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BFKL APPROACH AND DIPOLE PICTURE

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# BFKL approach and dipole picture

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## Abstract

Inter-relation of the BFKL approach and the colour dipole model is discussed. In the case of scattering of colourless objects the colour singlet BFKL kernel can be taken in the special representation called Möbius form. In the leading order (LO) it coincides with the kernel of the colour dipole model. In the next-to-leading order (NLO) the quark parts of the Möbius form and the colour dipole kernel are in accord with each other, but the gluon parts do not agree. Possible sources of this discrepancy are analyzed.

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In papers [1] – [4] with my collaborators R. Fiore, A.V. Grabovsky and A. Papa we investigated coordinate representation of the colour singlet BFKL kernel. The motivations were: to analyze relation between the BFKL approach and the dipole picture of high-energy scattering, to understand conformal properties of the BFKL kernel and to search more simple representations for it in the NLO.

The BFKL approach [5] gives the most common basis for the theoretical description of small- $x$  processes. The approach is based on the remarkable property of QCD – gluon Reggeization. The high-energy QCD can be reformulated in terms of the gauge-invariant effective field theory for Reggeized gluon interactions [6], so that the primary Reggeon in QCD is not the Pomeron, but the Reggeized gluon. Now the BFKL approach is well developed in the NLO. The BFKL kernel for the forward scattering (i.e. for  $t = 0$  and color singlet in the  $t$ -channel) is known for a long time [7]. Several years ago the kernel was found also for any fixed (not growing with energy) momentum transfer  $t$  and any possible color state in the  $t$ -channel [8]. All these results were obtained in the momentum representation.

The most interesting for phenomenological applications is the colourless exchange (colour singlet channel). Only this channel is considered below. Just this channel is described by the colour dipole model [9] and its non-linear generalization (BK equation) [10], formulated in the impact parameter space, and just in this channel the LO BFKL kernel has the remarkable property: it can be written in the Möbius form, invariant in regard to the conformal transformations of the transverse coordinates [11] and coincides with the kernel of the colour dipole model [1].

Generally speaking, even the colour singlet BFKL kernel  $\hat{K}$  is more general than the the dipole one. This is clear, because it can describe scattering not only of colourless objects, such as colour dipoles. However, when it is applied to the latter case, one can use the dipole and gauge invariance prop-

erties of targets and projectiles [11] and omit in  $\langle \vec{r}_1 \vec{r}_2 | \hat{K} | \vec{r}'_1 \vec{r}'_2 \rangle$  the terms proportional to  $\delta(\vec{r}_{1'2'})$ , as well as change the terms independent either of  $\vec{r}_1$  or of  $\vec{r}_2$  in such a way that the resulting kernel becomes conserving the dipole property, i.e. provides vanishing of cross-sections for scattering of zero-size dipoles. The coordinate representation of the kernel obtained in such a way is what we call Möbius form of the BFKL kernel.

In the NLO the Möbius form can be written as

$$\begin{aligned} \langle \vec{r}_1 \vec{r}_2 | \hat{K}_M | \vec{r}'_1 \vec{r}'_2 \rangle &= \delta(\vec{r}_{11'}) \delta(\vec{r}_{22'}) \int d\vec{\rho} g^0(\vec{r}_1, \vec{r}_2; \vec{\rho}) \\ &+ \delta(\vec{r}_{11'}) g(\vec{r}_1, \vec{r}_2; \vec{r}'_2) + \delta(\vec{r}_{22'}) g(\vec{r}_2, \vec{r}_1; \vec{r}'_1) + \frac{1}{\pi} g(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2) \end{aligned} \quad (1)$$

with the functions  $g$  turning into zero when their first two arguments coincide.

The first three terms contain ultraviolet singularities which cancel in their sum, as well as in the LO, with account of the dipole property of the target impact factors. The coefficient of  $\delta(\vec{r}_{11'}) \delta(\vec{r}_{22'})$  is written in the integral form in order to make the cancellation evident. The function  $g(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2)$  is absent in the LO because the LO kernel in the momentum space does not contain terms depending on all three independent momenta simultaneously.

In QCD the NLO kernel contains quark and gluon contributions. In ones turn, the quark contribution is divided into two pieces: non-Abelian (leading in  $N_c$ ) and Abelian (suppressed by  $N_c^{-2}$ ). The non-Abelian piece is the simplest one. In the BFKL framework it is known for a long time [12]. Two years ago it was found in the colour dipole model [13, 14]. A short time later we have found [1] its Möbius form which agrees with the results of Refs. [13, 14]. The Abelian part was calculated in the momentum representation many years ago in the framework of QED [15] and is very complicated. It turns out, however, that its Möbius form [2] is quite simple. This part contributes only to  $g(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2)$  and coincides with corresponding part of the quark contribution to the linearized BK kernel [14]. Moreover, it is conformal invariant. It could be especially interesting for the QED Pomeron.

The most important contribution to the BFKL kernel is the gluon one. In the momentum representation in the NLO for arbitrary momentum transfer it is extremely complicated [8]. The Möbius form of this contribution [3] turned out strikingly simple. It was found when the NLO gluon contribution to the dipole kernel was not yet known, so that comparison was impossible at that time. Therefore investigation of conformal properties became the first-priority problem. Evidently, in the NLO one can expect conformal invariance only in theories with non-running coupling constant. In [4] we

found momentum representation of the BFKL kernel and its Möbius form in Supersymmetric Yang-Mills theories. These theories contain gluons and  $n_M$  Majorana fermions in the adjoint representation of the colour group. For  $N$ -extended SUSY  $n_M = N$ . At  $N > 1$  besides fermions there are  $n_S$  scalar particles;  $n_S = 2$  at  $N = 2$  and  $n_S = 6$  at  $N = 4$ . In the momentum representation generalization of the BFKL kernel to SUSY Yang-Mills [4] in many respects is similar to the case of forward scattering [16]. The gluon contribution to the BFKL kernel is the same as in QCD. The fermion one can be obtained by change of the flavour coefficients:  $n_f \rightarrow n_M N_c$  for the non-Abelian part, and  $n_f \rightarrow -n_M N_c^3$  for the Abelian part. It appeared that the scalar analog of the non-Abelian quark contribution also can be obtained from this contribution by the same substitutions as in [16]. The only piece which can not be obtained by substitutions is the scalar analog of the Abelian quark contribution. For the Möbius form we obtained

$$\begin{aligned}
g(\vec{r}_1, \vec{r}_2; \vec{r}'_2) &= \frac{\alpha_s \left( \frac{4e^{-2C}}{\vec{r}^2} \right) N_c}{2\pi^2} \frac{\vec{r}_{12}^2}{\vec{r}_{22'}^2 \vec{r}_{12'}^2} \left[ 1 + \frac{\alpha_s N_c}{2\pi} \left( \frac{67}{18} - \zeta(2) - \frac{5n_M}{9} - \frac{2n_S}{9} \right. \right. \\
&+ \frac{\beta_0}{2N_c} \frac{\vec{r}_{12'}^2 - \vec{r}_{22'}^2}{\vec{r}_{12}^2} \ln \left( \frac{\vec{r}_{22'}^2}{\vec{r}_{12'}^2} \right) - \frac{1}{2} \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{22'}^2} \right) \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12'}^2} \right) \\
&+ \left. \left. \frac{\vec{r}_{12'}^2}{2\vec{r}_{12}^2} \ln \left( \frac{\vec{r}_{12'}^2}{\vec{r}_{22'}^2} \right) \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12'}^2} \right) \right] , \\
g^0(\vec{r}_1, \vec{r}_2; \vec{\rho}) &= -g(\vec{r}_1, \vec{r}_2; \rho) + \frac{3}{2} \frac{\vec{r}_{12}^2}{\vec{r}_{1\rho}^2 \vec{r}_{1\rho}^2} \ln \left( \frac{\vec{r}_{1\rho}^2}{\vec{r}_{12}^2} \right) \ln \left( \frac{\vec{r}_{2\rho}^2}{\vec{r}_{12}^2} \right) , \tag{2}
\end{aligned}$$

where

$$\beta_0 = \left( \frac{11}{3} - \frac{2n_M}{3} - \frac{n_S}{6} \right) N_c , \quad \alpha_s(\vec{q}^2) = \alpha_s(\mu^2) \left( 1 - \beta_0 \frac{\alpha_s(\mu^2) N_c}{4\pi} \ln \left( \frac{\vec{q}^2}{\mu^2} \right) \right) . \tag{3}$$

At  $N = 4$  the first coefficient of the  $\beta$ -function  $\beta_0 = 0$ , and  $\alpha_s$  does not run. Nevertheless, cancelation of terms violating conformal invariance is not complete.

The function  $g(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2)$  is not so simple (although it is incomparably simpler than the kernel in the momentum representation):

$$\begin{aligned}
g(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2) = & \frac{\alpha_s^2 N_c^2}{4\pi^3} \left[ \frac{1}{2\vec{r}_{1'2'}^4} \left( \frac{\vec{r}_{12'}^2, \vec{r}_{21'}^2}{d} \ln \left( \frac{\vec{r}_{12'}^2, \vec{r}_{21'}^2}{\vec{r}_{11'}^2, \vec{r}_{22'}^2} \right) - 1 \right) \left( 1 - n_M + \frac{n_S}{2} \right) \right. \\
& - \left( \frac{(4 - n_M) \vec{r}_{12}^2 \vec{r}'_{1'2'}}{4\vec{r}_{1'2'}^4} - \frac{1}{4\vec{r}_{11'}^2 \vec{r}'_{22'}^2} \left( \frac{\vec{r}_{12}^4}{d} - \frac{\vec{r}_{12}^2}{\vec{r}'_{1'2'}} \right) \right) \ln \left( \frac{\vec{r}_{12'}^2, \vec{r}_{21'}^2}{\vec{r}_{11'}^2, \vec{r}_{22'}^2} \right) \\
& + \frac{\ln \left( \frac{\vec{r}_{12}^2}{\vec{r}'_{1'2'}} \right)}{4\vec{r}_{11'}^2 \vec{r}'_{22'}^2} + \frac{\ln \left( \frac{\vec{r}_{12}^2 \vec{r}'_{1'2'}}{\vec{r}_{11'}^2 \vec{r}'_{22'}^2} \right)}{2\vec{r}_{12'}^2 \vec{r}'_{21'}^2} \left( \frac{\vec{r}_{12}^2}{2\vec{r}'_{1'2'}} + \frac{1}{2} - \frac{\vec{r}_{12}^2}{\vec{r}'_{1'2'}} \right) \\
& + \frac{\vec{r}_{12}^2 \ln \left( \frac{\vec{r}_{12}^2 \vec{r}'_{1'2'}}{\vec{r}_{12'}^2 \vec{r}'_{21'}^2} \right)}{4\vec{r}_{11'}^2 \vec{r}'_{22'}^2 \vec{r}_{12'}^2} + \frac{\ln \left( \frac{\vec{r}_{22'}}{\vec{r}_{12}} \right)}{2\vec{r}_{11'}^2 \vec{r}_{12'}^2} + \frac{\ln \left( \frac{\vec{r}_{12}^2 \vec{r}'_{1'2'}}{\vec{r}_{12'}^2 \vec{r}'_{22'}^2} \right)}{2\vec{r}_{11'}^2 \vec{r}'_{1'2'}^2} \\
& + \frac{\ln \left( \frac{\vec{r}_{12}^2 \vec{r}_{11'}}{\vec{r}_{22'}^2 \vec{r}'_{1'2'}} \right)}{2\vec{r}_{12'}^2 \vec{r}'_{1'2'}^2} + \frac{\vec{r}_{12}^2 \ln \left( \frac{\vec{r}_{11'}}{\vec{r}'_{1'2'}} \right)}{2\vec{r}_{11'}^2 \vec{r}'_{12'}^2 \vec{r}'_{22'}^2} + \frac{\vec{r}_{21'}^2 \ln \left( \frac{\vec{r}_{21'}^2 \vec{r}'_{1'2'}}{\vec{r}_{12}^2 \vec{r}'_{11'}} \right)}{2\vec{r}_{11'}^2 \vec{r}'_{22'}^2 \vec{r}'_{1'2'}^2} + (1 \leftrightarrow 2) \left. \right], \tag{4}
\end{aligned}$$

where  $d = \vec{r}_{12'}^2 \vec{r}_{21'}^2 - \vec{r}_{11'}^2 \vec{r}_{22'}^2$ . It also contains terms violating conformal invariance.

Now the gluon contribution to the BK kernel is known [17]. In the linear approximation it gives:

$$\begin{aligned}
g_{BC}(\vec{r}_1, \vec{r}_2; \vec{r}'_2) = & \frac{\alpha_s(\frac{1}{\bar{r}^2}) N_c}{2\pi^2} \frac{\vec{r}_{12}^2}{\vec{r}_{22'}^2 \vec{r}'_{12'}^2} \left[ 1 + \frac{\alpha_s N_c}{2\pi} \left( \frac{67}{18} - \zeta(2) - \frac{5n_f}{9N_c} \right. \right. \\
& \left. \left. + \frac{\beta_0}{2N_c} \frac{\vec{r}_{12}^2 - \vec{r}_{22'}^2}{\vec{r}_{12}^2} \ln \left( \frac{\vec{r}_{22'}}{\vec{r}_{12}} \right) - \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{22'}^2} \right) \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12'}^2} \right) \right] , \\
g_{BC}^0(\vec{r}_1, \vec{r}_2, \vec{\rho}) = & -g_B(\vec{r}_1, \vec{r}_2, \vec{\rho}) , \tag{5}
\end{aligned}$$

Here, besides the difference in the argument of the running coupling, which can be attributed to the difference in the renormalization conditions, there is the difference in  $g(\vec{r}_1, \vec{r}_2; \vec{r}'_2)$  and in the ratio of  $g$  and  $g^0$ . Even larger difference is in  $g(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2)$ :

$$\begin{aligned}
g_{BC}(\vec{r}_1, \vec{r}_2; \vec{r}'_1, \vec{r}'_2) = & \frac{\alpha_s^2 N_c^2}{4\pi^3} \left[ \frac{1}{\vec{r}_{1'2'}^4} \left( \frac{\vec{r}_{12'}^2, \vec{r}_{21'}^2}{d} \ln \left( \frac{\vec{r}_{12'}^2, \vec{r}_{21'}^2}{\vec{r}_{11'}^2, \vec{r}_{22'}^2} \right) - 1 \right) \left( 1 + \frac{n_f}{N_c^3} \right) \right. \\
& - \left( \frac{\vec{r}_{12}^2 \vec{r}'_{1'2'}}{\vec{r}_{1'2'}^4 d} \left( 2 + \frac{n_f}{2N_c^3} \right) - \frac{1}{4\vec{r}_{11'}^2 \vec{r}'_{22'}^2} \left( \frac{\vec{r}_{12}^4}{d} - \frac{\vec{r}_{12}^2}{\vec{r}'_{1'2'}} \right) \right. \\
& \left. \left. - \frac{1}{4\vec{r}_{12'}^2 \vec{r}'_{21'}^2} \left( \frac{\vec{r}_{12}^4}{d} - \frac{\vec{r}_{12}^2}{\vec{r}'_{1'2'}} \right) \right) \ln \left( \frac{\vec{r}_{12'}^2, \vec{r}_{21'}^2}{\vec{r}_{11'}^2, \vec{r}_{22'}^2} \right) \right] . \tag{6}
\end{aligned}$$

Summing up, the results are the following. The Möbius form of the BFKL kernel is greatly simplified in comparison with the kernel in the momentum representation. However, one has to remember that each iteration requires integration over two coordinates instead of one momentum. The quark part of the Möbius form agrees with corresponding part of the dipole kernel, and its Abelian piece is conformal invariant. The Abelian piece of the scalar particle contribution is also conformal invariant. But conformal invariance is broken even in the  $N = 4$  SUSY Yang-Mills theory and there is the discrepancy between the Möbius form of the BFKL kernel and the dipole kernel.

However, there is a chance to remove the discrepancy, as well as the violation of conformal invariance. This chance is concerned with ambiguities of the NLO kernel. In the BFKL approach scattering amplitudes are invariant under the operator transformation of the kernel

$$\hat{K} \rightarrow \hat{O}^{-1} \hat{K} \hat{O} \quad (7)$$

accompanied by corresponding transformations of impact factors. In the LO the kernel is fixed by the requirement of the conformal invariance of its Möbius form. But even after this transformations with  $\hat{O} = 1 + \hat{O}$ , where  $\hat{O} \sim g^2$ , are still possible. At that  $\hat{K} \rightarrow -[\hat{K}^B \hat{O}]$ . These transformations rearrange NLO corrections to the kernel and impact factors.

At first sight, there is one more ambiguity of the NLO kernel, related with a choice of the energy scale. But it was shown [18] that change of the energy scale can be compensated by corresponding redefinition of the impact factors. It means that the ambiguity in the energy scale is reduced to the transformation (7), with a specific form of  $\hat{O}$ .

To understand if it is possible to remove the discrepancy between the Möbius form of the BFKL kernel and the linearized BK kernel we considered forward scattering. Defining

$$\langle \vec{r} | \hat{K}_M | \vec{\rho} \rangle = \int \langle \vec{r}_1 \vec{r}_2 | \hat{K}_M | \vec{r}'_1 \vec{r}'_2 \rangle \delta(\vec{r}'_1 - \vec{r}'_2 - \vec{\rho}) d^2 r'_1 d^2 r'_2, \quad (8)$$

where  $\vec{r} = \vec{r}_1 - \vec{r}_2$ , we obtained

$$\begin{aligned} \langle \vec{r} | \hat{K}_M | \vec{r}' \rangle = & \frac{\alpha_s \left( \frac{4e^{-2C}}{\vec{r}^2} \right) N_c}{2\pi^2} \int \frac{d\vec{\rho} \vec{r}^2}{(\vec{r} - \vec{\rho})^2 \vec{\rho}^2} \left\{ \left( 2\delta(\vec{\rho} - \vec{r}') - \delta(\vec{r} - \vec{r}') \right) \left[ 1 + \frac{\alpha_s N_c}{4\pi} \left( \frac{67}{9} \right. \right. \right. \\ & \left. \left. \left. - 2\zeta(2) - \frac{10n_M}{9} - \frac{4n_S}{9} + \frac{\beta_0}{N_c} \frac{\vec{\rho}^2 - (\vec{r} - \vec{\rho})^2}{\vec{r}^2} \ln \left( \frac{(\vec{r} - \vec{\rho})^2}{\vec{\rho}^2} \right) \right] + 3\delta(\vec{r} - \vec{r}') \ln \left( \frac{\vec{\rho}^2}{\vec{r}^2} \right) \right\} \end{aligned}$$

$$\times \ln \left( \frac{(\vec{r} - \vec{\rho})^2}{\vec{r}^2} \right) \Bigg\} + \frac{\alpha_s^2 N_c^2}{4\pi^3} \frac{\vec{r}^2}{\vec{r}'^2} \left( f_1(\vec{r}, \vec{r}') + f_2(\vec{r}, \vec{r}') - \frac{1}{(\vec{r} - \vec{r}')^2} \ln^2 \left( \frac{\vec{r}^2}{\vec{r}'^2} \right) \right), \quad (9)$$

where

$$\begin{aligned} f_1(x, y) &= \frac{(x^2 - y^2)}{(x - y)^2 (x + y)^2} \left[ \ln \left( \frac{x^2}{y^2} \right) \ln \left( \frac{x^2 y^2 (x - y)^4}{(x^2 + y^2)^4} \right) + 2Li_2 \left( -\frac{y^2}{x^2} \right) \right. \\ &\quad \left. - 2Li_2 \left( -\frac{x^2}{y^2} \right) \right] - \left( 1 - \frac{(x^2 - y^2)^2}{(x - y)^2 (x + y)^2} \right) \left[ \int_0^1 - \int_1^\infty \frac{du}{(x - yu)^2} \ln \left( \frac{u^2 y^2}{x^2} \right) \right], \\ f_2(x, y) &= \frac{1}{8x^2 y^2} \left\{ \left[ (xy)^2 \left( 1 - \frac{3}{2} \left( \frac{y^2}{x^2} + \frac{x^2}{y^2} \right) \right) + (x^2 + y^2)^2 \right] \left( 1 - n_M + \frac{n_S}{2} \right) \right. \\ &\quad \left. - 16x^2 y^2 \left( 2 - \frac{n_M}{2} \right) \right\} \int_0^\infty dt \frac{\ln \left| \frac{1+t}{1-t} \right|}{y^2 + t^2 x^2} + \frac{3(xy)^2 - 2x^2 y^2}{16x^2 y^2} \left( \ln \frac{x^2}{y^2} \left( \frac{1}{y^2} - \frac{1}{x^2} \right) \right. \\ &\quad \left. + \frac{2}{x^2} + \frac{2}{y^2} \right). \quad (10) \end{aligned}$$

In the case of  $N = 4$  we have

$$\frac{\vec{r}'^2}{\vec{r}^2} \langle \vec{r} | \hat{K}_M | \vec{r}' \rangle = \frac{\vec{q}^2}{\vec{q}'^2} \langle \vec{q} | \hat{K} | \vec{q}' \rangle \Bigg|_{\vec{q} \rightarrow \vec{r}}, \quad (11)$$

where  $\langle \vec{q} | \hat{K} | \vec{q}' \rangle$  is the forward BFKL kernel in the momentum space [16]. In the QCD case, besides the difference in the argument of the coupling constant, the difference between the Möbius form of the forward BFKL kernel and the result of [17] is

$$\begin{aligned} \langle \vec{r} | \hat{K}_M - \hat{K}_{BC} | \vec{r}' \rangle &= \frac{\alpha_s^2 N_c^2}{4\pi^3} \left[ \frac{\vec{r}^2}{(\vec{r} - \vec{r}')^2 \vec{r}'^2} \left( -\ln^2 \left( \frac{\vec{r}'^2}{\vec{r}^2} \right) \right. \right. \\ &\quad \left. \left. + 2 \ln \left( \frac{\vec{r}'^2}{\vec{r}^2} \right) \ln \left( \frac{(\vec{r} - \vec{r}')^2}{\vec{r}^2} \right) \right) + \delta(\vec{r} - \vec{r}') 2\pi\zeta(3) \right], \quad (12) \end{aligned}$$

that corresponds to the difference in the eigenvalues

$$\omega(n, \gamma)_M - \omega(n, \gamma)_{BC} = \frac{\alpha_s^2 N_c^2}{4\pi^3} [2\chi'(n, \gamma)\chi(n, \gamma) + 2\pi\zeta(3)]. \quad (13)$$

The first term evidently can be eliminated by the transformation

$$\widehat{K} \rightarrow \widehat{K} + 2[\widehat{K}^B, \ln \widehat{q}^2 \widehat{K}^B], \quad (14)$$

corresponding to change of the energy scale. But the term with  $\zeta(3)$  can not be eliminated in such a way.

Thus, to remove the discrepancy between the Möbius form of the BFKL kernel and the linearized BK kernel obtained in [17] using the transformation (7) was found impossible. We have to add that in the BFKL approach the term with  $\zeta(3)$  passed through a great number of verifications.

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## References

- [1] V. S. Fadin, R. Fiore and A. Papa, Nucl. Phys. **B769** (2007) 108 [arXiv:hep-ph/0612284].
- [2] V.S. Fadin, R. Fiore and A. Papa, Phys. Lett. **B647** (2007) 179 [arXiv:hep-ph/0701075].
- [3] V. S. Fadin, R. Fiore, A. V. Grabovsky and A. Papa, Nucl. Phys. **B784** (2007) 49 [arXiv:0705.1885 [hep-ph]].
- [4] V. S. Fadin and R. Fiore, Phys. Lett. **B661** (2008) 139 [arXiv:0712.3901 [hep-ph]].
- [5] Fadin, V. S. Kuraev, E. A. and Lipatov, L. N., Phys. Lett. **B60** (1975) 50; Kuraev, E. A., Lipatov, L. N. and Fadin, V. S., Sov. Phys. JETP **44** (1976) 443 [Zh. Eksp. Teor. Fiz. **71** (1976) 840]; Sov. Phys. JETP **45** (1977) 199 [Zh. Eksp. Teor. Fiz. **72** (1977) 377]; Balitsky, I. I. and Lipatov, L. N., Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].
- [6] Lipatov, L. N., Nucl. Phys. **B452** (1995) 369.
- [7] Fadin, V. S. and Lipatov, L. N., Phys. Lett. **B429** (1998) 127 [arXiv:hep-ph/9802290]; Ciafaloni, M. and Camici, G., Phys. Lett. **B430** (1998) 349 [arXiv:hep-ph/9803389].
- [8] Fadin, V. S. and Fiore, R., Phys. Lett. **B610** (2005) 61 [Erratum-ibid. **B621** (2005) 61] [arXiv:hep-ph/0412386]; Fadin, V. S. and Fiore, R., Phys. Rev. **D72** (2005) 014018 [arXiv:hep-ph/0502045].
- [9] Nikolaev, N.N. and Zakharov, B.G., Z. Phys. **C64** (1994) 631; A. H. Mueller, Nucl. Phys. **B415** (1994) 373.
- [10] I. Balitsky, Nucl. Phys. **B463** (1996) 99 [arXiv:hep-ph/9509348]; Y. V. Kovchegov, Phys. Rev. **D60** (1999) 034008 [arXiv:hep-ph/9901281].
- [11] Lipatov, L. N., Adv. Ser. Direct. High Energy Phys. **5** (1989) 411.
- [12] V. S. Fadin, R. Fiore and A. Papa, Phys. Rev. **D60** (1999) 074025 [arXiv:hep-ph/9812456].
- [13] Y. V. Kovchegov and H. Weigert, Nucl. Phys. **A784** (2007) 188 [arXiv:hep-ph/0609090].

- [14] I. Balitsky, Phys. Rev. **D75** (2007) 014001 [arXiv:hep-ph/0609105].
- [15] V. N. Gribov, L. N. Lipatov and G. V. Frolov, Sov. J. Nucl. Phys. **12** (1971) 543 [Yad. Fiz. **12** (1970) 994]; H. Cheng and T. T. Wu, Phys. Rev. **D1** (1970) 2775.
- [16] A. V. Kotikov and L. N. Lipatov, Nucl. Phys. **B582** (2000) 19.
- [17] I. Balitsky and G. A. Chirilli, Phys. Rev. **D77** (2008) 014019 [arXiv:0710.4330 [hep-ph]].
- [18] V. S. Fadin, arXiv:hep-ph/9807527.

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**BFKL approach and dipole picture**

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