will call it the “standard kernel”. Recently it was shown [13] that there exist an operator \hat{U} such that the transformation (1) applied to this standard kernel gives a kernel with quasi-conformal Möbius form, which agrees with the form obtained in Ref. [14] in the colour dipole approach. It turns out that this form is quite simple. It is unbelievably simple in comparison with the form of the standard kernel [3]. Evidently, the question arose about the relation between these two forms.

This question is not trivial not only because the Möbius form is defined in the coordinate space, whereas the standard kernel was calculated in the momentum space. Remind that the Möbius representation is defined on a special class of functions. Therefore at the first sight it seemed impossible to reconstruct the complete operator from its Möbius form. However, due to the gauge invariance of the BFKL kernel, it is not so. It was shown [15] for any gauge invariant two-particle operator that it is possible to restore the complete operator from its Möbius form and the restoration is unique up to terms which do not contribute to the operator matrix elements, because of symmetry and gauge invariance of the wave functions.

Therefore, it is in principle possible to restore the complete BFKL kernel from its quasi-conformal Möbius form. Since this form is quite simple, one can hope for simplicity of the complete kernel in the momentum space too. Evidently this kernel differs from the standard kernel found in Ref. [3], but is connected with the last one by the transformation (1). However, the direct restoration is not easy. It includes the Fourier transformation of the Möbius form from coordinate to momentum space and, although this form is very compact, the transformation is intricate since it contains complicated integrals.

Instead, one can try to find the difference between the standard kernel and the one restored from the quasi-conformal Möbius form. Our paper is devoted to the solution of this problem. The difference under investigation is given by the second term in the transformation (1). For the operator \hat{U} , both Möbius form and complete representation in the momentum space are known now [15]. The same is true for $\hat{\mathcal{K}}^{(B)}$. We are looking for the difference in the momentum space. It can be found using for the calculation of the commutator in the transformation (1) both \hat{U} and $\hat{\mathcal{K}}^{(B)}$ in this space. Alternatively, it is possible to calculate the commutator in the coordinate Möbius space and then to restore its complete form in the momentum space

using the method developed in Ref. [15]. We use both these ways, on one side for cross-checking the obtained result, on the other for a demonstration of the efficiency of the method of restoration of complete operators from their Möbius forms, developed in Ref. [15].

The paper is organized as follows. In the next Section we calculate the commutator in the transformation (1) directly in the momentum space. In Section 3 this commutator is calculated firstly in the coordinate Möbius space and then the obtained result is used for restoration of the complete form of the commutator in the momentum space. The last Section contains our conclusions. The integrals used in the calculations are presented in the Appendix.

2 Direct calculation of the difference in momentum space

We adopt the notation used in Ref. [15] and put the space-time dimension D equal to 4, so that states $|\vec{q}\rangle$ with definite two-dimensional transverse Reggeon momentum \vec{q} and states $|\vec{r}\rangle$ with definite Reggeon impact parameter \vec{r} are normalized as follows:

$$\langle \vec{q} | \vec{q}' \rangle = \delta(\vec{q} - \vec{q}'), \quad \langle \vec{r} | \vec{r}' \rangle = \delta(\vec{r} - \vec{r}'), \quad \langle \vec{r} | \vec{q} \rangle = \frac{e^{i\vec{q}\vec{r}}}{2\pi}. \quad (2)$$

As it was shown in Ref. [13], the quasi-conformal kernel $\hat{\mathcal{K}}^{QC}$ can be obtained from the kernel calculated in Ref. [3] by the transformation (1), namely,

$$\hat{\mathcal{K}}^{QC} = \hat{\mathcal{K}} - \alpha_s [\hat{\mathcal{K}}^{(B)}, \hat{U}]. \quad (3)$$

It is worthwhile to note here that the kernel $\hat{\mathcal{K}}$ is defined in such a way that in the LO its Möbius form is conformal invariant. Therefore one has (see Ref. [15] for details)

$$\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}} | \vec{q}'_1, \vec{q}'_2 \rangle = \delta(\vec{q}_1 + \vec{q}_2 - \vec{q}'_1 - \vec{q}'_2) \frac{1}{\vec{q}_1^2 \vec{q}_2^2} K(\vec{q}_1, \vec{q}'_1; \vec{q}), \quad (4)$$

where $\vec{q} = \vec{q}_1 + \vec{q}_2 = \vec{q}'_1 + \vec{q}'_2$ and $K(\vec{q}_1, \vec{q}'_1; \vec{q})$ is the symmetric kernel

$$K(\vec{q}_1, \vec{q}'_1; \vec{q}) = K(\vec{q}'_1, \vec{q}_1; \vec{q}), \quad (5)$$

defined in Ref. [12] and calculated in Ref. [3]. Its real part K_r satisfies the gauge invariance conditions

$$K_r(\vec{0}, \vec{q}'_1; \vec{q}) = K_r(\vec{q}_1, \vec{0}; \vec{q}) = K_r(\vec{q}, \vec{q}'_1; \vec{q}) = K_r(\vec{q}_1, \vec{q}; \vec{q}). \quad (6)$$

Our goal is to find in the momentum space an explicit form for the commutator in Eq. (3). In this Section it is done using the known expressions in this space for the LO kernel $\hat{\mathcal{K}}^{(B)}$ and the operator \hat{U} .

The kernel $\hat{\mathcal{K}}^{(B)}$ can be presented as follows

$$\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}}^{(B)} | \vec{q}'_1, \vec{q}'_2 \rangle = \delta(\vec{q}_{11'} + \vec{q}_{22'}) \frac{\alpha_s N_c}{2\pi^2} \left[R(\vec{q}_1, \vec{q}_2; \vec{k}) - \delta(\vec{k}) \int d\vec{l} V(\vec{q}_1, \vec{q}_2; \vec{l}) \right], \quad (7)$$

where $\vec{k} = \vec{q}_{11'} = -\vec{q}_{22'}$ (here and below $\vec{a}_{ij'} = \vec{a}_i - \vec{a}'_j$, $\vec{a}_{ij} = \vec{a}_i - \vec{a}_j$, $\vec{a}_{i'j'} = \vec{a}'_i - \vec{a}'_j$),

$$R(\vec{q}_1, \vec{q}_2; \vec{k}) = \frac{2}{\vec{k}^2} - 2 \frac{\vec{q}_1 \vec{k}}{\vec{k}^2 \vec{q}_1^2} + 2 \frac{\vec{q}_2 \vec{k}}{\vec{k}^2 \vec{q}_2^2} - 2 \frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1^2 \vec{q}_2^2} \quad (8)$$

and

$$V(\vec{q}_1, \vec{q}_2; \vec{l}) = \frac{2}{\vec{l}^2} - \frac{\vec{l}(\vec{l} - \vec{q}_1)}{\vec{l}^2(\vec{l} - \vec{q}_1)^2} - \frac{\vec{l}(\vec{l} - \vec{q}_2)}{\vec{l}^2(\vec{l} - \vec{q}_2)^2}. \quad (9)$$

Note that the term $2/\vec{l}^2$ in $V(\vec{q}_1, \vec{q}_2; \vec{l})$ leads to the divergence of the integral over $d\vec{l}$ in the second term of Eq. (7), which represents the virtual part of the kernel. It is a well known infrared divergence which cancels with the divergence coming from the term $2/\vec{k}^2$ in the part with $R(\vec{q}_1, \vec{q}_2; \vec{k})$ in Eq. (7) (real part), when $\hat{\mathcal{K}}^{(B)}$ acts on some state. In the commutator $[\hat{\mathcal{K}}^{(B)}, \hat{U}]$ there are no problems with these divergences at all, because they cancel separately in the virtual and real parts.

The gauge invariance properties for R look as follows:

$$R(\vec{q}_1, \vec{q}_2; \vec{q}_1) = R(\vec{q}_1, \vec{q}_2; -\vec{q}_2) = 0, \\ (\vec{q}_1^2 \vec{q}_2^2 R(\vec{q}_1, \vec{q}_2; \vec{k}))|_{\vec{q}_1=0} = (\vec{q}_1^2 \vec{q}_2^2 R(\vec{q}_1, \vec{q}_2; \vec{k}))|_{\vec{q}_2=0} = 0. \quad (10)$$

An explicit form of the operator \hat{U} in the momentum space was found in Ref. [15]. Omitting terms which do not contribute to the commutator in Eq. (3), we have

$$\langle \vec{q}_1, \vec{q}_2 | \alpha_s \hat{U} | \vec{q}'_1, \vec{q}'_2 \rangle = \delta(\vec{q}_{11'} + \vec{q}_{22'}) \frac{\alpha_s N_c}{4\pi^2} R_u(\vec{q}_1, \vec{q}_2; \vec{k}) \\ - \frac{\alpha_s \beta_0}{8\pi} \ln(\vec{q}_1^2 \vec{q}_2^2) \delta(\vec{q}_{11'}) \delta(\vec{q}_{22'}), \quad (11)$$

where β_0 is the first coefficient of the Gell-Mann–Low function,

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f \quad (12)$$

and

$$\begin{aligned}
R_u(\vec{q}_1, \vec{q}_2; \vec{k}) &= \frac{1}{\vec{q}_1^2} \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2^2}{\vec{k}^2 \vec{q}^2} \right) + \frac{1}{\vec{q}_2^2} \ln \left(\frac{\vec{q}_2'^2 \vec{q}_1^2}{\vec{k}^2 \vec{q}^2} \right) + \frac{1}{\vec{k}^2} \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2'^2}{\vec{q}_1^2 \vec{q}_2^2} \right) \\
&\quad - 2 \frac{\vec{q}_1 \vec{k}}{\vec{k}^2 \vec{q}_1^2} \ln \left(\frac{\vec{q}_1'^2}{\vec{k}^2} \right) + 2 \frac{\vec{q}_2 \vec{k}}{\vec{k}^2 \vec{q}_2^2} \ln \left(\frac{\vec{q}_2'^2}{\vec{k}^2} \right) - 2 \frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1^2 \vec{q}_2^2} \ln \left(\frac{\vec{q}^2}{\vec{k}^2} \right). \quad (13)
\end{aligned}$$

Note that R_u has the same gauge invariance properties as R :

$$\begin{aligned}
R_u(\vec{q}_1, \vec{q}_2; \vec{q}_1) &= R_u(\vec{q}_1, \vec{q}_2; -\vec{q}_2) = 0, \\
(\vec{q}_1^2 \vec{q}_2^2 R_u(\vec{q}_1, \vec{q}_2; \vec{k}))|_{\vec{q}_1=0} &= (\vec{q}_1^2 \vec{q}_2^2 R_u(\vec{q}_1, \vec{q}_2; \vec{k}))|_{\vec{q}_2=0} = 0. \quad (14)
\end{aligned}$$

Indeed, these properties are required to conserve the gauge invariance in the transformation (1).

Another important property of R_u is the absence of either infrared, or ultraviolet non-integrable singularities, thus leading to convergence of the integral

$$\int \frac{d\vec{k}_1 d\vec{k}_2}{\pi} \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) R_u(\vec{q}_1 - \vec{k}_1, \vec{q}_2 + \vec{k}_1; \vec{k}_2) = -\ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right). \quad (15)$$

The calculation of this integral and of the integrals appearing below is described in the Appendix. The result (15) follows from (A.1) and (A.4) with a subsequent elementary integration over l .

Having Eqs. (7) and (11), it is quite straightforward to write the commutator $[\hat{\mathcal{K}}^{(B)}, \hat{U}]$ in the form

$$\begin{aligned}
\langle \vec{q}_1, \vec{q}_2 | \alpha_s [\hat{\mathcal{K}}^{(B)}, \hat{U}] | \vec{q}_1', \vec{q}_2' \rangle &= \delta(\vec{q}_{11'} + \vec{q}_{22'}) \frac{\alpha_s^2 N_c^2}{8\pi^3} \left[\frac{\beta_0}{2N_c} \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2'^2}{\vec{q}_1^2 \vec{q}_2^2} \right) R(\vec{q}_1, \vec{q}_2; \vec{k}) \right. \\
&\quad \left. + \int \frac{d\vec{l}}{\pi} \left(V(\vec{q}_1', \vec{q}_2'; \vec{l}) - V(\vec{q}_1, \vec{q}_2; \vec{l}) \right) R_u(\vec{q}_1, \vec{q}_2; \vec{k}) + F(\vec{q}_1, \vec{q}_2; \vec{k}) \right], \quad (16)
\end{aligned}$$

where

$$\begin{aligned}
F(\vec{q}_1, \vec{q}_2; \vec{k}) &= \int \frac{d\vec{k}_1 d\vec{k}_2}{\pi} \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) \mathcal{F}(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2), \quad \mathcal{F}(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2) = \\
&= R(\vec{q}_1, \vec{q}_2; \vec{k}_1) R_u(\vec{q}_1 - \vec{k}_1, \vec{q}_2 + \vec{k}_1; \vec{k}_2) - R_u(\vec{q}_1, \vec{q}_2; \vec{k}_1) R(\vec{q}_1 - \vec{k}_1, \vec{q}_2 + \vec{k}_1; \vec{k}_2). \quad (17)
\end{aligned}$$

The infrared divergent pieces in the virtual parts entering the integral over $d\vec{l}$ in Eq. (16) cancel, and one can easily obtain (see (A.5))

$$\int \frac{d\vec{l}}{\pi} \left(V(\vec{q}'_1, \vec{q}'_2; \vec{l}) - V(\vec{q}_1, \vec{q}_2; \vec{l}) \right) = \ln \left(\frac{\vec{q}'_1{}^2 \vec{q}'_2{}^2}{\vec{q}_1{}^2 \vec{q}_2{}^2} \right). \quad (18)$$

Unfortunately, the calculation of $F(\vec{q}_1, \vec{q}_2; \vec{k})$ is not so easy, both because of the presence of a great number of terms in $\mathcal{F}(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2)$ and of the complexity of the integration. One of the reasons of this complexity is the singularity of $R(\vec{q}_1, \vec{q}_2; \vec{k})$ at $\vec{k}^2 = 0$. Of course, this singularity disappears in $F(\vec{q}_1, \vec{q}_2; \vec{k})$, Eq. (17). To make this evident, let us write

$$R(\vec{q}_1, \vec{q}_2; \vec{k}) = \frac{2}{\vec{k}^2} + R_f(\vec{q}_1, \vec{q}_2; \vec{k}),$$

$$R_f(\vec{q}_1, \vec{q}_2; \vec{k}) = -2 \frac{\vec{q}_1 \vec{k}}{\vec{k}^2 \vec{q}_1^2} + 2 \frac{\vec{q}_2 \vec{k}}{\vec{k}^2 \vec{q}_2^2} - 2 \frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1^2 \vec{q}_2^2}, \quad (19)$$

and divide $F(\vec{q}_1, \vec{q}_2; \vec{k})$, Eq. (17), into three pieces:

$$F(\vec{q}_1, \vec{q}_2; \vec{k}) = \sum_{i=1}^3 F_i(\vec{q}_1, \vec{q}_2; \vec{k}), \quad (20)$$

where

$$F_1(\vec{q}_1, \vec{q}_2; \vec{k}) = \int \frac{d\vec{k}_1}{\pi} R_f(\vec{q}_1, \vec{q}_2; \vec{k}_1) R_u(\vec{q}_1 - \vec{k}_1, \vec{q}_2 + \vec{k}_1; \vec{k} - \vec{k}_1), \quad (21)$$

$$F_2(\vec{q}_1, \vec{q}_2; \vec{k}) = - \int \frac{d\vec{k}_1}{\pi} R_u(\vec{q}_1, \vec{q}_2; \vec{k}_1) R_f(\vec{q}_1 - \vec{k}_1, \vec{q}_2 + \vec{k}_1; \vec{k} - \vec{k}_1), \quad (22)$$

$$F_3(\vec{q}_1, \vec{q}_2; \vec{k}) = \int \frac{d\vec{k}_1}{\pi} \frac{2}{\vec{k}_1^2} \left(R_u(\vec{q}_1 - \vec{k}_1, \vec{q}_2 + \vec{k}_1; \vec{k} - \vec{k}_1) - R_u(\vec{q}_1, \vec{q}_2; \vec{k} - \vec{k}_1) \right). \quad (23)$$

Now all the three pieces have no infrared singularities, the first two of them because of the absence of singularities in the integrands, and the last one because of the evident cancellation between the two terms with R_u in Eq. (23) at $\vec{k}_1 = 0$. The integration of the first piece can be performed with the help of Eqs. (15), (A.8), (A.9) and (A.10) and gives

$$\begin{aligned}
F_1(\vec{q}_1, \vec{q}_2; \vec{k}) &= \left(\frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1^2 \vec{q}_2^2} - \frac{1}{\vec{q}_1^2} \right) \ln \left(\frac{\vec{q}^2 \vec{q}_1^2}{\vec{q}_2^2 \vec{k}^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2^2}{\vec{q}^2 \vec{k}^2} \right) \\
&+ \frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1^2 \vec{q}_2^2} \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right) + 4 \left(\frac{[\vec{q}_2 \times \vec{k}]}{\vec{q}_2^2 \vec{k}^2} - \frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1^2 \vec{k}^2} \right) [\vec{q}_1 \times \vec{k}] I_{\vec{k}, \vec{q}_1'} \\
&+ 2 \frac{[\vec{q}_1 \times \vec{q}_2]^2}{\vec{q}_1^2 \vec{q}_2^2} I_{\vec{q}_1, \vec{q}_2} + 2 \frac{[\vec{q}_1 \times \vec{q}_2][\vec{q}_1' \times \vec{q}_2']}{\vec{q}_1^2 \vec{q}_2^2} I_{\vec{q}_1', \vec{q}_2'} + \vec{q}_1 \leftrightarrow -\vec{q}_2. \quad (24)
\end{aligned}$$

Here

$$I_{\vec{p}, \vec{q}} = \int_0^1 \frac{dx}{(\vec{p} + x\vec{q})^2} \ln \left(\frac{\vec{p}^2}{x^2 \vec{q}^2} \right) \quad (25)$$

is the di-logarithmic function with high symmetry,

$$I_{\vec{p}, \vec{q}} = I_{-\vec{p}, -\vec{q}} = I_{\vec{q}, \vec{p}} = I_{\vec{p}, -\vec{p}-\vec{q}}. \quad (26)$$

The representation exhibiting these properties [16] is

$$I_{\vec{p}, \vec{q}} = \int_0^1 \int_0^1 \int_0^1 \frac{dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)}{(\vec{p}^2 x_1 + \vec{q}^2 x_2 + (\vec{p} + \vec{q})^2 x_3)(x_1 x_2 + x_1 x_3 + x_2 x_3)}. \quad (27)$$

Other useful representations are

$$\begin{aligned}
I_{\vec{p}, \vec{q}} &= \int_0^1 \frac{dx}{a(1-x) + bx - cx(1-x)} \ln \left(\frac{a(1-x) + bx}{cx(1-x)} \right) \\
&= \int_0^1 dx \int_0^1 dz \frac{1}{cx(1-x)z + (b(1-x) + ax)(1-z)}, \quad (28)
\end{aligned}$$

where $a = \vec{p}^2$, $b = \vec{q}^2$, $c = (\vec{p} + \vec{q})^2$.

Note that F_1 must turn into zero at $\vec{q}_1' = 0$ or $\vec{q}_2' = 0$ due to the gauge invariance of R_u . It is easy to see from Eq. (24) that this property is fulfilled.

Unfortunately, neither F_2 nor F_3 possess such property. Moreover, the separation (20) destroys the good behaviour of $R(\vec{q}_1, \vec{q}_2; \vec{k})$ in the ultraviolet region, so that the integrals (22) and (23) diverge at large \vec{k}_1^2 and we have to introduce an ultraviolet cut-off Λ^2 for them. The loss of gauge invariance and ultraviolet convergence of the integrals makes them more complex. Using (A.9)–(A.14) we obtain

$$F_2(\vec{q}_1, \vec{q}_2; \vec{k}) = \frac{1}{\vec{q}_1^2} \left(\ln \left(\frac{\vec{q}_2^2}{\vec{q}^2} \right) \ln \left(\frac{\Lambda^4 \vec{q}^4 \vec{q}_1^2}{\vec{q}_1'^6 \vec{q}_2'^2 \vec{q}_2^2} \right) + \ln \left(\frac{\vec{q}_1'^2}{\vec{k}^2} \right) \ln \left(\frac{\vec{q}_1^2 \vec{q}_2^2}{\vec{k}^2 \vec{q}_2'^2} \right) \right) +$$

$$\begin{aligned}
& + \frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1'^2 \vec{q}_2'^2} \left(\ln^2 \left(\frac{\Lambda^2}{\vec{q}^2} \right) - \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right) - \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2'^2}{\vec{q}^4} \right) \right. \\
& \quad \left. - \ln \left(\frac{\vec{k}^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{k}^2 \vec{q}_1'^2}{\vec{q}_1'^4} \right) \right) + 2 \frac{\vec{q}_1 \vec{k}}{\vec{q}_1'^2 \vec{k}^2} \ln \left(\frac{\vec{q}^2}{\vec{q}_2'^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2'^2}{\vec{q}_1'^2 \vec{q}_2'^2} \right) \\
& \quad + 4 \left(\frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1'^2 \vec{k}^2} - \frac{[\vec{q}_2 \times \vec{k}]}{\vec{q}_2'^2 \vec{k}^2} - \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_2'^2 \vec{q}_2'^2} \right) [\vec{q}_1 \times \vec{k}] I_{\vec{k}, \vec{q}_1'} \\
& \quad \quad - 2 \left(\frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1'^2 \vec{k}^2} + \frac{[\vec{q}_2 \times \vec{k}]}{\vec{q}_2'^2 \vec{k}^2} + \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_2'^2 \vec{q}_2'^2} \right) \\
& \quad \quad \times \left([\vec{q}_1 \times \vec{q}_2] I_{\vec{q}_1, \vec{q}_2} - [\vec{q}_1' \times \vec{q}_2'] I_{\vec{q}_1', \vec{q}_2'} \right) + \vec{q}_1 \leftrightarrow -\vec{q}_2 . \tag{29}
\end{aligned}$$

The result for $F_3(\vec{q}_1, \vec{q}_2; \vec{k})$ can be obtained using Eqs. (A.8), (A.13)–(A.16) and reads

$$\begin{aligned}
F_3(\vec{q}_1, \vec{q}_2; \vec{k}) &= \frac{1}{\vec{q}_1'^2} \left(\ln \left(\frac{\vec{q}^2}{\vec{q}_2'^2} \right) \ln \left(\frac{\Lambda^4 \vec{q}^2}{\vec{q}_1'^4 \vec{q}_2'^2} \right) - 2 \ln \left(\frac{\vec{q}_1'^2}{\vec{k}^2} \right) \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \right) - \frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1'^2 \vec{q}_2'^2} \\
& \times \left(\ln^2 \left(\frac{\Lambda^2}{\vec{q}^2} \right) - 2 \ln^2 \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) - \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right) + 2 \ln \left(\frac{\vec{k}^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \right) \\
& \quad - 4 \frac{\vec{q}_1 \vec{k}}{\vec{q}_1'^2 \vec{k}^2} \ln \left(\frac{\vec{k}^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) - \frac{2}{\vec{k}^2} \ln^2 \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \\
& \quad + 2 \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1'^2 \vec{q}_2'^2} \left(2 [\vec{q}_1 \times \vec{k}] I_{\vec{k}, \vec{q}_1'} - [\vec{q}_1' \times \vec{q}_2'] I_{\vec{q}_1', \vec{q}_2'} \right) + \vec{q}_1 \leftrightarrow -\vec{q}_2 . \tag{30}
\end{aligned}$$

From the Eq. (20) and the definitions (21)–(23) it follows

$$\begin{aligned}
F(\vec{q}_1, \vec{q}_2; \vec{k}) &= \frac{2}{\vec{q}_1'^2} \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2'^2}{\vec{q}^2 \vec{k}^2} \right) + \frac{2 \vec{q}_1 \vec{q}_2}{\vec{q}_1'^2 \vec{q}_2'^2} \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{k}^2}{\vec{q}^2} \right) \\
& + 2 \frac{\vec{q}_1 \vec{k}}{\vec{q}_1'^2 \vec{k}^2} \left(\ln \left(\frac{\vec{q}^2}{\vec{q}_2'^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{q}_2'^2}{\vec{q}_1'^2 \vec{q}_2'^2} \right) + 2 \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{q}_1'^2}{\vec{k}^2} \right) \right) - \frac{2}{\vec{k}^2} \ln^2 \left(\frac{\vec{q}_1'^2}{\vec{q}_1'^2} \right) \\
& - 2 \left(\frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1'^2 \vec{q}_2'^2} + 2 \frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1'^2 \vec{k}^2} \right) ([\vec{q}_1 \times \vec{q}_2] I_{\vec{q}_1, \vec{q}_2} - [\vec{q}_1' \times \vec{q}_2'] I_{\vec{q}_1', \vec{q}_2'}) + \vec{q}_1 \leftrightarrow -\vec{q}_2 . \tag{31}
\end{aligned}$$

The definition (17) and the properties (10) and (14) of R and R_u , respectively, secure the gauge invariance of F :

$$F(\vec{q}_1, \vec{q}_2; \vec{q}_1) = F(\vec{q}_1, \vec{q}_2; -\vec{q}_2) = 0,$$

$$(\vec{q}_1^2 \vec{q}_2^2 F(\vec{q}_1, \vec{q}_2; \vec{q}_1))|_{\vec{q}_1=0} = (\vec{q}_1^2 \vec{q}_2^2 F(\vec{q}_1, \vec{q}_2; \vec{q}_1))|_{\vec{q}_2=0} = 0. \quad (32)$$

The fulfilment of these properties can be easily seen from Eq. (31).

Finally, Eq. (16) together with Eqs. (13), (18) and (31) gives

$$\begin{aligned} & \langle \vec{q}_1, \vec{q}_2 | \alpha_s [\hat{\mathcal{K}}^{(B)}, \hat{U}] | \vec{q}'_1, \vec{q}'_2 \rangle = \\ & = \delta(\vec{q}_{11'} + \vec{q}_{22'}) \frac{\alpha_s^2 N_c^2}{8\pi^3} \left[-\frac{\beta_0}{2N_c} R(\vec{q}_1, \vec{q}_2; \vec{k}) \ln \left(\frac{\vec{q}'_1{}^2 \vec{q}'_2{}^2}{\vec{q}_1^2 \vec{q}_2^2} \right) \right. \\ & + \frac{\vec{q}'_1{}^2}{\vec{q}_1^2 \vec{k}^2} \ln \left(\frac{\vec{q}_1^2 \vec{q}'_2{}^2}{\vec{q}_2^2 \vec{q}_1^2} \right) \ln \left(\frac{\vec{q}_2^2 \vec{q}'_1{}^2}{\vec{q}^2 \vec{k}^2} \right) + \frac{\vec{q}'_2{}^2}{\vec{q}_2^2 \vec{k}^2} \ln \left(\frac{\vec{q}_2^2 \vec{q}'_1{}^2}{\vec{q}_1^2 \vec{q}_2^2} \right) \ln \left(\frac{\vec{q}_1^2 \vec{q}'_2{}^2}{\vec{q}^2 \vec{k}^2} \right) \\ & \left. - 4 \left(\frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1^2 \vec{q}_2^2} + \frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1^2 \vec{k}^2} + \frac{[\vec{q}_2 \times \vec{k}]}{\vec{q}_2^2 \vec{k}^2} \right) ([\vec{q}_1 \times \vec{q}_2] J_{\vec{q}_1, \vec{q}_2} - [\vec{q}'_1 \times \vec{q}'_2] J_{\vec{q}'_1, \vec{q}'_2}) \right]. \quad (33) \end{aligned}$$

3 Use of Möbius space

Since the result (33) was derived by means of lengthy and intricate calculations, we want to obtain it in a quite independent way, starting from the Möbius forms of the kernel $\hat{\mathcal{K}}^{(B)}$ and of the operator \hat{U} , calculating their commutator and restoring the complete commutator (33) in the momentum space from its Möbius form. Simultaneously, the efficiency of the method of restoration developed in Ref. [15] will be demonstrated. Here this alternative derivation is illustrated.

As it is known [5], the Möbius form of the kernel $\hat{\mathcal{K}}^{(B)}$ coincides with the kernel of the colour dipole model [6] and can be written as

$$\begin{aligned} & \langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}_M^{(B)} | \vec{r}'_1 \vec{r}'_2 \rangle = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{r}_0 g(\vec{r}_1, \vec{r}_2, \vec{r}_0) \\ & \times \left[\delta(\vec{r}_{11'}) \delta(\vec{r}_{2'0}) + \delta(\vec{r}_{1'0}) \delta(\vec{r}_{22'}) - \delta(\vec{r}_{11'}) \delta(\vec{r}_{22'}) \right], \quad (34) \end{aligned}$$

where

$$g(\vec{r}_1, \vec{r}_2, \vec{r}_0) = g(\vec{r}_2, \vec{r}_1, \vec{r}_0) = \frac{\vec{r}_{12}^2}{\vec{r}_{10}^2 \vec{r}_{20}^2}. \quad (35)$$

The Möbius form of the operator U was found in Ref. [15]. Omitting the term with $\hat{\mathcal{K}}^{(B)}$, which does not contribute to the commutator in (3), one has

$$\begin{aligned} \langle \vec{r}_1 \vec{r}_2 | \alpha_s \hat{U}_M | \vec{r}'_1 \vec{r}'_2 \rangle &= \frac{\alpha_s N_c}{4\pi^2} \left[\delta(\vec{r}_{11'}) V_1(\vec{r}_1, \vec{r}_2, \vec{r}'_2) + \delta(\vec{r}_{22'}) V_1(\vec{r}_2, \vec{r}_1, \vec{r}'_1) \right. \\ &+ \left. \frac{1}{\pi} V_3(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) \right] + \frac{\alpha_s \beta_0}{8\pi^2} \left[\delta(\vec{r}_{11'}) \left(\frac{1}{\vec{r}_{22'}^2} - \frac{1}{\vec{r}_{12'}^2} \right) + \delta(\vec{r}_{22'}) \left(\frac{1}{\vec{r}_{11'}^2} - \frac{1}{\vec{r}_{21'}^2} \right) \right], \end{aligned} \quad (36)$$

where

$$V_1(\vec{r}_1, \vec{r}_2, \vec{r}'_2) = \frac{\vec{r}_{12}^2}{\vec{r}_{12'}^2 \vec{r}_{22'}^2} \ln \left(\frac{\vec{r}_{12}^2}{\vec{r}_{22'}^2} \right) + \frac{1}{\vec{r}_{22'}^2} \ln \left(\frac{\vec{r}_{22'}^2}{\vec{r}_{12'}^2} \right), \quad (37)$$

$$V_3(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) = V_3(\vec{r}_2, \vec{r}_1, \vec{r}'_2, \vec{r}'_1) = \frac{1}{\vec{r}_{1'2'}^2} \left[\frac{2\vec{r}_{11'} \vec{r}_{22'}}{\vec{r}_{11'}^2 \vec{r}_{22'}^2} - \frac{\vec{r}_{11'} \vec{r}_{12'}}{\vec{r}_{11'}^2 \vec{r}_{12'}^2} - \frac{\vec{r}_{21'} \vec{r}_{22'}}{\vec{r}_{21'}^2 \vec{r}_{22'}^2} \right]. \quad (38)$$

The treatment of the term with β_0 in \hat{U} can be performed quite easily in the momentum space (see Eq. (16)), so that in the following we will omit this term, denoting the remaining part of \hat{U} as \hat{U}^s . With the notation (34)–(38) the Möbius form for the commutator $[\hat{\mathcal{K}}^{(B)}, \hat{U}^s]$ can be presented as

$$\begin{aligned} \langle \vec{r}_1 \vec{r}_2 | \alpha_s [\hat{\mathcal{K}}^{(B)}, \hat{U}^s]_M | \vec{r}'_1 \vec{r}'_2 \rangle &= \frac{\alpha_s^2 N_c^2}{8\pi^3} \left[\delta(\vec{r}_{11'}) J(\vec{r}_1, \vec{r}_2, \vec{r}'_2) + \frac{1}{\pi} F(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) \right. \\ &+ \left. \frac{1}{\pi} I(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) \right] + 1 \leftrightarrow 2, \end{aligned} \quad (39)$$

where $1 \leftrightarrow 2$ means the substitution $\vec{r}_1 \leftrightarrow \vec{r}_2$, $\vec{r}'_1 \leftrightarrow \vec{r}'_2$. The first two terms in the square brackets in Eq. (39) come from the term with $V_1(\vec{r}_1, \vec{r}_2, \vec{r}'_2)$ in Eq. (36) and are written as

$$\begin{aligned} J(\vec{r}_1, \vec{r}_2, \vec{r}'_2) &= \int \frac{d\vec{r}_0}{\pi} [g(\vec{r}_1, \vec{r}_2, \vec{r}_0) V_1(\vec{r}_1, \vec{r}_0, \vec{r}'_2) - V_1(\vec{r}_1, \vec{r}_2, \vec{r}_0) g(\vec{r}_1, \vec{r}_0, \vec{r}'_2) \\ &- (g(\vec{r}_1, \vec{r}_2, \vec{r}_0) - g(\vec{r}_1, \vec{r}'_2, \vec{r}_0)) V_1(\vec{r}_1, \vec{r}_2, \vec{r}'_2)] \end{aligned} \quad (40)$$

and

$$F(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) = g(\vec{r}_2, \vec{r}_1, \vec{r}'_1) V_1(\vec{r}'_1, \vec{r}_2, \vec{r}'_2) - V_1(\vec{r}_1, \vec{r}_2, \vec{r}'_2) g(\vec{r}'_2, \vec{r}_1, \vec{r}'_1). \quad (41)$$

The last term in the square brackets in Eq. (39) related with V_3 is presented in the form

$$\begin{aligned}
I(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) &= \int \frac{d\vec{r}_0}{\pi} \left[\left(\frac{1}{\vec{r}_{10}^2} - \frac{\vec{r}_{10}\vec{r}_{20}}{\vec{r}_{10}^2\vec{r}_{20}^2} \right) V_3(\vec{r}_1, \vec{r}_0, \vec{r}'_1, \vec{r}'_2) + \left(\frac{1}{\vec{r}_{20}^2} - \frac{\vec{r}_{10}\vec{r}_{20}}{\vec{r}_{10}^2\vec{r}_{20}^2} \right) \right. \\
&\times \left(V_3(\vec{r}_1, \vec{r}_0, \vec{r}'_1, \vec{r}'_2) - V_3(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) \right) - \frac{1}{\vec{r}_{02'}^2} \left(V_3(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}_0) \frac{\vec{r}_{01'}^2}{\vec{r}_{1'2'}^2} \right. \\
&\left. \left. - V_3(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) \right) - \frac{\vec{r}_{1'0}\vec{r}_{2'0}}{\vec{r}_{1'0}^2\vec{r}_{2'0}^2} V_3(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) \right]. \quad (42)
\end{aligned}$$

Note that each of the J, F, I functions independently turns into zero at $\vec{r}_{12} = 0$. In contrast to the function F , which is given explicitly by Eq. (41), the functions J and I are expressed in terms of the integrals (40) and (42), respectively. The integrals are not very intricate, although their calculation is complicated by the ultraviolet divergences existing in separate terms. The integrands in (40) and (42) are written in such a way so as to make the cancellation evident. The results of the integration (which can be performed by the method described in the Appendix) are very simple:

$$J(\vec{r}_1, \vec{r}_2, \vec{r}'_2) = \left(\frac{2(\vec{r}_{12'}\vec{r}_{22'})}{\vec{r}_{12'}^2\vec{r}_{22'}^2} - \frac{1}{\vec{r}_{12'}^2} \right) \ln \left(\frac{\vec{r}_{12'}^2}{\vec{r}_{12}^2} \right) \ln \left(\frac{\vec{r}_{22'}^2}{\vec{r}_{12}^2} \right) - \frac{1}{\vec{r}_{22'}^2} \ln^2 \left(\frac{\vec{r}_{12'}^2}{\vec{r}_{12}^2} \right) \quad (43)$$

and

$$\begin{aligned}
I(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2) &= \frac{1}{\vec{r}_{12'}^2} \left(\frac{(\vec{r}_{11'}\vec{r}_{22'})}{\vec{r}_{11'}^2\vec{r}_{22'}^2} \ln \left(\frac{\vec{r}_{21'}^2\vec{r}_{12'}^2}{\vec{r}_{12}^2\vec{r}_{22'}^2} \right) + \frac{(\vec{r}_{11'}\vec{r}_{12'})}{\vec{r}_{11'}^2\vec{r}_{12'}^2} \ln \left(\frac{\vec{r}_{12'}^4\vec{r}_{12}^2}{\vec{r}_{11'}^2\vec{r}_{22'}^4} \right) \right. \\
&\left. + \frac{(\vec{r}_{22'}\vec{r}_{21'})}{\vec{r}_{22'}^2\vec{r}_{21'}^2} \ln \left(\frac{\vec{r}_{12}^2}{\vec{r}_{21}^2} \right) + \frac{(\vec{r}_{12'}\vec{r}_{21'})}{\vec{r}_{12'}^2\vec{r}_{21'}^2} \ln \left(\frac{\vec{r}_{11'}^2\vec{r}_{22'}^2}{\vec{r}_{12}^2\vec{r}_{12'}^2} \right) \right). \quad (44)
\end{aligned}$$

Note that the property of turning into zero at $\vec{r}_{12} = 0$ is conserved after integration. Thus, the Möbius form of the commutator given by Eqs. (39), (41), (43) and (44) is rather simple and does not contain special functions. Having this form one can find the complete commutator in the momentum space $\langle \vec{q}_1, \vec{q}_2 |_{\alpha_s} [\hat{\mathcal{K}}^{(B)}, \hat{U}] | \vec{q}'_1, \vec{q}'_2 \rangle$ according to the prescriptions of Ref. [15]. We write it in the form

$$\begin{aligned}
\langle \vec{q}_1, \vec{q}_2 |_{\alpha_s} [\hat{\mathcal{K}}^{(B)}, \hat{U}] | \vec{q}'_1, \vec{q}'_2 \rangle &= \delta(\vec{q}_{11'} + \vec{q}_{22'}) \frac{\alpha_s^2 N_c^2}{8\pi^3} \left[\frac{\beta_0}{4N_c} \ln \left(\frac{\vec{q}_1^2}{\vec{q}'_1{}^2} \right) R(\vec{q}_1, \vec{q}_2; \vec{k}) \right. \\
&\left. + F(\vec{q}_2, \vec{q}_2; \vec{k}) + J(\vec{q}_2, \vec{q}_2; \vec{k}) + I(\vec{q}_2, \vec{q}_2; \vec{k}) + \vec{q}_1 \leftrightarrow -\vec{q}_2 \right]. \quad (45)
\end{aligned}$$

Here $R(\vec{q}_1, \vec{q}_2; \vec{k})$ is given by Eq. (8) and

$$\begin{aligned}
F(\vec{q}_1, \vec{q}_2; \vec{k}) &= \frac{1}{\pi} \left\langle \int \frac{d\vec{r}'_{11'}}{2\pi} \frac{d\vec{r}'_{22'}}{2\pi} d\vec{r}'_{1'2'} e^{-i(\vec{q}_1 \vec{r}'_{11'} + \vec{q}_2 \vec{r}'_{22'} + \vec{k} \vec{r}'_{1'2'})} F(\vec{r}'_1, \vec{r}'_2, \vec{r}'_1, \vec{r}'_2) \right\rangle \\
&= \left(\frac{1}{\vec{q}_1^2} \ln \left(\frac{\vec{q}_1^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{q}_2^2}{\vec{q}^2} \right) + \frac{1}{2\vec{k}^2} \ln \left(\frac{\vec{q}_1^2}{\vec{q}_1'^2} \right) \ln \left(\frac{\vec{q}_2^2}{\vec{q}_2'^2} \right) - \frac{(\vec{q}_1 \vec{k})}{\vec{q}_1^2 \vec{k}^2} \right. \\
&\quad \times \left(\ln \left(\frac{\vec{q}_2'^2}{\vec{q}_2^2} \right) \ln \left(\frac{\vec{q}_1'^2}{\vec{k}^2} \right) + \ln \left(\frac{\vec{q}^2}{\vec{q}_2^2} \right) \ln \left(\frac{\vec{q}_2^2 \vec{q}_1'^2}{\vec{q}_1^2 \vec{q}_2'^2} \right) \right) + \vec{q}_1 \leftrightarrow -\vec{q}_2 \Big) + \frac{(\vec{q}_1 \vec{q}_2)}{\vec{q}_1^2 \vec{q}_2^2} \\
&\quad \times \left(\ln \left(\frac{\vec{q}^2}{\vec{q}_1^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{k}^2}{\vec{q}_1^2 \vec{q}_2^2} \right) + \ln \left(\frac{\vec{q}_2'^2}{\vec{k}^2} \right) \ln \left(\frac{\vec{q}^2}{\vec{q}_1'^2} \right) \right) - 2 \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1^2 \vec{q}_2^2} [\vec{q}_1 \times \vec{k}] I_{\vec{k}, \vec{q}_1'} \\
&\quad - 2 \left(\frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1^2 \vec{k}^2} + \frac{[\vec{q}_2 \times \vec{k}]}{\vec{q}_2^2 \vec{k}^2} + \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1^2 \vec{q}_2^2} \right) ([\vec{q}_1 \times \vec{q}_2] I_{\vec{q}_1, \vec{q}_2} - [\vec{q}_1' \times \vec{q}_2'] I_{\vec{q}_1', \vec{q}_2'}) , \tag{46}
\end{aligned}$$

$$\begin{aligned}
J(\vec{q}_2, \vec{q}_2; \vec{k}) &= \frac{1}{\pi} \left\langle \int \frac{d\vec{r}'_{11'}}{2\pi} \frac{d\vec{r}'_{22'}}{2\pi} d\vec{r}'_{1'2'} e^{-i(\vec{q}_1 \vec{r}'_{11'} + \vec{q}_2 \vec{r}'_{22'} + \vec{k} \vec{r}'_{1'2'})} \delta(\vec{r}'_{11'}) J(\vec{r}'_1, \vec{r}'_2, \vec{r}'_2) \right\rangle \\
&\quad \left(\frac{1}{\vec{q}_2^2} + 2 \frac{(\vec{q}_2 \vec{k})}{\vec{q}_2^2 \vec{k}^2} \right) \ln \left(\frac{\vec{q}_2^2}{\vec{q}_2'^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{k}^2} \right) - \frac{1}{\vec{k}^2} \ln^2 \left(\frac{\vec{q}_2^2}{\vec{q}_2'^2} \right) , \tag{47}
\end{aligned}$$

$$\begin{aligned}
I(\vec{q}_1, \vec{q}_2; \vec{k}) &= \frac{1}{\pi} \left\langle \int \frac{d\vec{r}'_{11'}}{2\pi} \frac{d\vec{r}'_{22'}}{2\pi} d\vec{r}'_{1'2'} e^{-i(\vec{q}_1 \vec{r}'_{11'} + \vec{q}_2 \vec{r}'_{22'} + \vec{k} \vec{r}'_{1'2'})} F(\vec{r}'_1, \vec{r}'_2, \vec{r}'_1, \vec{r}'_2) \right\rangle \\
&= \frac{1}{2\vec{q}_1^2} \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right) + \frac{1}{2\vec{q}_2^2} \left(\ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right) - 2 \ln \left(\frac{\vec{q}_1'^2}{\vec{q}_1^2} \right) \ln \left(\frac{\vec{k}^2}{\vec{q}_1'^2} \right) \right) \\
&\quad - \frac{(\vec{q}_1 \vec{q}_2)}{\vec{q}_1^2 \vec{q}_2^2} \left(\ln \left(\frac{\vec{q}^2}{\vec{q}_1^2} \right) \ln \left(\frac{\vec{q}_1'^2 \vec{k}^2}{\vec{q}_1^2 \vec{q}_2^2} \right) + \ln \left(\frac{\vec{q}_2'^2}{\vec{k}^2} \right) \ln \left(\frac{\vec{q}^2}{\vec{q}_1'^2} \right) \right) \\
&\quad + 2 \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1^2 \vec{q}_2^2} [\vec{q}_1 \times \vec{k}] I_{\vec{k}, \vec{q}_1'} - 2 \frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1^2 \vec{q}_2^2} [\vec{q}_1' \times \vec{q}_2'] I_{\vec{q}_1', \vec{q}_2'} . \tag{48}
\end{aligned}$$

In these equalities the symbols $\langle \dots \rangle$ mean adding to the direct Fourier transform terms that depend only on \vec{q}_1 and \vec{q}_2 (and do not depend on \vec{k}) and terms that are antisymmetric with respect to the substitution $\vec{q}_1 \leftrightarrow -\vec{q}_2$. These terms are fixed by the requirement of the gauge invariance and the symmetry of the kernel, according to Ref. [15].

Equalities (46)–(48) can be derived using formulas given in the Appendices of Ref. [8] and of the present paper. The substitution of these equalities in Eq. (45) gives the same result as Eq. (33).

4 Conclusion

The simplicity of the Möbius form of the quasi-conformal NLO BFKL kernel suggested to use just this form for finding the kernel in the momentum space. The way to do that was not evident, and even the possibility to do it seemed doubtful, because the Möbius form is defined on a special class of functions in the coordinate space. However, it was shown [15] that such possibility exists due to the gauge invariance of the kernel and the way to obtain the kernel in the momentum space from its Möbius form was elaborated. But technically obtaining it turned out to be not easy.

In this paper we found in the momentum space the difference between the standard BFKL kernel, defined according to the prescriptions given in Ref. [12] and calculated in Ref. [3], and the quasi-conformal BFKL kernel. This difference turned out to be rather simple. The most natural conclusion is that the simplicity of the Möbius form of the quasi-conformal kernel is caused mainly by using the impact parameter space. The other possibility is that the quasi-conformal kernel can be written in simple form also in the transverse momentum space. If this is true, the standard kernel of Ref. [3] could result itself in a much simpler form. We plan to check this possibility using both the representation of Ref. [3] and the representation in terms of integrals in the transverse momentum space of Ref. [17].

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Appendix

The two-dimensional integrals of Section 2 were calculated choosing appropriate integration vectors and performing firstly the integration over azimuthal angles. It is convenient to make this integration using “helical” vector components “ \pm ” instead of the Cartesian ones “ x, y ”, $a^\pm = a_x \pm ia_y$. Denoting the integration vector as \vec{l} , we have $l^\pm = le^{\pm i\phi}$, where ϕ is its azimuthal angle and l is its modulus. The integration over ϕ can be performed using the representation $2(\vec{a} - \vec{l})(\vec{b} - \vec{l}) = (a^+ - l^+)(b^- - l^-) + (a^- - l^-)(b^+ - l^+)$ and the expansion of the integrands in positive or negative powers of l^\pm at various values of l . Thus one can easily obtain

$$\int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \ln(\vec{a} - \vec{l})^2 = \theta(\vec{a}^2 - \vec{l}^2) \ln \vec{a}^2 + \theta(\vec{a}^2 - \vec{l}^2) \ln \vec{l}^2, \quad (\text{A.1})$$

$$\int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \frac{1}{a^\pm - l^\pm} = \frac{\theta(\vec{a}^2 - \vec{l}^2)}{a^\pm}, \quad (\text{A.2})$$

$$\int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \frac{1}{(a^\pm - l^\pm)(b^\mp - l^\mp)} = \frac{\theta(\vec{a}^2 - \vec{l}^2)}{a^\pm b^\mp - \vec{l}^2} + \frac{\theta(\vec{l}^2 - \vec{b}^2)}{\vec{l}^2 - a^\pm b^\mp}. \quad (\text{A.3})$$

In particular, one has from Eq. (A.3)

$$\begin{aligned} \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \frac{2(\vec{l}(\vec{a} - \vec{l}))}{\vec{l}^2(\vec{a} - \vec{l})^2} &= \theta(\vec{l}^2 - \vec{a}^2) \frac{-2}{\vec{l}^2}, \\ \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \frac{2((\vec{a} - \vec{l})(\vec{b} - \vec{l}))}{(\vec{a} - \vec{l})^2(\vec{b} - \vec{l})^2} &= \\ &= \left(\theta(\vec{a}^2 - \vec{l}^2) - \theta(\vec{b}^2 - \vec{l}^2) \right) \left(\frac{1}{a^+ b^- - \vec{l}^2} + \frac{1}{a^- b^+ - \vec{l}^2} \right). \end{aligned} \quad (\text{A.4})$$

The result (15) follows from Eqs. (A.1) and (A.4) with the subsequent elementary integration over l . Since the integral consists of several terms, which are not ultraviolet convergent when taken separately, it is convenient to calculate them introducing an ultraviolet cut-off Λ . Using Eq. (A.4), one can also easily obtain

$$\int \frac{d\vec{l}}{\pi} \left(\frac{(\vec{l}(\vec{a} - \vec{l}))}{\vec{l}^2(\vec{a} - \vec{l})^2} - \frac{(\vec{l}(\vec{b} - \vec{l}))}{\vec{l}^2(\vec{b} - \vec{l})^2} \right) = \ln \left(\frac{\vec{b}^2}{\vec{a}^2} \right), \quad (\text{A.5})$$

that gives the result (18).

Though we use the ultraviolet cut-off Λ (which is supposed tending to infinity) for separate integrals, it is possible to shift the integration vectors in them, since these integrals have only logarithmic divergence. Therefore, with an appropriate choice of \vec{l} , in all integrals of Section 2 the integration over ϕ can be performed using Eqs. (A.2) and (A.3). But sometimes it is more convenient to use Eq. (A.1) as, for example, in the integral

$$\begin{aligned} & \int \frac{d\vec{l}}{\pi} \theta(\Lambda^2 - \vec{l}^2) \frac{1}{(\vec{a} - \vec{l})^2} \ln \left(\frac{\vec{l}^2}{\vec{a}^2} \right) = \\ & = \int \frac{d\vec{l}}{\pi} \theta(\Lambda^2 - \vec{l}^2) \frac{1}{\vec{l}^2} \ln \frac{(\vec{a} - \vec{l})^2}{\vec{a}^2} = \frac{1}{2} \ln^2 \left(\frac{\Lambda^2}{\vec{a}^2} \right). \end{aligned} \quad (\text{A.6})$$

Using Eq. (A.3), we obtain:

$$\begin{aligned} & \int \frac{d\vec{l}}{\pi} \frac{1}{(\vec{a} - \vec{l})^+} \frac{1}{(\vec{b} - \vec{l})^-} \ln \left(\frac{\vec{l}^2}{\mu^2} \right) \theta(\Lambda^2 - \vec{l}^2) = \\ & = \frac{1}{2} \ln \left(\frac{\Lambda^2}{(\vec{a} - \vec{b})^2} \right) \ln \left(\frac{\Lambda^2 (\vec{a} - \vec{b})^2}{\mu^4} \right) \\ & + \frac{1}{2} \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{b}^2} \right) \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{a}^2} \right) + \frac{a^+ b^- - a^- b^+}{2} I_{\vec{a}, -\vec{b}}, \end{aligned} \quad (\text{A.7})$$

where $I_{\vec{a}, \vec{b}}$ is defined in Eq. (25) (see also Eqs. (27) and (28)). In fact, all integrals of Section 2 can be calculated using this one. In particular, the integral (A.6) can be obtained from the integral (A.7) as the limit $\vec{b} \rightarrow \vec{a}$ at $\mu^2 = \vec{a}^2$. The integrals (A.4) and (A.5) also can be found using the part of the integral (A.7) proportional to $\ln \mu^2$. We find also

$$\begin{aligned} & \int \frac{d\vec{l}}{\pi} \frac{2}{(\vec{a} - \vec{l})^2} \frac{(\vec{b} - \vec{l})}{(\vec{b} - \vec{l})^2} \ln \left(\frac{\vec{l}^2}{\vec{a}^2} \right) = \frac{(\vec{a} - \vec{b})}{(\vec{a} - \vec{b})^2} \ln \left(\frac{\vec{a}^2}{\vec{b}^2} \right) \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{a}^2} \right) \\ & + 2 \frac{[(\vec{a} - \vec{b}) \times [\vec{a} \times \vec{b}]]}{(\vec{a} - \vec{b})^2} I_{\vec{a}, -\vec{b}}, \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned}
& \int \frac{d\vec{l}}{\pi} \frac{(\vec{c}-\vec{l})}{(\vec{c}-\vec{l})^2} \frac{2((\vec{a}-\vec{l})(\vec{b}-\vec{l}))}{(\vec{a}-\vec{l})^2(\vec{b}-\vec{l})^2} \ln\left(\frac{\vec{l}^2}{\mu^2}\right) = \frac{(\vec{c}-\vec{b})}{(\vec{c}-\vec{b})^2} \left[\ln\left(\frac{\vec{a}^2}{\mu^2}\right) \ln\left(\frac{(\vec{c}-\vec{a})^2}{(\vec{b}-\vec{a})^2}\right) \right. \\
& + \frac{1}{2} \ln\left(\frac{c^2}{\vec{a}^2}\right) \ln\left(\frac{(\vec{a}-\vec{c})^2}{\vec{b}^2}\right) - \frac{1}{2} \ln\left(\frac{b^2}{\vec{a}^2}\right) \ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{c}^2}\right) \left. + \frac{(\vec{c}-\vec{a})}{(\vec{c}-\vec{a})^2} \left[\ln\left(\frac{\vec{b}^2}{\mu^2}\right) \right. \right. \\
& \times \ln\left(\frac{(\vec{c}-\vec{b})^2}{(\vec{a}-\vec{b})^2}\right) + \frac{1}{2} \ln\left(\frac{c^2}{\vec{b}^2}\right) \ln\left(\frac{(\vec{b}-\vec{c})^2}{\vec{a}^2}\right) - \frac{1}{2} \ln\left(\frac{a^2}{\vec{b}^2}\right) \ln\left(\frac{(\vec{b}-\vec{a})^2}{\vec{c}^2}\right) \left. \right] \\
& + \left(\frac{[(\vec{c}-\vec{b}) \times [\vec{a} \times \vec{b}]]}{(\vec{c}-\vec{b})^2} + \frac{[(\vec{c}-\vec{a}) \times [\vec{b} \times \vec{a}]]}{(\vec{c}-\vec{a})^2} \right) I_{\vec{a},-\vec{b}} + \frac{[(\vec{c}-\vec{b}) \times [\vec{c} \times \vec{a}]]}{(\vec{c}-\vec{b})^2} I_{\vec{c},-\vec{a}} \\
& \quad + \frac{[(\vec{c}-\vec{a}) \times [\vec{c} \times \vec{b}]]}{(\vec{c}-\vec{a})^2} I_{\vec{c},-\vec{b}}. \tag{A.9}
\end{aligned}$$

The result (24) for $F_1(\vec{q}_1, \vec{q}_2; \vec{k})$ was obtained using Eqs. (15), (A.8) and (A.9) with its particular cases, such as

$$\begin{aligned}
& \int \frac{d\vec{l}}{\pi} \frac{(\vec{a}-\vec{l})}{(\vec{a}-\vec{l})^2} \frac{2(\vec{l}(\vec{b}-\vec{l}))}{\vec{l}^2(\vec{b}-\vec{l})^2} \ln\left(\frac{\vec{l}^2}{\mu^2}\right) = -\frac{1}{2} \frac{(\vec{a}-\vec{b})}{(\vec{a}-\vec{b})^2} \ln\left(\frac{\vec{a}^2}{\vec{b}^2}\right) \ln\left(\frac{\vec{a}^2 \vec{b}^2}{\mu^4}\right) \\
& - \frac{1}{2} \frac{\vec{a}}{\vec{a}^2} \ln\left(\frac{\vec{a}^2 \vec{b}^2}{\mu^4}\right) \ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{b}^2}\right) - \frac{[\vec{a} \times [\vec{a} \times \vec{b}]]}{\vec{a}^2} I_{\vec{a},-\vec{b}}. \tag{A.10}
\end{aligned}$$

To obtain $F_2(\vec{q}_1, \vec{q}_2; \vec{k})$, Eq. (29), we used

$$\begin{aligned}
& \int \frac{d\vec{l}}{\pi} \theta(\Lambda^2 - \vec{l}^2) \frac{2((\vec{a}-\vec{l})(\vec{b}-\vec{l}))}{(\vec{a}-\vec{l})^2(\vec{b}-\vec{l})^2} \ln\left(\frac{\vec{l}^2}{\mu^2}\right) = \ln\left(\frac{\Lambda^2(\vec{a}-\vec{b})^2}{\mu^4}\right) \ln\left(\frac{\Lambda^2}{(\vec{a}-\vec{b})^2}\right) \\
& \quad + \ln\left(\frac{\vec{a}^2}{(\vec{a}-\vec{b})^2}\right) \ln\left(\frac{\vec{b}^2}{(\vec{a}-\vec{b})^2}\right), \tag{A.11} \\
& \int \frac{d\vec{l}}{\pi} \frac{1}{\vec{l}^2} \frac{2((\vec{a}-\vec{l})(\vec{b}-\vec{l}))}{(\vec{a}-\vec{l})^2(\vec{b}-\vec{l})^2} \ln\left(\frac{(\vec{c}-\vec{l})^2}{\vec{c}^2}\right) = \\
& = \frac{1}{\vec{a}^2 \vec{b}^2} \left[(\vec{a}\vec{b}) \left(\ln\left(\frac{(\vec{c}-\vec{a})^2}{\vec{c}^2}\right) \ln\left(\frac{(\vec{c}-\vec{b})^2}{\vec{c}^2}\right) \right) \right.
\end{aligned}$$

$$\begin{aligned}
& -\ln\left(\frac{(\vec{c}-\vec{a})^2}{\vec{c}^2}\right)\ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{a}^2}\right)-\ln\left(\frac{(\vec{c}-\vec{b})^2}{\vec{c}^2}\right)\ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{b}^2}\right) \\
& +2([\vec{a}\times\vec{b}][\vec{a}\times\vec{c}])I_{\vec{a},-\vec{c}}+2([\vec{a}\times\vec{b}][\vec{c}\times\vec{b}])I_{\vec{b},-\vec{c}} \\
& +2([\vec{a}\times\vec{b}][(\vec{a}-\vec{c})\times(\vec{b}-\vec{c})])I_{\vec{a}-\vec{c},\vec{c}-\vec{b}} \Big], \tag{A.12}
\end{aligned}$$

in particular,

$$\begin{aligned}
& \int\frac{d\vec{l}}{\pi}\frac{1}{(\vec{a}-\vec{l})^2}\frac{2(\vec{l}(\vec{b}-\vec{l}))}{\vec{l}^2(\vec{b}-\vec{l})^2}\ln\left(\frac{\vec{l}^2}{\vec{a}^2}\right)= \\
& =\frac{1}{\vec{a}^2(\vec{a}-\vec{b})^2}\left[(\vec{a}(\vec{a}-\vec{b}))\ln\left(\frac{\vec{a}^2}{\vec{b}^2}\right)\ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{b}^2}\right)-2[\vec{a}\times\vec{b}]^2I_{\vec{a}-\vec{b},-\vec{a}}\right], \tag{A.13}
\end{aligned}$$

and Eq. (A.9) with its particular cases (A.10) and

$$\begin{aligned}
& \int\frac{d\vec{l}}{\pi}\frac{\vec{l}}{\vec{l}^2}\frac{2((\vec{a}-\vec{l})(\vec{b}-\vec{l}))}{(\vec{a}-\vec{l})^2(\vec{b}-\vec{l})^2}\ln\left(\frac{\vec{l}^2}{\mu^2}\right)=\frac{\vec{b}}{\vec{b}^2}\left[\ln\left(\frac{\vec{a}^2}{\mu^2}\right)\ln\left(\frac{\vec{a}^2}{(\vec{b}-\vec{a})^2}\right)\right. \\
& \left.+\frac{1}{2}\ln\left(\frac{(\vec{b}-\vec{a})^2}{\vec{a}^2}\right)\ln\left(\frac{\vec{a}^2}{\vec{b}^2}\right)\right]+\frac{\vec{a}}{\vec{a}^2}\left[\ln\left(\frac{\vec{b}^2}{\mu^2}\right)\ln\left(\frac{\vec{b}^2}{(\vec{a}-\vec{b})^2}\right)\right. \\
& \left.+\frac{1}{2}\ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{b}^2}\right)\ln\left(\frac{\vec{b}^2}{\vec{a}^2}\right)\right]+\left(\frac{[\vec{b}\times[\vec{a}\times\vec{b}]]}{\vec{b}^2}+\frac{[\vec{a}\times[\vec{b}\times\vec{a}]]}{\vec{a}^2}\right)I_{\vec{a},-\vec{b}}. \tag{A.14}
\end{aligned}$$

The result (30) for $F_3(\vec{q}_1, \vec{q}_2; \vec{k})$ can be obtained using Eqs. (A.8), (A.13), (A.14),

$$\int\frac{d\vec{l}}{\pi}\frac{1}{\vec{l}^2(\vec{a}-\vec{l})^2}\ln\left(\frac{(\vec{b}-\vec{l})^2(\vec{a}-\vec{b}-\vec{l})^2}{\vec{b}^2(\vec{a}-\vec{b})^2}\right)=\frac{1}{\vec{a}^2}\ln^2\left(\frac{(\vec{a}-\vec{b})^2}{\vec{b}^2}\right) \tag{A.15}$$

and

$$\begin{aligned}
& \int \frac{d\vec{l}}{\pi} \frac{\theta(\Lambda^2 - \vec{l}^2)}{(\vec{c} - \vec{l})^2} \left(\frac{2((\vec{a} - \vec{l})(\vec{b} - \vec{l}))}{(\vec{a} - \vec{l})^2(\vec{b} - \vec{l})^2} - 2\frac{((\vec{a} - \vec{c})(\vec{b} - \vec{c}))}{(\vec{a} - \vec{c})^2(\vec{b} - \vec{c})^2} \right) \ln \left(\frac{\vec{l}^2}{\mu^2} \right) = \\
& = \frac{((\vec{a} - \vec{c})(\vec{b} - \vec{c}))}{(\vec{a} - \vec{c})^2(\vec{b} - \vec{c})^2} \left[\ln \left(\frac{\Lambda^2}{(\vec{a} - \vec{b})^2} \right) \ln \left(\frac{\Lambda^2(\vec{a} - \vec{b})^2}{\mu^4} \right) \right. \\
& + \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{a}^2} \right) \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{b}^2} \right) - \ln \left(\frac{\Lambda^2}{(\vec{a} - \vec{c})^2} \right) \ln \left(\frac{\Lambda^2(\vec{a} - \vec{c})^2}{\mu^4} \right) \\
& \quad - \ln \left(\frac{(\vec{a} - \vec{c})^2}{\vec{a}^2} \right) \ln \left(\frac{(\vec{a} - \vec{c})^2}{\vec{c}^2} \right) \\
& \left. - \ln \left(\frac{\Lambda^2}{(\vec{c} - \vec{b})^2} \right) \ln \left(\frac{\Lambda^2(\vec{c} - \vec{b})^2}{\mu^4} \right) - \ln \left(\frac{(\vec{c} - \vec{b})^2}{\vec{c}^2} \right) \ln \left(\frac{(\vec{c} - \vec{b})^2}{\vec{b}^2} \right) \right] \\
& + 2 \left(\frac{[(\vec{a} - \vec{c}) \times (\vec{b} - \vec{c})]}{(\vec{a} - \vec{c})^2(\vec{b} - \vec{c})^2} \left([\vec{a} \times \vec{b}]I_{\vec{a}, -\vec{b}} - [\vec{a} \times \vec{c}]I_{\vec{a}, -\vec{c}} - [\vec{c} \times \vec{b}]I_{\vec{c}, -\vec{b}} \right) \right). \tag{A.16}
\end{aligned}$$

Let us present also the integral

$$\begin{aligned}
& \int \frac{d\vec{l}}{\pi} \frac{2}{(\vec{a} - \vec{l})^2} \frac{l_i(b - \vec{l})_j}{\vec{l}^2(\vec{b} - \vec{l})^2} \ln \left(\frac{\vec{l}^2}{\vec{a}^2} \right) = \\
& = \frac{b_i(a - b)_j + (a - b)_i b_j - \delta_{ij}(\vec{b}(\vec{a} - \vec{b}))}{2\vec{b}^2(\vec{a} - \vec{b})^2} \ln \left(\frac{\vec{a}^2}{\vec{b}^2} \right) \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{a}^2} \right) \\
& + \frac{b_i a_j - a_i b_j + \delta_{ij}(\vec{a}(\vec{a} - \vec{b}))}{2\vec{a}^2(\vec{a} - \vec{b})^2} \ln \left(\frac{\vec{a}^2}{\vec{b}^2} \right) \ln \left(\frac{(\vec{a} - \vec{b})^2}{\vec{b}^2} \right) + \frac{I_{\vec{a}, -\vec{b}}}{(\vec{a} - \vec{b})^2} \\
& \times \left(\frac{1}{\vec{b}^2} \left([\vec{b} \times [\vec{a} \times \vec{b}]]_i (\vec{a} - \vec{b})_j + (\vec{a} - \vec{b})_i [\vec{b} \times [\vec{a} \times \vec{b}]]_j - \delta_{ij}([\vec{b} \times [\vec{a} \times \vec{b}]](\vec{a} - \vec{b})) \right) \right. \\
& \left. + \frac{1}{\vec{a}^2} \left((\vec{a} - \vec{b})_i [\vec{a} \times [\vec{a} \times \vec{b}]]_j - [\vec{a} \times [\vec{a} \times \vec{b}]]_i (\vec{a} - \vec{b})_j - \delta_{ij}([\vec{a} \times [\vec{a} \times \vec{b}]](\vec{a} - \vec{b})) \right) \right) \tag{A.17}
\end{aligned}$$

which is more general than the integral (A.10) and can appear in decompositions of the integrands for F_i different from ours, and the integrals

$$\begin{aligned} & \int \frac{d\vec{l}}{\pi} \frac{2}{\vec{l}^2} \left(\frac{(\vec{a} - \vec{l})}{(\vec{a} - \vec{l})^2} \left(\frac{(\vec{b} - \vec{l})}{(\vec{b} - \vec{l})^2} - \frac{\vec{b}}{\vec{b}^2} \right) \right) \ln \left(\frac{\vec{l}^2}{\vec{q}^2} \right) = \\ & = \frac{(\vec{a}\vec{b})}{\vec{a}^2 \vec{b}^2} \ln \left(\frac{\vec{a}^2 \vec{b}^2}{\vec{q}^4} \right) \ln \left(\frac{\vec{b}^2}{(\vec{a} - \vec{b})^2} \right) + \frac{2[\vec{a} \times \vec{b}]^2}{\vec{a}^2 \vec{b}^2} I_{\vec{a}, -\vec{b}}, \end{aligned} \quad (\text{A.18})$$

$$\begin{aligned} & \int \frac{d\vec{l}}{\pi} \frac{2}{\vec{l}^2} \left(\frac{(\vec{b} - \vec{l})}{(\vec{b} - \vec{l})^2} - \frac{\vec{b}}{\vec{b}^2} \right) \ln \left(\frac{(\vec{a} - \vec{l})^2}{\vec{l}^2} \right) = \frac{\vec{b}}{\vec{b}^2} \ln \left(\frac{\vec{a}^2}{\vec{b}^2} \right) \ln \left(\frac{\vec{b}^2}{(\vec{a} - \vec{b})^2} \right) \\ & + 2 \frac{[\vec{b} \times [\vec{a} \times \vec{b}]]}{\vec{b}^2} I_{\vec{a}, \vec{b} - \vec{a}}, \end{aligned} \quad (\text{A.19})$$

which also can be useful.

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**Difference between standard
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