Siberian Branch of Russian Academy of Science
BUDKER INSTITUTE OF NUCLEAR PHYSICS

V.S. Fadin and R. Fiore

# NON-FORWARD NLO BFKL KERNEL

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**Non-forward NLO BFKL kernel**[1]

*V.S. Fadin*† *and R. Fiore*‡

† Budker Institute of Nuclear Physics
and Novosibirsk State University
630090 Novosibirsk, Russia

‡ Dipartimento di Fisica, Università della Calabria
and Istituto Nazionale di Fisica Nucleare,
Gruppo collegato di Cosenza
I-87036 Arcavacata di Rende, Cosenza, Italy

**Abstract**

Details of the calculation of the non-forward BFKL kernel at next-to-leading order (NLO) are offered. Specifically we show the calculation of the two-gluon production contribution. This contribution was the last missing part of the kernel. Together with the NLO gluon Regge trajectory, the NLO contribution of one-gluon production and the contribution of quark-antiquark production which were found before it defines the kernel completely for any colour state in the $t$-channel, in particular the Pomeron kernel presented recently.



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† *email address:* fadin@inp.nsk.su
‡ *email address:* fiore@cs.infn.it

# 1 Introduction

Talking about the BFKL kernel one usually has in mind the kernel of the BFKL equation [1] for the case of forward scattering, i.e. for the momentum transfer $t = 0$ and vacuum quantum numbers in the $t$-channel. However, the BFKL approach is not limited to this particular case and, what is more, from the beginning it was developed for arbitrary $t$ and for all possible $t$-channel colour states. Initially it was done in the leading logarithmic approximation (LLA), which means summation of terms of the type $[\alpha_s \ln s]^n$ ($\alpha_s$ is the QCD coupling constant and $s$ is the squared c.m.s. energy). This approximation can provide only qualitative results, since it does not fix scales, neither for energy nor for transverse momenta determining the running coupling constant $\alpha_s$. Therefore calculation of radiative corrections to LLA seems to be a daily need. Unfortunately, till now it is not completed, although the forward BFKL kernel in the NLO was found already five years ago [2].

The problem of the development of the BFKL approach in the next-to leading approximation (NLA) is naturally divided into two parts, in compliance with the representation of scattering amplitudes in this approach by the convolution of the impact factors of interacting particles with the Green's function of two Reggeized gluons in the $t$-channel. The impact factors describing the scattering of particles by the Reggeized gluons contain all the dependence on the nature of the particles and are energy independent. All the dependence on energy is defined by the universal (i.e. process independent) Green's function, which is determined by the BFKL kernel. For a consistent description of scattering amplitudes one needs to know the impact factors with the same accuracy as the kernel. Especially interesting is the highly virtual photon impact factor, because it can be calculated from the "first principles" in perturbative QCD. Unfortunately, this calculation turned out to be a very complicated problem, which is not yet solved, although a noticeable progress has been reached here [3]. Recently an important step was done finding the solution of a related problem: the NLO impact factor for the transition of a virtual photon in a light vector meson was calculated in the case of $t = 0$ and longitudinal polarizations [4]. The NLO impact factors are known also at parton level (i.e. for quarks and gluons) [5].

The calculation of the NLO BFKL kernel for the non-forward scattering was not completed until recently. We remind that for any colour group representation $\mathcal{R}$ in the $t$-channel the kernel is given by the the sum of "virtual" and "real" parts [6]. The "virtual" part is universal (i.e. it does not depend on $\mathcal{R}$) and is expressed through the NLO gluon Regge trajectory [7]. The "real" part is related to the particle production in Reggeon-Reggeon collisions and consists of one-gluon, two-gluon and quark-antiquark contributions. The first contribution is expressed through the effective Reggeon-Reggeon-gluon NLO vertex [8]. Apart from a colour coefficient, it also is universal and known [10]. Each of last two contributions is written as a sum of two terms with depending on $R$ coefficients, at that only one of these terms enters in the kernel for the antisymmetric colour octet representation $\mathcal{R} = 8_a$ (gluon channel), whereas the kernel for the colour singlet representation $\mathcal{R} = 1$ (Pomeron channel) contains both terms. For the case of quark-antiquark production both these terms are known [9]. Instead, only the piece related to the gluon channel was known for the case of two-gluon production [10]. Note that for scattering of physical (colourless) particles only the Pomeron channel exists. Nevertheless the gluon channel plays an important role. It is caused by the possibility to use this channel for a check of self-consistency, and, finally, for a proof of the gluon Reggeization (see Ref. [11] and references therein).

Thus, the two-gluon production contribution was the only missing piece in the the non-forward BFKL kernel. Now it is calculated and the Pomeron kernel is known [12]. Here we present the details of the calculation of the two-gluon contributions and the non-forward BFKL kernel at NLO for all possible colour states in the $t$-channel. Since the quark contribution to the non-forward kernel is known [9] for any $\mathcal{R}$, we shall consider in the following only the gluon contribution, i.e. we shall work in pure gluodynamics.

In the next Section we present the gluon piece of the gluon trajectory, the general form of the "real" contribution to the kernel and its part related to one-gluon production. In Section 3 we derive the contribution to the kernel from the two-gluon production and define the "symmetric" part of this contribution. The colour group relations used in this Section are given in Appendix A. The "symmetric" part of the two-gluon contribution is considered in Section 4. The three pieces contributing to this part are calculated in Appendices B, C and D, respectively. Finally, in Section 5 the non-forward kernel is discussed.

## 2 The "virtual" and "one-gluon" parts of the kernel

As usual, we utilize the Sudakov decomposition of momenta, denoting $p_1$ and $p_2$ the light-cone vectors close to the initial particle momenta $p_A$ and $p_B$ respectively, so that $2p_1p_2 = (p_A + p_B)^2 = s$. We use the conventional dimensional regularization with the space-time dimension $D = 4 + 2\epsilon$ and the normalization adopted in Ref. [6]. The BFKL equation for the Mellin transform $G_\omega$ of the Green's function $G$ is written as

$$\omega G_\omega^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}r}{\vec{r}^{\,2}(\vec{r}-\vec{q})^2}\mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{r}; \vec{q})\, G_\omega^{(\mathcal{R})}(\vec{r}, \vec{q}_2; \vec{q})\,. \quad (2.1)$$

Here $q_i$ and $q_i' \equiv q_i - q\,,\ (i = 1 \div 2)$ are the Reggeon (Reggeized gluon) momenta, $q \simeq q_\perp$ is the momentum transfer; $q^2 \simeq q_\perp^2 = -\vec{q}^{\,2} = t$; the vector sign is used for denoting components of momenta transverse to the $p_1p_2$ plane. The BFKL kernel $\mathcal{K}^{(\mathcal{R})}$ has the form

$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-\vec{q}_1^{\,\prime\,2}\right)\right]\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q})\,, \quad (2.2)$$

i.e. it is given by the sum of the "virtual" part, determined by the gluon Regge trajectory $\omega(t)$ (actually the trajectory is $j(t) = 1 + \omega(t)$), and the "real" part, related to particle production in Reggeon-Reggeon collisions. In the limit $\epsilon \to 0$ [7] the trajectory is given by

$$\omega(t) = \omega^{(1)}(t)\left\{1 + \frac{\omega^{(1)}(t)}{4}\left[\frac{11}{3} + \left(2\zeta(2) - \frac{67}{9}\right)\epsilon + \left(\frac{404}{27} - 2\zeta(3)\right)\epsilon^2\right]\right\}\,, \quad (2.3)$$

where $\omega^{(1)}(t)$ is the one-loop contribution, whose expression is

$$\omega^{(1)}(t) = \frac{g^2N_ct}{2(2\pi)^{D-1}}\int\frac{d^{D-2}r}{\vec{r}^{\,2}\vec{r}^{\,\prime\,2}} = -g^2\frac{N_c\Gamma(1-\epsilon)}{(4\pi)^{D/2}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon\,. \quad (2.4)$$

Here and in the following $\vec{r}^{\,\prime} = \vec{r} - \vec{q}$ , $N_c$ is the number of colors, $\Gamma(x)$ is the Euler function, $\zeta(n)$ is the Riemann zeta function, ($\zeta(2) = \pi^2/6$) and $g$ is the bare coupling constant, concerned with the renormalized coupling $g_\mu$ in the $\overline{MS}$ scheme through the relation

$$g = g_\mu\mu^{-\epsilon}\left[1 + \frac{11}{3}\frac{\bar{g}_\mu^2}{2\epsilon}\right]\ ;\quad \bar{g}_\mu^2 = \frac{g_\mu^2N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\,. \quad (2.5)$$

The "real" part $\mathcal{K}_r^{(\mathcal{R})}$ of the kernel is related to real particle production in Reggeon-Reggeon collisions. It can be presented in the form [6]

$$\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \mathcal{K}_r^{(\mathcal{R})\Lambda}(\vec{q}_1, \vec{q}_2; \vec{q}) - \frac{1}{2}\int \frac{d^{D-2}r}{\vec{r}^{\,2}\vec{r}^{\,\prime\,2}} \mathcal{K}_r^{(\mathcal{R})B}(\vec{q}_1, \vec{r}; \vec{q})$$

$$\times \mathcal{K}_r^{(\mathcal{R})B}(\vec{r}, \vec{q}_2; \vec{q}) \,\ln\left(\frac{s_\Lambda^2}{(\vec{q}_1-\vec{r})^2(\vec{q}_2-\vec{r})^2}\right) \,, \tag{2.6}$$

where the "non-subtracted" kernel $\mathcal{K}_r^{(\mathcal{R})\Lambda}$ is

$$\mathcal{K}_r^{(\mathcal{R})\Lambda}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{\langle bb'|\hat{\mathcal{P}}_\mathcal{R}|aa'\rangle}{n_\mathcal{R}} \sum_J \int \gamma_{ab}^J(q_1, q_2) \left(\gamma_{a'b'}^J(q_1', q_2')\right)^* \frac{d\phi_J}{2(2\pi)^{D-1}} \,. \tag{2.7}$$

Here $\hat{\mathcal{P}}_\mathcal{R}$ is the operator for projection of two-gluon colour states on the representation $\mathcal{R}$; $a, a'$ and $b, b'$ are Reggeon colour indices; $n_\mathcal{R}$ is the number of independent states in $\mathcal{R}$; $\gamma_{ab}^J(q_1, q_2)$ is the effective vertex for production of the state $J$ in the collision of Reggeons with momenta $q_1 = \beta p_1 + q_{1\perp}, \;\; q_2 = -\alpha p_2 + q_{2\perp}$; $d\phi_J$ is the corresponding phase space element; the sum is over all possible states $J$. For a state $J$ consisting of particles with momenta $k_i$, with the total momentum $k = q_1 - q_2$, we have

$$d\phi_J = \frac{dk^2}{2\pi}\theta(s_\Lambda - k^2)(2\pi)^D \delta^D(k - \sum_i k_i) \prod_i \frac{d^{D-1}k_i}{(2\pi)^{D-1}\, 2\epsilon_i} \,. \tag{2.8}$$

The intermediate parameter $s_\Lambda$ in Eq. (2.6) must be taken tending to infinity. The second term in the R.H.S. of Eq. (2.6) appears only at the NLO and serves for subtraction of the large $k^2$ contribution, in order to avoid a double counting of this region. At the leading order (LO) only one-gluon production does contribute, so that $k^2 = 0$, Eq. (2.7) does not depended on $s_\Lambda$ and gives the kernel in the leading (Born) order:

$$\mathcal{K}_r^{(\mathcal{R})\mathcal{B}}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{\langle bb'|\hat{\mathcal{P}}_\mathcal{R}|aa'\rangle}{2n_\mathcal{R}(2\pi)^{D-1}} \sum_G \gamma_{ab}^G(q_1, q_2) \left(\gamma_{a'b'}^G(q_1', q_2')\right)^*$$

$$= \frac{g^2 N_c c_\mathcal{R}}{(2\pi)^{D-1}} \left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} + \vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{(\vec{q}_1 - \vec{q}_2)^2} - \vec{q}^{\,2}\right) \,. \tag{2.9}$$

Here

$$c_\mathcal{R} = \frac{Tr\left(\hat{\mathcal{P}}_\mathcal{R} T^d \bigotimes T^{d*}\right)}{N_c n_R} = \frac{\langle bb'|\hat{\mathcal{P}}_\mathcal{R}|aa'\rangle}{N_c n_\mathcal{R}} T_{ab}^d T_{b'a'}^d \tag{2.10}$$

are the group coefficients, $T^d$ are the colour group generators in the adjoint representation, $T^d_{ab} = -if_{dab}$, $f_{dab}$ are the group structure constants. The projection operators and the coefficients $c_{\mathcal{R}}$ for all possible representations $\mathcal{R}$ are given, for completeness, in the Appendix A. As it was already noted, the most interesting representations are the colour singlet (Pomeron channel, $\mathcal{R} = 1$ ) and the antisymmetric colour octet (gluon channel $\mathcal{R} = 8_a$). Respectively, we have for them

$$c_1 = 1\,, \qquad c_{8_a} = \frac{1}{2}\,. \tag{2.11}$$

Note that for the symmetric colour octet $\mathcal{R} = 8_s$ we have $c_{8_s} = c_{8_a}$, so that at LO the BFKL kernels for the symmetric and anti-symmetric octet representations coincide. Since the trajectory $\omega(t)$ is the eigenvalue of $\mathcal{K}^{(8_a)}$ ("bootstrap" of the gluon Reggeization), the same is true for $\mathcal{K}^{(8_s)}$ (trajectory degeneration). However, generally speaking, it does not mean that amplitudes with $\mathcal{R} = 8_s$ in the $t$-channel have the Reggeized form (with the same trajectory but positive signature), because such a form requires impact factors proportional to the eigenfunction corresponding to the eigenvalue $\omega(t)$. It turns out that for parton scattering amplitudes in LLA it is just the case.

Having the one-gluon production vertices at NLO [8], one can easily calculate the one-gluon contribution to the kernel with the NLO accuracy. Retaining only terms giving non-vanishing contributions in the $\epsilon \to 0$ limit after integration of the kernel over $d^{D-2}k$ in a neighbourhood of the singular point $\vec{k} = \vec{q}_1 - \vec{q}_2 = 0$, we have [10]

$$\mathcal{K}_G^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{g^2 N_c c_{\mathcal{R}}}{(2\pi)^{D-1}} \left\{ \left( \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} + \vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2} \right) \right.$$
$$\times \left( \frac{1}{2} + \frac{g^2 N_c \Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \left[ -(\vec{k}^{\,2})^{\epsilon} \left( \frac{2}{\epsilon^2} - \pi^2 + 4\,\epsilon\,\zeta(3) \right) - \ln^2\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \right] \right)$$
$$+ \frac{g^2 N_c \Gamma(1-\epsilon)}{6(4\pi)^{2+\epsilon}} \left( \left[ \frac{\vec{q}_1^{\,\prime\,2} - \vec{q}_2^{\,\prime\,2}}{\vec{q}_1^{\,2} - \vec{q}_2^{\,2}} - \frac{\vec{k}^{\,2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})^2} \left( \vec{q}_1^{\,2} + \vec{q}_2^{\,2} + 4\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime} - 2\vec{q}^{\,2} \right) \right] \right.$$
$$\times \left[ \frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2} - \vec{q}_2^{\,2}} \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) - \vec{q}_1^{\,2} - \vec{q}_2^{\,2} \right] + 11 \left[ \frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2} - \vec{q}_2^{\,2}} + \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} - \vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} \right.$$
$$\left.\left.\left. - \frac{\vec{q}_1^{\,2} + \vec{q}_2^{\,2}}{\vec{q}_1^{\,2} - \vec{q}_2^{\,2}}\, \vec{q}^{\,2} \right] \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) - 2\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime} \right) \right\} + \left( \vec{q}_i \leftrightarrow \vec{q}_i^{\,\prime} \right). \tag{2.12}$$

For arbitrary $D$ this part of the kernel can be found in the last of Refs. [8] (see there Eq. (4.10)). Note that the exchange $\vec{q}_i \leftrightarrow \vec{q}_i^{\,\prime}$ implies also $\vec{q} \leftrightarrow -\vec{q}$.

Since the colour structure of the one-gluon production vertex is not changed at NLO, the coefficients $c_{\mathcal{R}}$ here are the same as in Eq. (2.9).

The remarkable properties of the kernel, subsequent from general arguments, are

$$\mathcal{K}_r^{(\mathcal{R})}(0,\vec{q}_2;\vec{q}\,) = \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,0;\vec{q}\,) = \mathcal{K}_r^{(\mathcal{R})}(\vec{q},\vec{q}_2;\vec{q}\,) = \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q};\vec{q}\,) = 0 \quad (2.13)$$

and

$$\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}\,) = \mathcal{K}_r^{(\mathcal{R})}(-\vec{q}_1^{\,\prime},-\vec{q}_2^{\,\prime};\vec{q}\,) = \mathcal{K}_r^{(\mathcal{R})}(-\vec{q}_2,-\vec{q}_1;-\vec{q}\,)\ . \quad (2.14)$$

Properties (2.13) mean that the kernel turns into zero at zero transverse momenta of the Reggeons and appear as a consequence of the gauge invariance. Properties (2.14) are a consequence of the crossing invariance and the gluon identity.

The symmetry properties (2.14) of $\mathcal{K}_G^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q})$ are evident from Eq. (2.12). Instead properties (2.13) are not so evident, but can be easily checked.

# 3 The two-gluon production contribution

The new states which appear in the sum over $J$ in Eq. (2.7) at NLO are the two-gluon ones. For these (and only for these) states the integral in Eq. (2.7) is logarithmically divergent at large $k^2$. In this region the two-gluon production vertex factorizes into the product of the one-gluon vertices (see below), so that the dependence on $s_\Lambda$ in Eq. (2.6) is cancelled [6].

Lorentz- and gauge- invariant representation of the two-gluon production vertex has been obtained in Ref. [13]. It has the form

$$\gamma_{ij}^{G_1G_2}(q_1,q_2) = g^2\, \overset{*}{e}{}_1^{\mu}\, \overset{*}{e}{}_2^{\nu}\, \Big[\big(T^{d_1}T^{d_2}\big)_{ij} A_{\mu\nu}(k_1,k_2) + \big(T^{d_2}T^{d_1}\big)_{ij} A_{\nu\mu}(k_2,k_1)\Big]\ , \quad (3.1)$$

where $e_i$ and $d_i$ are gluon polarization vectors and colour indices, respectively. Note that the tensor $A_{\mu\nu}(k_1,k_2)$ depends not only on $k_1,k_2$, as it is explicitly indicated, but on $p_1,q_1$ and $p_2,q_2$ as well. The explicit expression of the tensor is given in Ref. [13]. Its important property is the Abelian-type gauge invariance

$$k_1^\mu A_{\mu\nu}(k_1,k_2) = k_2^\nu A_{\mu\nu}(k_1,k_2) = 0\ . \quad (3.2)$$

Another important property of the tensor $A_{\mu\nu}(k_1,k_2)$ is its transformation law under simultaneous exchange $(p_1,q_1,q,i) \longleftrightarrow (p_2,-q_2,-q,j)$, which we will call $L \longleftrightarrow R$ exchange:

$$A_{\mu\nu}(k_1, k_2)|_{L\longleftrightarrow R} = A_{\nu\mu}(k_2, k_1) \ . \tag{3.3}$$

This property and the representation (3.1) guarantee that the vertex $\gamma_{ij}^{G_1G_2}(q_1, q_2)$ is invariant with respect to the $L \longleftrightarrow R$ exchange, as it must be. Taking into account the representation (2.7), this invariance provides the symmetry of the non-subtracted kernel with respect to the exchange $\vec{q}_1 \leftrightarrow -\vec{q}_2$ , $\vec{q} \leftrightarrow -\vec{q}$ .

We use the Sudakov decomposition for the produced gluon momenta $k_1$ and $k_2$ $(k = k_1 + k_2 = q_1 - q_2)$ in the form $(i = 1, 2)$

$$k_i = \beta_i p_1 + \alpha_i p_2 + k_{i\perp} \ , \ \ s\alpha_i\beta_i = -k_{i\perp}^2 = \vec{k}_i^{\ 2} \ , \ \ \beta_i = x_i\beta \ , \ \ x_1 + x_2 = 1 \ . \tag{3.4}$$

We find that it is convenient to use the same light-cone gauge $e_i p_2 = e_i k_i = 0$ for both gluons, so that we put

$$e_{=} e_{i\perp} - \frac{(e_{i\perp}k_{i\perp})}{(k_i p_2)} p_2 \ . \tag{3.5}$$

In this gauge the vertex takes the form

$$\gamma_{ij}^{G_1G_2}(q_1, q_2) = 4g^2 \ \overset{*\alpha}{e}_{1\perp} \overset{*\beta}{e}_{2\perp}$$

$$\times \left[ \left(T^{d_1}T^{d_2}\right)_{ij} b_{\alpha\beta}(q_1; k_1, k_2) + \left(T^{d_2}T^{d_1}\right)_{ij} b_{\beta\alpha}(q_1; k_2, k_1) \right] \ , \tag{3.6}$$

with the tensor

$$4b^{\alpha\beta}(q_1; k_1, k_2) = (g_\perp^{\alpha\mu} - \frac{k_{1\perp}^\alpha p_2^\mu}{(p_2k_1)})(g_\perp^{\beta\nu} - \frac{k_{2\perp}^\beta p_2^\nu}{(p_2k_2)}) A_{\mu\nu}(k_1, k_2) \ , \tag{3.7}$$

The explicit form of this tensor in terms of the Sudakov variables has been found in Ref. [14]:

$$b^{\alpha\beta}(q_1; k_1, k_2) = \frac{1}{2} g_\perp^{\alpha\beta} \left[ \frac{1}{k^2} \left( 2q_{1\perp}\Lambda_\perp + q_{1\perp}^2 \frac{\Lambda_\perp(2x_1x_2k_\perp - \Lambda_\perp(x_1 - x_2))}{\Sigma} \right) \right.$$

$$\left. - x_2 \frac{q_{1\perp}^2 - 2q_{1\perp}k_{1\perp}}{\tilde{t}_1} \right] - \frac{x_2 k_{1\perp}^\alpha q_{1\perp}^\beta - x_1 q_{1\perp}^\alpha (q_1 - k_1)_\perp^\beta}{x_1 \tilde{t}_1} - \frac{q_{1\perp}^2 k_{1\perp}^\alpha (q_1 - k_1)^\beta}{k_{1\perp}^2 \tilde{t}_1}$$

$$- \frac{x_1 q_{1\perp}^\alpha \Lambda_\perp^\beta + x_2 \Lambda_\perp^\alpha q_{1\perp}^\beta}{x_1 x_2 k^2} - \frac{x_1 q_{1\perp}^2 k_{1\perp}^\alpha k_{2\perp}^\beta}{k_{1\perp}^2 \Sigma} - \frac{q_{1\perp}^2}{k^2 \Sigma} (\Lambda_\perp^\alpha k_{2\perp}^\beta + k_{1\perp}^\alpha \Lambda_\perp^\beta) \ , \tag{3.8}$$

where $g_\perp^{\mu\nu}$ is the metric tensor in the transverse plane:

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{(p_1 p_2)} \ . \tag{3.9}$$

Moreover, the following positions hold:

$$\Lambda_\perp = (x_2 k_1 - x_1 k_2)_\perp \ , \ \ \Sigma = -(x_1 k_{2\perp}^2 + x_2 k_{1\perp}^2) = -\Lambda_\perp^2 - x_1 x_2 k_\perp^2 \ ,$$

$$k^2 = -\frac{\Lambda_\perp^2}{x_1 x_2} \ , \ \ \tilde{t}_1 = (q_1 - k_1)^2 = \frac{1}{x_1}(x_1(q_1 - k_1)_\perp^2 + x_2 k_{1\perp}^2) \ ,$$

$$\tilde{t}_2 = (q_1 - k_2)^2 = \frac{1}{x_2}(x_2(q_1 - k_2)_\perp^2 + x_1 k_{2\perp}^2) \ . \tag{3.10}$$

Note that on account of Eq. (3.7) the tensors $b^{\alpha\beta}(q_1; k_1, k_2)$ and $b^{\beta\alpha}(q_1'; k_2, k_1)$ contain in the denominators "extra" powers of $x_1$ and $x_2$, in comparison with "naive" expectations based on the consideration of Feynman diagrams.

We use the notations which coincide with those adopted in Ref. [10]. Note, however, that in Ref. [10] two different gauges were used for the two gluons: the gauge (3.5) for the second gluon (that with momentum $k_2$), and the gauge similar to the gauge (3.5) with $p_2$ replaced by $p_1$ for the first one (that with momentum $k_1$). Therefore our tensors are related to those used in Ref. [10] by the equalities

$$b^{\alpha\beta}(q_1; k_1, k_2) = \Omega^{\alpha\gamma}(k_1) c_\gamma{}^\beta(k_1, k_2) \ , \ \ b^{\beta\alpha}(q_1; k_2, k_1) = \Omega^{\beta\gamma}(k_2) {c'}_\gamma{}^\alpha(k_2, k_1) \ , \tag{3.11}$$

where the tensors

$$\Omega^{\alpha\beta}(k) = g_\perp^{\alpha\beta} - 2\frac{k_\perp^\alpha k_\perp^\beta}{k_\perp^2} \ , \tag{3.12}$$

with the property

$$\Omega^{\alpha\gamma}(k) \Omega_\gamma{}^\beta(k) = g_\perp^{\alpha\beta} \ , \tag{3.13}$$

realize the transformations between the gauges with the gauge fixing vectors $p_1$ and $p_2$.

Let us consider the behaviour of the vertex (3.6) in the multi-Regge kinematics, i.e. in the limits $x_1 \to 1$ , $x_2 \to 0$ and $x_1 \to 0$ , $x_2 \to 1$. The first of them corresponds to the case when the first gluon is much closer to the particle $A$ in rapidity space than the second gluon. Therefore in this limit the two-gluon production vertex must be factorized as

$$\gamma_{ij}^{G_1 G_2}(q_1, q_2) = \gamma_{il}^{G_1}(q_1, q_1 - k_1) \frac{1}{(q_1 - k_1)_\perp^2} \gamma_{lj}^{G_2}(q_1 - k_1, q_2) \ , \tag{3.14}$$

where $\gamma_{ij}^{G}(q_1,q_2)$ is the one-gluon production vertex. Indeed, at $x_1=1$ , $x_2=0$ we have

$$\Sigma=-k_{2\perp}^2\ ,\ \ x_2k^2=-k_{2\perp}^2\ ,\ \ \tilde{t}_1=(q_1-k_1)_\perp^2\ ,\ \ x_2\tilde{t}_2=k_{2\perp}^2\ , \tag{3.15}$$

that gives

$$b^{\beta\alpha}(q_1;k_2,k_1)|_{x_1=1}=0\ , \tag{3.16}$$

so that

$$\gamma_{ij}^{G_1G_2}(q_1,q_2)=4g^2\ \overset{*}{e}{}_{1\perp}^{\ \alpha}\overset{*}{e}{}_{2\perp}^{\ \beta}\ \left(T^{d_1}T^{d_2}\right)_{ij}b_{\alpha\beta}(q_1;k_1,k_2)|_{x_1=1}\ , \tag{3.17}$$

where

$$b^{\alpha\beta}(q_1;k_1,k_2)|_{x_1=1}$$
$$=\frac{1}{(q_1-k_1)_\perp^2}\left[q_{1\perp}-\frac{q_{1\perp}^2}{k_{1\perp}^2}k_{1\perp}\right]^\alpha\left[q_{1\perp}-k_{1\perp}-\frac{(q_{1\perp}-k_{1\perp})^2}{k_{2\perp}^2}k_{2\perp}\right]^\beta\ . \tag{3.18}$$

Since in the gauge (3.5) we have

$$\gamma_{ij}^{G_1}(q_1,q_1-k_1)=-2g\ \overset{*}{e}{}_{1\perp}^{\ \alpha}\ T_{ij}^{d_1}\left[q_{1\perp}-\frac{q_{1\perp}^2}{k_{1\perp}^2}k_{1\perp}\right]_\alpha\ , \tag{3.19}$$

we see that the factorization property (3.14) is fulfilled. Moreover, with account of the result (3.18), from this property it follows that

$$\left(\frac{2g^2N_cc_{\mathcal{R}}}{(2\pi)^{D-1}}\right)^2\left(b^{\alpha\beta}(q_1;k_1,k_2)b_{\alpha\beta}(q_1';k_1,k_2)\right)|_{x_1=1}=$$
$$\frac{\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1,\vec{q}_1-\vec{k}_1;\vec{q}\right)\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1-\vec{k}_1,\vec{q}_2;\vec{q}\right)}{(\vec{q}_1-\vec{k}_1)^2(\vec{q}_1{}'-\vec{k}_1)^2}\ . \tag{3.20}$$

This equality can be obtained also directly from Eq. (3.18) using the expression (2.9) for the LO kernel.

In the second limit, i.e. $x_1=0$ , $x_2=1$ we get

$$\Sigma=-k_{1\perp}^2\ ,\ \ x_1k^2=-k_{1\perp}^2\ ,\ \ x_1\tilde{t}_1=k_{1\perp}^2\ ,\ \ \tilde{t}_2=(q_1-k_2)_\perp^2\ ,\ \ b^{\alpha\beta}(q_1;k_1,k_2)=0\ ,$$

$$b^{\beta\alpha}(q_1;k_2,k_1)|_{x_2=1}$$
$$=\frac{1}{(q_1-k_2)_\perp^2}\left[q_{1\perp}-\frac{q_{1\perp}^2}{k_{2\perp}^2}k_{2\perp}\right]^\beta\left[q_{1\perp}-k_{2\perp}-\frac{(q_{1\perp}-k_{2\perp})^2}{k_{1\perp}^2}k_{1\perp}\right]^\alpha \tag{3.21}$$

and

$$\gamma_{ij}^{G_1G_2}(q_1,q_2) = \gamma_{il}^{G_2}(q_1,q_1-k_2)\frac{1}{(q_1-k_2)_\perp^2}\gamma_{lj}^{G_1}(q_1-k_2,q_2)\ . \qquad (3.22)$$

We have also

$$\left(\frac{g^2N_c c_{\mathcal{R}}}{2(2\pi)^{D-1}}\right)^2\left(b^{\alpha\beta}(q_1;k_2,k_1)b_{\alpha\beta}(q_1';k_2,k_1)\right)|_{x_2=1}$$
$$= \frac{\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1,\vec{q}_1-\vec{k}_2;\vec{q}\right)\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1{}'-\vec{k}_2,\vec{q}_2;\vec{q}\right)}{(\vec{q}_1-\vec{k}_2)^2(\vec{q}_1{}'-\vec{k}_2)^2}\ . \qquad (3.23)$$

With the help of Eqs. (2.7) and (3.6) the two-gluon contribution to the “non-subtracted" kernel is presented in the form

$$\mathcal{K}_{GG}^{(\mathcal{R})\Lambda}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) = 8g^4N_c^2\int\left(a_RF_a(k_1,k_2)+b_RF_b(k_1,k_2)\right)\frac{d\phi_{GG}}{(2\pi)^{D-1}}\ , \qquad (3.24)$$

where the group coefficients $a_R$ and $b_R$ are defined as

$$a_R = \frac{Tr\left(\hat{\mathcal{P}}_{\mathcal{R}}(T^{d_1}T^{d_2})\bigotimes(T^{d_1}T^{d_2})^*\right)}{N_c^2n_R}\ ,\ \ b_R = \frac{Tr\left(\hat{\mathcal{P}}_{\mathcal{R}}(T^{d_1}T^{d_2})\bigotimes(T^{d_2}T^{d_1})^*\right)}{N_c^2n_R}\ , \qquad (3.25)$$

and the functions $F_a$ and $F_b$ as

$$F_a(k_1,k_2) = b^{\alpha\beta}(q_1;k_1,k_2)b_{\alpha\beta}(q_1';k_1,k_2)+b^{\beta\alpha}(q_1;k_2,k_1)b_{\beta\alpha}(q_1';k_2,k_1)\ , \qquad (3.26)$$
$$F_b(k_1,k_2) = b^{\alpha\beta}(q_1;k_1,k_2)b_{\beta\alpha}(q_1';k_2,k_1)+b^{\beta\alpha}(q_1;k_2,k_1)b_{\alpha\beta}(q_1';k_1,k_2)\ . \qquad (3.27)$$

It is easy to see that $a_R = c_R^2$, since

$$Tr\left(\hat{\mathcal{P}}_{\mathcal{R}}(T^{d_1}T^{d_2})\bigotimes(T^{d_1}T^{d_2})^*\right) = Tr\left(\hat{\mathcal{P}}_{\mathcal{R}}(T^{d_1}\bigotimes T^{d_1*})(T^{d_2}\bigotimes T^{d_2*})\right) =$$
$$= Tr\left(\hat{\mathcal{P}}_{\mathcal{R}}T^{d_1}\bigotimes T^{d_1*}\hat{\mathcal{P}}_{\mathcal{R}}T^{d_2}\bigotimes T^{d_2*}\right)$$
$$= \frac{1}{n_R}Tr\left(\hat{\mathcal{P}}_{\mathcal{R}}T^{d_1}\bigotimes T^{d_1*}\right)\left(\hat{\mathcal{P}}_{\mathcal{R}}T^{d_2}\bigotimes T^{d_2*}\right)\ . \qquad (3.28)$$

This relation is important for the cancellation of the $s_\Lambda$–dependence in the kernel (2.6). Due to the result (2.11) it gives, in particular,

$$a_0 = 1\ ,\ \ a_{8_a} = a_{8_s} = \frac{1}{4}\ . \qquad (3.29)$$

For the coefficients $b_R$, using the relation

$$(T^{d_1}T^{d_2})\bigotimes(T^{d_1}T^{d_2})^* - (T^{d_1}T^{d_2})\bigotimes(T^{d_2}T^{d_1})^*$$

$$= if^{dd_2d_1}(T^{d_1}T^{d_2})\bigotimes T^{d*} = \frac{N_c}{2}T^d\bigotimes T^{d*} \ , \tag{3.30}$$

with account of Eq. (2.10) we obtain

$$b_R = a_R - \frac{1}{2}c_R = c_R\left(c_R - \frac{1}{2}\right) \ , \tag{3.31}$$

i.e. $b_1 = 1/2$, whereas for both symmetric and antisymmetric colour octet representations the coefficients $b_R$ are zero. This is especially important for the antisymmetric case, since the vanishing of $b_{8_a}$ is crucial for the gluon Reggeization. Note that the vanishing of $b_{8_s}$ means that in pure gluodynamics the kernels for both octet representations coincide at the NLO as well as at the LO.

In terms of the variables $k_{i\perp}$ and $x_i$ the phase space element $d\phi_{GG}$ can be written as

$$d\phi_{GG} = \frac{dx_1dx_2}{4x_1x_2}\delta(1-x_1-x_2)\frac{d^{D-2}k_1d^{D-2}k_2}{(2\pi)^{(D-1)}}\delta^{D-2}(k_\perp-k_{1\perp}-k_{2\perp})\theta(s_\Lambda-k^2) \ . \tag{3.32}$$

Here the identity of the final gluons is taken into account by the factor 1/2!, so that integration must be performed over all the phase space; $k_\perp = q_{1\perp}-q_{2\perp} = q'_{1\perp}-q'_{2\perp}$ and $k^2$ must be expressed in terms of $x_i$ and $k_{i\perp}$ (see Eq. (3.10)). We recall that the parameter $s_\Lambda$ must be taken tending to infinity before the limit $\epsilon \to 0$. From the expression (3.8) one can see that the tensors $b^{\alpha\beta}$ fall down as $1/\vec{k}_i^{\,2}$ at $\vec{k}_i^{\,2} \to \infty$ at fixed $x_i$. Therefore the integral over $\vec{k}_i$ in Eq. (3.24) is well convergent in the ultraviolet region, so that the restrictions imposed by the theta-function can be written as

$$x_i \geq \frac{\vec{k}_i^{\,2}}{s_\Lambda} \ . \tag{3.33}$$

Let us discuss the properties (2.13) and (2.14) of the two-gluon contribution to the kernel (2.2). As for the subtraction term, its properties (2.13), related to gauge invariance, follow directly from the corresponding properties of the “Born" kernel (2.9). Properties (2.14) are provided by the appropriate symmetries expressed by Eq. (2.9) and the invariance of the logarithm and the integration measure $d^{D-2}r/(\vec{r}^{\,2}\vec{r}^{\,\prime 2})$ in Eq. (2.6) under the exchanges $\vec{q}_i \leftrightarrow -\vec{q}_i^{\,\prime}$ , $\vec{r} \leftrightarrow -\vec{r}^{\,\prime}$, as well as $\vec{q}_1 \leftrightarrow -\vec{q}_2$ , $\vec{q} \leftrightarrow -\vec{q}$ , $\vec{r} \leftrightarrow -\vec{r}$ .

Turn now to the non-subtracted contribution (3.24). Since its "gauge-invariance" properties (2.13) are provided (see Eqs. (3.26) and (3.27)) by the conversion into zero of the tensor $b^{\alpha_1\alpha_2}(q_1;k_1,k_2)$ at $q_{1\perp}=0$ and at $q_{2\perp}=(q_1-k_1-k_2)_\perp=0$, they can be easily seen from the representation (3.8). The symmetry with respect to the exchanges $\vec{q}_i \leftrightarrow -\vec{q}_i^{\,\prime}$ can be also easily seen from the representation (3.24), taking into account Eqs. (3.26) and (3.27). As it was already discussed, the symmetry relative to the exchanges $\vec{q}_1 \leftrightarrow -\vec{q}_2$ , $\vec{q} \leftrightarrow -\vec{q}$ follows from the general representation (2.7) and the invariance of the vertex $\gamma_{ij}^{G_1G_2}(q_1,q_2)$ with regard to the $L \longleftrightarrow R$ exchange. However, it is not easy to derive this symmetry from the representation (3.24) using Eqs. (3.26), (3.27) and the tensor $b_{\alpha\beta}(q_1;k_1,k_2)$ presented in Eq. (3.8). The matter is that the gauge (3.5) breaks the symmetry between $p_2$ and $p_1$, so that a transformation law of this tensor under the $L \longleftrightarrow R$ exchange has not a simple form like that of Eq. (3.3). Moreover, the choice of the variables $x_i$ also destroys the symmetry between $p_1$ and $p_2$. To restore the symmetry one has to introduce the variables $y_i$, defined as

$$y_i = \frac{\alpha_i}{\alpha} , \quad y_1 + y_2 = 1 , \tag{3.34}$$

concerned with $x_i$ by the relation

$$\frac{y_1}{y_2} = \frac{x_2}{x_1}\frac{k_{1\perp}^2}{k_{2\perp}^2} . \tag{3.35}$$

Using this relation and taking into account that

$$k^2 = \frac{(x_2\vec{k}_1 - x_1\vec{k}_2)^2}{x_1x_2} = \frac{(y_2\vec{k}_1 - y_1\vec{k}_2)^2}{y_1y_2} , \tag{3.36}$$

it is easy to see that the phase space element $d\phi_{GG}$ (3.32) is invariant under the exchange $x_i \leftrightarrow y_i$. Note that in terms of the Sudakov variables the $L \longleftrightarrow R$ exchange means

$$\beta_i \leftrightarrow \alpha_i , \quad q_{1\perp} \leftrightarrow -q_{2\perp} , \quad q_\perp \leftrightarrow -q_\perp \tag{3.37}$$

or, in terms of $x_i$ and $y_i$,

$$x_i \leftrightarrow y_i , \quad q_{1\perp} \leftrightarrow -q_{2\perp} , \quad q_\perp \leftrightarrow -q_\perp . \tag{3.38}$$

As it was already mentioned, under this exchange the transformation law for the tensor $b^{\alpha\beta}$ is not the same as for $A^{\mu\nu}$ of Eq. (3.3), because of the choice of the gauge (3.5). It is not difficult to understand that the transformation low must be

$$b_{\alpha\beta}(q_1; k_1, k_2)|_{L\longleftrightarrow R} = \Omega^{\alpha}{}_{\gamma}(k_1)\Omega^{\beta}{}_{\delta}(k_2) b_{\delta\gamma}(q_1; k_2, k_1) \ , \tag{3.39}$$

where the tensors $\Omega^{\alpha\beta}$ (see the definition (3.12)) take into account the gauge change under the $L \leftrightarrow R$ exchange. One can check directly using Eq. (3.8) that the property (3.39) is indeed fulfilled. Together with that of Eq. (3.13), this property demonstrates once more that the functions $F_a$ and $F_b$, defined by Eqs. (3.26) and (3.27) respectively, are invariant with respect to the $L \leftrightarrow R$ exchange. This means that, due to the invariance of the phase space element discussed above, the contribution (3.24) is symmetric under the replacement $\vec{q}_1 \leftrightarrow -\vec{q}_2, \ \ \vec{q} \leftrightarrow -\vec{q}$.

From Eqs. (2.6), (3.20), (3.23) and (3.26) it follows that the subtraction term can be written as

$$\frac{g^4 N_c^2 c_R^2}{(2\pi)^{D-1}} \int \frac{d^{2+2\epsilon} k_1}{(2\pi)^{D-1}} \left( F_a(k_1, k_2)|_{x_1=1} + F_a(k_1, k_2)|_{x_2=1} \right) \ln\left(\frac{s_\Lambda^2}{k_1^2 k_2^2}\right)$$
$$= 8 g^4 N_c^2 a_R \int \frac{d\phi_{GG}}{(2\pi)^{D-1}} \left( x_1 F_a(k_1, k_2)|_{x_1=1} + x_2 F_a(k_1, k_2)|_{x_2=1} \right)$$
$$+ \frac{g^4 N_c^2 a_R}{(2\pi)^{D-1}} \int \frac{d^{2+2\epsilon} k_1}{(2\pi)^{D-1}} \left( F_a(k_1, k_2)|_{x_1=1} - F_a(k_1, k_2)|_{x_2=1} \right) \ln\left(\frac{k_2^2}{k_1^2}\right) \ . \tag{3.40}$$

Here the equality $a_R = c_R^2$, the expression (3.32) for the phase space element and the restrictions given by the inequality (3.33) on $x_i$ were taken into account. Note that the second integral in the R.H.S of Eq. (3.40) is completely antisymmetric with respect to the substitution $\vec{q}_1 \leftrightarrow -\vec{q}_2, \ \ \vec{q} \leftrightarrow -\vec{q}$. Therefore the subtraction term can be obtained by symmetrization of the first integral. Consequently, using the definition

$$\left(\frac{f(x)}{x(1-x)}\right)_+ \equiv \frac{1}{x}[f(x) - f(0)] + \frac{1}{(1-x)}[f(x) - f(1)] \ , \tag{3.41}$$

we can write the two-gluon contribution to the kernel (2.6) in the limit $s_\Lambda \to \infty$ in the form

$$\mathcal{K}_{GG}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{2 g^4 N_c^2}{(2\pi)^{D-1}} \hat{\mathcal{S}} \int_0^1 dx \int \frac{d^{2+2\epsilon} k_1}{(2\pi)^{D-1}} \left( \frac{a_R F_a(k_1, k_2) + b_R F_b(k_1, k_2)}{x(1-x)} \right)_+ , \tag{3.42}$$

where $x \equiv x_1$ and the operator $\hat{\mathcal{S}}$ symmetrizes with respect to the substitution $\vec{q}_1 \leftrightarrow -\vec{q}_2, \ \ \vec{q} \leftrightarrow -\vec{q}$. It was used here that $F_b(k_1, k_2)|_{x_1=0} =$

$F_b(k_1,k_2)|_{x_1=1} = 0$, according to the definition (3.27) and the properties $b^{\alpha\beta}(q_1;k_1,k_2)|_{x_1=0} = b^{\beta\alpha}(q_1;k_2,k_1)|_{x_1=1} = 0$ .

Since the coefficient $b_R$ is equal to zero for an octet representation, the first term in Eq. (3.42) is determined by the two-gluon contribution to the octet kernel, which is already calculated [10]. Therefore, our task is to calculate the second term. However, we have found that to calculate just this term is not the most convenient way because of the rather complicated form of the convolution $F_b$. Instead of this we find more convenient to calculate the "symmetric" contribution

$$\mathcal{K}^{(s)}_{GG}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{2g^4N_c^2}{(2\pi)^{D-1}}\hat{\mathcal{S}}\int_0^1 dx \int \frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}\left(\frac{F_s(k_1,k_2)}{x(1-x)}\right)_+ , \tag{3.43}$$

where the function

$$F_s(k_1,k_2) = F_a(k_1,k_2) + F_b(k_1,k_2) \tag{3.44}$$

is given by the convolution

$$(b^{\alpha\beta}(q_1;k_1,k_2) + b^{\beta\alpha}(q_1;k_2,k_1))(b_{\alpha\beta}(q_1';k_1,k_2) + b_{\beta\alpha}(q_1';k_2,k_1)) \ . \tag{3.45}$$

Taking into account that $a_8 = 1/4$, we can present the two-gluon contribution to the kernel for any representation $R$ as

$$\mathcal{K}^{(R)}_{GG}(\vec{q}_1,\vec{q}_2;\vec{q}) = 4(a_R - b_R)\mathcal{K}^{(8)}_{GG}(\vec{q}_1,\vec{q}_2;\vec{q}) + b_R\mathcal{K}^{(s)}_{GG}(\vec{q}_1,\vec{q}_2;\vec{q}). \tag{3.46}$$

# 4 The "symmetric" contribution to the kernel

Calculation of $\mathcal{K}^{(s)}_{GG}(\vec{q}_1,\vec{q}_2;\vec{q})$ seems to be more convenient since the sum

$$b^{\alpha\beta}(q_1;k_1,k_2) + b^{\beta\alpha}(q_1;k_2,k_1) = \frac{q_{1\perp}^2 k_{1\perp}^\alpha k_{2\perp}^\beta}{k_{1\perp}^2 k_{2\perp}^2}$$

$$-\frac{1}{2}g_\perp^{\alpha\beta}x_2\frac{q_{1\perp}^2 - 2q_{1\perp}k_{1\perp}}{\tilde{t}_1} - \frac{x_2k_{1\perp}^\alpha q_{1\perp}^\beta - x_1q_{1\perp}^\alpha(q_1-k_1)_\perp^\beta}{x_1\tilde{t}_1} - \frac{q_{1\perp}^2k_{1\perp}^\alpha(q_1-k_1)_\perp^\beta}{k_{1\perp}^2\tilde{t}_1}$$

$$-\frac{1}{2}g_\perp^{\alpha\beta}x_1\frac{q_{1\perp}^2 - 2q_{1\perp}k_{2\perp}}{\tilde{t}_2} - \frac{x_1q_{1\perp}^\alpha k_{2\perp}^\beta - x_2(q_1-k_2)_\perp^\alpha q_{1\perp}^\beta}{x_2\tilde{t}_2} - \frac{q_{1\perp}^2(q_1-k_2)_\perp^\alpha k_{2\perp}^\beta}{k_{2\perp}^2\tilde{t}_2} , \tag{4.1}$$

looks simpler than $b^{\alpha\beta}(q_1;k_1,k_2)$ and $b^{\beta\alpha}(q_1;k_2,k_1)$ taken separately. Note, however, that $F_s(k_1,k_2)$ does not turn into zero at the points $x_1 = 0$ and

$x_2 = 0$, in contrast to $F_b(k_1, k_2)$, so that the prescription (3.41) is necessary in Eq. (3.43) to remove the "Regge divergencies", i.e. the divergencies at $x \equiv x_1 = 0$ and $x = 1$. We shall use the decompositions

$$b^{\alpha\beta}(q_1; k_1, k_2) + b^{\beta\alpha}(q_1; k_2, k_1) = b_0^{\alpha\beta} + b_1^{\alpha\beta} + b_2^{\alpha\beta} \ ,$$

$$b^{\alpha\beta}(q_1'; k_1, k_2) + b^{\beta\alpha}(q_1'; k_2, k_1) = b_0'^{\ \alpha\beta} + b_1'^{\ \alpha\beta} + b_2'^{\ \alpha\beta} \ , \tag{4.2}$$

with

$$b_0^{\alpha\beta} = \frac{q_{1\perp}^2 k_{1\perp}^\alpha k_{2\perp}^\beta}{k_{1\perp}^2 k_{2\perp}^2} \ , \tag{4.3}$$

$$b_1^{\alpha\beta} = -\frac{1}{2} g_\perp^{\alpha\beta} x_2 \frac{q_{1\perp}^2 - 2q_{1\perp}k_{1\perp}}{\tilde{t}_1} - \frac{x_2 k_{1\perp}^\alpha q_{1\perp}^\beta - x_1 q_{1\perp}^\alpha (q_1 - k_1)_\perp^\beta}{x_1 \tilde{t}_1}$$
$$- \frac{q_{1\perp}^2 k_{1\perp}^\alpha (q_1 - k_1)_\perp^\beta}{k_{1\perp}^2 \tilde{t}_1} \tag{4.4}$$

and

$$b_2^{\alpha\beta} = -\frac{1}{2} g_\perp^{\alpha\beta} x_1 \frac{q_{1\perp}^2 - 2q_{1\perp}k_{2\perp}}{\tilde{t}_2} - \frac{x_1 q_{1\perp}^\alpha k_{2\perp}^\beta - x_2 (q_1 - k_2)_\perp^\alpha q_{1\perp}^\beta}{x_2 \tilde{t}_2}$$
$$- \frac{q_{1\perp}^2 (q_1 - k_2)_\perp^\alpha k_{2\perp}^\beta}{k_{2\perp}^2 \tilde{t}_2} \ . \tag{4.5}$$

The tensor $b_0^{\alpha\beta}$ does not depend on $x$ at all; as far as $b_{1,2}^{\alpha\beta}$ is concerned, it is easy to obtain

$$b_1^{\alpha\beta}|_{x=0} = -\frac{k_{1\perp}^\alpha q_{1\perp}^\beta}{k_{1\perp}^2} \ , \quad b_2^{\alpha\beta}|_{x=0} = \frac{(q_1 - k_2)_\perp^\alpha q_{1\perp}^\beta}{(q_1 - k_2)_\perp^2} - \frac{q_{1\perp}^2 (q_1 - k_2)_\perp^\alpha k_{2\perp}^\beta}{k_{2\perp}^2 (q_1 - k_2)_\perp^2} \ , \tag{4.6}$$

$$b_1^{\alpha\beta}|_{x=1} = \frac{q_{1\perp}^\alpha (q_1 - k_1)_\perp^\beta}{(q_1 - k_1)_\perp^2} - \frac{q_{1\perp}^2 k_{1\perp}^\alpha (q_1 - k_1)_\perp^\beta}{k_{1\perp}^2 (q_1 - k_1)_\perp^2} \ , \quad b_2^{\alpha\beta}|_{x=1} = -\frac{q_{1\perp}^\alpha k_{2\perp}^\beta}{k_{2\perp}^2} \ . \tag{4.7}$$

Note also that at fixed $x$ the tensors $b_i^{\alpha\beta}$ have infrared singularities. The singularities of $b_0^{\alpha\beta}$ are evident from Eq. (4.3). As for $b_i^{\alpha\beta}$ with $i = 1, 2$, they are singular at $k_{i\perp}$, where

$$b_1^{\alpha\beta}|_{k_{1\perp}\to 0} = -\frac{k_{1\perp}^\alpha q_{1\perp}^\beta}{k_{1\perp}^2} \ , \quad b_2^{\alpha\beta}|_{k_{2\perp}\to 0} = -\frac{q_{1\perp}^\alpha k_{2\perp}^\beta}{k_{2\perp}^2} \ . \tag{4.8}$$

Accordingly to the composition (4.2), we present $F_s(k_1, k_2)$ in the form

$$F_s(k_1,k_2) = (A_0 + A_1 + A_2 + A_3) + (k_1 \leftrightarrow k_2)\ . \qquad (4.9)$$

Note that, since the total convolution (4.9) includes the terms obtained by the substitution $(k_1 \leftrightarrow k_2)$ from $A_i$, the choice of $A_i$ is not unique. We get (in the following in this Section only transverse momenta are used and we omit the $\perp$ sign; pay attention, however, that the Minkowski metric is used)

$$A_0 = \frac{1}{2} b_0^{\alpha\beta} b'_{0\ \alpha\beta} \ = \frac{q_1^2 q_1'^2}{2k_1^2 k_2^2}\ , \qquad (4.10)$$

$$A_1 = b_1^{\alpha\beta} b'_{0\ \alpha\beta} + b_0^{\alpha\beta} b'_{1\ \alpha\beta} \ = -\frac{q_1'^2}{k_1^2 k_2^2 \tilde{t}_1}\left[\frac{x_2}{2}(k_1k_2)(q_1^2 - 2q_1k_1)\right.$$

$$\left. +\frac{x_2}{x_1}k_1^2(q_1k_2) + (q_1^2 - (q_1k_1))(k_2(q_1-k_1))\right] + (q_1 \leftrightarrow q_1')\ , \qquad (4.11)$$

$$A_2 = b_1^{\alpha\beta} b'_{1\ \alpha\beta} \ = \frac{1}{2\tilde{t}_1\tilde{t}_1'}\left[\frac{(D-2)}{4}x_2^2(q_1^2 - 2q_1k_1)(q_1'^2 - 2q_1'\, k_1) + x_2(q_1'^2 - 2q_1'k_1)\right.$$

$$\times\left((q_1k_1)(\frac{1}{x_1} + \frac{q_1^2}{k_1^2}) - 2q_1^2\right) + \frac{x_2^2}{x_1^2}k_1^2(q_1q_1'\ ) + (q_1q_1'\ )((q_1-k_1)(q_1'-k_1))$$

$$-2\frac{x_2}{x_1}(k_1q_1)(q_1'(q_1-k_1)) + \frac{q_1'^2}{k_1^2}\left(2\frac{x_2}{x_1}k_1^2(q_1(q_1'\ - k_1))\right.$$

$$\left.\left. +(q_1^2 - 2(q_1k_1))((q_1-k_1)(q_1'\ - k_1))\right)\right] + (q_i \leftrightarrow q_i')\ , \qquad (4.12)$$

As for $A_3$, its definition is not so simple. Actually $A_3$ is constructed from $b_1^{\alpha\beta} b'_{2\ \alpha\beta}$. Note that this convolution is invariant with respect to the simultaneous substitution $k_1 \leftrightarrow k_2$ and $q_1 \leftrightarrow q_1'$. As for the two terms in $b_1^{\alpha\beta} b'_{2\ \alpha\beta}$, which are obtained from each other by this substitution, we have taken one of them in an unchanged form, whereas in the other we have performed the substitution $k_1 \leftrightarrow k_2$, so that it can be obtained from the first term by the substitution $q_1 \leftrightarrow q_1'$. Of course, this procedure is not unique. We define $A_3$ as

$$A_3 = \frac{1}{2\tilde{t}_1\tilde{t}_2'}\left[\frac{(D-2)}{4}x_1x_2(q_1^2 - 2q_1k_1)(q_1'^2 - 2q_1'\, k_2) + (q_1'^2 - 2q_1'k_2)\right.$$

$$\times\left((q_1k_1)(1+\frac{x_1q_1^2}{k_1^2})-2x_1q_1^2\right)+(q_1k_2)(q_1'k_1)+(q_1(q_1'-k_2))(q_1'(q_1-k_1))$$

$$+2q_1'^2\frac{x_2(q_1k_2)(q_1'\,k_1-k_2k_1)-x_1(q_1k_2-k_1k_2)(q_1q_1'-q_1k_2)}{x_1k_2^2}$$

$$\left.-\frac{2x_2}{x_1}(q_1q_1')(k_1(q_1'-k_2))+\frac{q_1^2q_1'^2}{k_1^2k_2^2}(q_1k_2-k_1k_2)(q_1'\,k_1-k_2k_1)\right]+(q_i\leftrightarrow q_i')\ . \tag{4.13}$$

From Eqs. (3.43) and (4.9) it follows that we can present $\mathcal{K}_{GG}^{(s)}(\vec{q}_1,\vec{q}_2;\vec{q})$ as

$$\mathcal{K}_{GG}^{(s)}(\vec{q}_1,\vec{q}_2;\vec{q})=\frac{4g^4N_c^2}{(2\pi)^{D-1}}\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\hat{\mathcal{S}}(\mathcal{J}_0+\mathcal{J}_1+\mathcal{J}_2+\mathcal{J}_3)\ , \tag{4.14}$$

where

$$\mathcal{J}_i=\int_0^1dx\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left(\frac{A_i+(k_1\leftrightarrow k_2)}{x(1-x)}\right)_+\ . \tag{4.15}$$

Since $A_0$ does not depend on $x$, the integral $\mathcal{J}_0$ evidently is equal to zero according to the definition (3.41). The integrals $\mathcal{J}_i$ with $i\neq 0$ will be discussed below. Here we note that $\mathcal{K}_{GG}^{(s)}(\vec{q}_1,\vec{q}_2;\vec{q})$ is finite in the limit $\epsilon\to 0$. Indeed, let us consider the terms in $A_i$ having non-integrable infrared singularities in the limit $\epsilon\to 0$ . These terms can be easily obtained using Eqs. (4.3) and (4.8), as well as the explicit expressions (4.11)-(4.13):

$$A_1|_{sing}=-\frac{q_1'^2(q_1k_2)}{k_1^2k_2^2}+(q_1\leftrightarrow q_1'),\ \ A_2|_{sing}=\frac{(q_1q_1')}{2k_1^2}+(q_1\leftrightarrow q_1'),\ A_3|_{sing}=0. \tag{4.16}$$

We see that the infrared singular parts of $A_i$ do not depend on $x$. On the other hand for $A_i|_{x=0}$ and $A_i|_{x=1}$ we find (it can be done using Eqs. (4.3), (4.6) and (4.7) as well as Eqs. (4.11)-(4.13) )

$$A_1|_{x=0}=-\frac{q_1'^2(q_1k_2)}{k_1^2k_2^2}+(q_i\leftrightarrow q_i')\ ,$$

$$A_1|_{x=1}=-\frac{q_1'^2}{k_1^2k_2^2(q_1-k_1)^2}(q_1^2-(q_1k_1))(k_2(q_1-k_1))+(q_i\leftrightarrow q_i')\ , \tag{4.17}$$

$$A_2|_{x=0}=\frac{(q_1q_1')}{2k_1^2}+(q_i\leftrightarrow q_i')\ ,$$

$$A_2|_{x=1}=\frac{(q_1-k_1)(q_1'\ -k_1)}{2(q_1-k_1)^2(q_1'\ -k_1)^2}\left[(q_1q_1'\ )+\frac{q_1'^2}{k_1^2}\left(q_1^2-2(k_1q_1)\right)\right]+(q_i\leftrightarrow q_i')\ , \tag{4.18}$$

$$A_3|_{x=0} = -\frac{(k_1(q_1' - k_2))}{k_1^2(q_1' - k_2)^2}\left[(q_1q_1') - q_1'^2\frac{(q_1k_2)}{k_2^2}\right] + (q_i \leftrightarrow q_i'), \quad A_3|_{x=1} = 0 \ . \tag{4.19}$$

Comparing these expressions with the results (4.16), we see that they do not contain new (i.e. different from $A_i|_{sing}$) non-integrable infrared singularities in the limit $\epsilon \to 0$. Therefore such singularities are absent in $(A_i/[x(1-x)])_+$.

## 4.1 Calculation of $\mathcal{J}_1$

In order to calculate the integral $\mathcal{J}_1$ it is suitable to first integrate over $x$ and then over $k_1$. Using the invariance of the integration measure with respect to the exchange $k_1 \leftrightarrow k_2$, after the first integration (here and below we omit in integrands terms giving zero after the subsequent integration) we obtain

$$\mathcal{J}_1 = \frac{{q_1'}^{\,2}}{2}\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{k_2^2}\ln\left(\frac{(\,q_1 - \ k_1)^2}{k_1^2}\right)$$

$$\times\left(\frac{q_1^2\ q_2^2}{k_1^2(\,q_1 - \ k_1)^2} - \frac{q_2^2}{(\,q_1 - \ k_1)^2} - \frac{q_2^2 + 2q_2k_1}{k_1^2}\right) \ + \ (q_i \leftrightarrow q_i') \ . \tag{4.20}$$

Note that the singularities of separate terms in the integrand at $k_1 = 0$, $k_2 = 0$ and $q_1 - k_1 = 0$ cancel each other.

Details of the calculation of this integral are given in Appendix B. At arbitrary $\epsilon$ we find

$$\mathcal{J}_1 = \frac{{q_1'}^{\,2}}{2}\int_0^1 dx\int_0^1 dy y^{\epsilon-1}$$

$$\times\left[\left(\frac{(q_1^2 - k^2)(1-x)(1-y(1-x)) - q_2^2(x - y(1-x^2))}{(-(k^2(1-x) + q_2^2x)(1-y) - q_1^2x(1-x)y)^{1-\epsilon}\, x(1-x)}\right)_+ + \ln\left(\frac{x}{1-x}\right)\right.$$

$$\left.\times\frac{(1-\epsilon)q_1^2q_2^2}{(-(k^2(1-x) + q_2^2x)(1-y) - q_1^2x(1-x)y)^{2-\epsilon}}\right] \ + \ (q_i \leftrightarrow q_i') \ . \tag{4.21}$$

The integral cannot be expressed in terms of elementary functions not only at arbitrary $D$, but even in the limit $\epsilon \to 0$. In this limit we have

$$\mathcal{J}_1 = \frac{{q_1'}^{\,2}}{2}\left(\frac{(k^2 - q_1^2 - q_2^2)^2 - 4q_1^2q_2^2}{2k^2}I(k^2, q_2^2, q_1^2)\right.$$

$$\left. + \frac{k^2 + q_2^2 - q_1^2}{2k^2}\ln\left(\frac{k^2}{q_2^2}\right)\ln\left(\frac{q_1^2}{q_2^2}\right)\right) \ + \ (q_i \leftrightarrow q_i') \ , \tag{4.22}$$

where

$$I(a,b,c) = \int_0^1 \frac{dx}{a(1-x)+bx-cx(1-x)} \ln\left(\frac{a(1-x)+bx}{cx(1-x)}\right) . \tag{4.23}$$

Note that the integral $I(a,b,c)$ is invariant with respect to any permutation of its arguments, as it can be seen from the representation [15]

$$I(a,b,c) = \int_0^1 \int_0^1 \int_0^1 \frac{dx_1 dx_2 dx_3 \delta(1-x_1-x_2-x_3)}{(ax_1+bx_2+cx_3)(x_1x_2+x_1x_3+x_2x_3)} . \tag{4.24}$$

In particular, $I(k^2, q_2^2, q_1^2)$ does not change under the substitution $q_1 \leftrightarrow -q_2$.

## 4.2 Calculation of $\mathcal{J}_2$

The order of integration used for the calculation of $\mathcal{J}_1$ (first over $x$ and then over $k_1$) is suitable for the calculation of $\mathcal{J}_2$ as well. Details of the integration are given in Appendix C. The result of the integration over $x$ can be presented as

$$\mathcal{J}_2 = \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \left\{ \left[ \frac{1}{(q_1-k_1)^2-k_1^2} \left( (1+\epsilon)k_1^2 - q_1^2 \left( 2 - \frac{(k_1q_1)}{k_1^2} \right) \right) \right.\right.$$
$$\left. + \frac{(q_1-k_1)(q_1'-k_1)}{(q_1-k_1)^2} \left( \frac{q_1^2}{k_1^2} - \frac{q^2}{2(q_1'-k_1)^2} \right) - \frac{(q_1q_1')}{k_1^2} \right] \ln\left( \frac{(q_1-k_1)^2}{k_1^2} \right)$$
$$+ \left[ \left( \frac{q^2}{2} \left( 1 + \frac{(q_1-k_1)(q_1'-k_1)}{(q_1-k_1)^2} \right) \right) - \frac{1+\epsilon}{2}(q_1-k_1)^2 \right]$$
$$\left. \times \frac{1}{(q_1-k_1)^2-(q_1'-k_1)^2} \ln\left( \frac{(q_1-k_1)^2}{(q_1'-k_1)^2} \right) \right\} + (q_1 \leftrightarrow q_1') . \tag{4.25}$$

At arbitrary $\epsilon$ the integration in Eq. (4.25) gives

$$\mathcal{J}_2 = \frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)} \left( (-q_1^2)^{1+\epsilon} + (-q_1'^2)^{1+\epsilon} - (-q^2)^{1+\epsilon} \right)$$
$$\times \left( \frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)} - \psi(1+\epsilon) + \psi(1+2\epsilon) \right) - \frac{q^2}{2} I_+(q_1, q_1') , \tag{4.26}$$

where

$$I_+(q_1, q_1') = -\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{(q_1-k_1)(q_1'-k_1)}{(q_1-k_1)^2(q_1'-k_1)^2} \ln\left( \frac{(q_1-k_1)^2(q_1'-k_1)^2}{(k_1^2)^2} \right) . \tag{4.27}$$

This last integral cannot be expressed in terms of elementary functions at arbitrary $\epsilon$. In the limit $\epsilon \to 0$, instead, at fixed nonzero $q$ it becomes

$$I_+(q_1, q'_1) = -\ln\left(\frac{q_1^2}{q^2}\right)\ln\left(\frac{{q'_1}^2}{q^2}\right) . \tag{4.28}$$

Note that for $I_+(q_1, q'_1)$ the limits $\epsilon \to 0$ and $q \to 0$ are noninterchangeable. However, it does not matter for $\mathcal{J}_2$, where $I_+(q_1, q'_1)$ enters with the coefficient $\sim q^2$. At $\epsilon \to 0$ we obtain

$$\mathcal{J}_2 = (q_1^2 + {q'_1}^2 - q^2)\left(-\frac{11}{6\epsilon} + \frac{67}{18} - \zeta(2)\right) - \frac{q^2}{2}\ln\left(\frac{q_1^2}{q^2}\right)\ln\left(\frac{{q'_1}^2}{q^2}\right)$$

$$+\frac{11}{6}\left(q_1^2\ln(-q_1^2) + {q'_1}^2\ln(-{q'_1}^2) - q^2\ln(-q^2)\right) . \tag{4.29}$$

Appearance of the pole at $\epsilon = 0$ in $\mathcal{J}_2$ does not contradict the statement that non-integrable infrared singularities in the limit $\epsilon \to 0$ are absent in $(A_i/[x(1-x)])_+$. The pole term comes from the ultraviolet region. Indeed, one can see from the expression (4.12) for $A_2$ that, after averaging over the azimuthal angle at $D = 4$ in the limit of large $|k_1|$, we have

$$\left(\frac{\overline{A}_2}{x(1-x)}\right)_+ \simeq \frac{q_1 q'_1}{k_1^2}(x(1-x) - 2) . \tag{4.30}$$

The pole term in Eq. (4.29) is just the doubled result of the integration of the expression (4.30) over $dxd^{2+2\epsilon}k_1/\pi$. Evidently, the ultraviolet divergency is artificial and appears to be the result of the separation of $F_s(k_1, k_2)$ as shown in Eq. (4.9). Indeed, as it can be easily proved from formulas (4.12) and (4.13), the terms leading to such divergencies cancel in the sum $A_2 + A_3 + (k_1 \leftrightarrow k_2)$. This means that the pole term in $\mathcal{J}_2$ is cancelled by an analogous term in $\mathcal{J}_3$ (see below).

## 4.3 Calculation of $\mathcal{J}_3$

The integral $\mathcal{J}_3$ is much more complicated than the preceding ones. As a consequence, the trick of integrating first over $x$, applied before, cannot be used in the calculation of $\mathcal{J}_3$, because it leads to terms with the denominators containing a third power of $k_1$. Such terms cannot be integrated over $k_1$ by known methods. This complexity is connected with non-planarity of diagrams represented by $\mathcal{J}_3$, which is seen from the denominator $\tilde{t}_1\tilde{t}_2$ related to the cross-box diagram. The complexity of contributions of the cross-box diagrams is well known since the calculation of the non-forward kernel for the QED Pomeron [17] which was found only in the form of two-dimensional integral. In QCD the situation is greatly worse because of the existence of cross-pentagon and cross-hexagon diagrams in addition to QED-type cross-box diagrams. It requires the use of additional Feynman parameters. At arbitrary $D$ no integration over these parameters at all can be done in elementary functions. It occurs, however, that in the limit $\epsilon \to 0$ the integration over additional Feynman parameters can be performed, so that the result can be written as two-dimensional integral, as well as in QED. Details of the calculation are given in Appendix D. The result is

$$\mathcal{J}_3 = \frac{11}{12}(q_1^2 + {q'_1}^2 - q^2)\left(\frac{1}{\epsilon} + \ln(-k^2) + 1\right) + J(q_1, q_2; q) + (q_i \leftrightarrow q'_i)$$

with

$$J(q_1, q_2; q) = \int_0^1 dx \int_0^1 dz \left\{ q_1 q'_1 \left( (2 - x_1 x_2) \ln\left(\frac{Q^2}{-k^2}\right) - \frac{2}{x_1} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)\right.$$
$$+\frac{1}{Q^2}\left(\frac{x_1 x_2}{2}(q_1^2 - 2q_1 r_1)(q_1'^2 - 2q'_1 r_2) + q_1'^2 q_1 (r_1 - 2q'_1) + 4x_1 q_1^2 (q'_1 r_2)\right.$$
$$+q'_1 q_1 (q'_1 q_1 - q'_1 r_1 - q_1 r_2) + 2(q'_1 r_1)(q_1 r_2) - 2(q'_1 r_2)(q_1 r_1) + \frac{2}{x_1}\left(q_1'^2 q_1 r_2\right.$$
$$\left.\left. - x_2 q_1 q'_1 \left((q'_1 - r_2) r_1\right)\right)\right) - \frac{2}{Q_0^2 x_1}\left(z(1-z) q_2'^2 q_1 q'_1 + q_1'^2 (z q_1 k + (1-z) q_1 q'_1)\right)$$
$$+q_1'^2 \left[\left(q_1(x_2 q'_1 + q_2) - \frac{x_2}{x_1} q_1 (q'_1 + k)\right)\frac{1}{r_2^2}\ln\left(\frac{Q^2}{\mu_2^2}\right) + \frac{1}{x_1} q_1 (q'_1 + k)\frac{1}{r_0^2}\ln\left(\frac{Q_0^2}{\mu_0^2}\right)\right)$$
$$-\frac{1}{\mu_2^2 Q^2}\left(2\frac{x_2}{x_1}(q_1 r_2) q'_1 k + x_2 (q'_1 r_2)(q_2^2 - k^2) + 2(q_2 r_2) q_1 q\right) + \frac{2}{\mu_0^2 Q_0^2}\frac{1}{x_1}(q_1 r_0) q'_1 k$$
$$+\frac{1}{r_2^2}\left(\frac{1}{r_2^2}\ln\left(\frac{Q^2}{\mu_2^2}\right) - \frac{1}{Q^2}\right)\left(2\frac{x_2}{x_1}(q_1 r_2)(q'_1 + k) r_2 - 2((x_2 q'_1 + q_2) r_2) q_1 r_2\right)$$
$$-\frac{1}{r_0^2}\left(\frac{1}{r_0^2}\ln\left(\frac{Q_0^2}{\mu_0^2}\right) - \frac{1}{Q_0^2}\right)\left(2\frac{1}{x_1}(q_1 r_0)(q'_1 + k) r_0\right)$$
$$-\frac{q_1^2}{d}\left((q_2 k)(q'_2 k)\left(\frac{1}{k^2} + \frac{Q^2}{d}\mathcal{L}\right) + (q_2 r_2)(q'_2 k)\left(\frac{1}{\mu_2^2} - \frac{\mu_1^2}{d}\mathcal{L}\right) + (q_2 k)(q'_2 r_1)\right.$$
$$\left.\left.\times\left(\frac{1}{\mu_1^2} - \frac{\mu_2^2}{d}\mathcal{L}\right) - (q_2 r_2)(q'_2 r_1)\left(\frac{1}{Q^2} + \frac{k^2}{d}\mathcal{L}\right) - \frac{(q_2 q'_2)}{2}\mathcal{L}\right)\right]\right\}. \qquad (4.31)$$

Here

$$r_1 = zxq_1 + (1-z)(xk - (1-x)q_2'), \quad r_2 = z((1-x)k - xq_2) + (1-z)(1-x)q_1';$$

$$Q^2 = -x(1-x)(q_1^2 z + q_1'^2(1-z)) - z(1-z)(q_2^2 x + q_2'^2(1-x) - q^2 x(1-x)),$$

$$\mu_i^2 = Q^2 - r_i^2, \; r_0 = zk + (1-z)q_1', \; Q_0^2 = -z(1-z)q_2'^2, \; \mu_0^2 = -zk^2 - (1-z)q_1'^2,$$

$$d = \mu_1^2\mu_2^2 + k^2Q^2 = z(1-z)x(1-x)\left((k^2 - q_1^2 - q_2'^2)(k^2 - q_1'^2 - q_2^2) + k^2q^2\right)$$

$$+ q_1^2 q_2^2 xz(x+z-1) + q_1'^2 q_2'^2 (1-x)(1-z)(1-x-z), \quad \mathcal{L} = \ln\left(\frac{\mu_1^2\mu_2^2}{-k^2Q^2}\right). \quad (4.32)$$

# 5 Non-forward kernel

In pure gluodynamics, which is considered here, the part $\mathcal{K}_r^{(\mathcal{R})}$ of the BFKL kernel (2.2), related to the production of real particles, for any representation $\mathcal{R}$ is given by the sum of one-gluon $\mathcal{K}_G^{(\mathcal{R})}$ and two-gluon $\mathcal{K}_{GG}^{(\mathcal{R})}$ contributions. Using for the last of them the decomposition (3.46) we have

$$\mathcal{K}_r^{(\mathcal{R})} = \mathcal{K}_G^{(\mathcal{R})} + 4(a_R - b_R)\mathcal{K}_{GG}^{(8)} + b_R\mathcal{K}_{GG}^{(s)} \;, \quad (5.1)$$

where the colour group coefficients $a_R$ and $b_R$ are defined in Eqs. (3.25) and the one-gluon contribution $\mathcal{K}_G^{(\mathcal{R})}$ is given by Eq. (2.12) (for arbitrary $D$ see Eq. (4.10) in the last of Refs. [8]). The two-gluon contribution for the octet channel $\mathcal{K}_{GG}^{(8)}$ was calculated in the second of Refs. [10] (see there Eqs. (61) and (63) for arbitrary $D$ and for $D = 4$, respectively). The calculation of the "symmetric" contribution $\mathcal{K}_{GG}^{(s)}$ performed in this paper solves the problem of finding the expression of the non-forward BFKL kernel for all possible colour states in the $t$-channel. This contribution is determined by Eq. (4.14), where $\mathcal{J}_0 = 0$ and $\mathcal{J}_i$, for $i = 1 \div 3$, are given by Eqs. (4.21), (4.26), (D.8) and (4.22), (4.29), (4.31) for arbitrary $D$ and for $D = 4$ correspondingly. Note that everywhere in these formulas the bare coupling constant $g$ is used. The transition to the renormalized coupling $g_\mu$ in the $\overline{MS}$ scheme takes place by means of Eq. (2.5).

For the most important colour singlet case, using $c_1 = a_1 = 1$ and $c_8 = b_1 = 1/2$, from Eqs. (5.1) and (2.12) we obtain

$$\mathcal{K}_r^{(1)} = 2\mathcal{K}_r^{(8)} + \frac{1}{2}\mathcal{K}_{GG}^{(s)} \;. \quad (5.2)$$

Because of the significance of this case let us consider it in more detail. We present the kernel $\mathcal{K}_r^{(1)}$ obtained from the above stated sources in the limit $D = 4 + 2\epsilon \longrightarrow 4$ as sum of two parts,

$$\mathcal{K}_r^{(1)} = \mathcal{K}_r^{sing} + \mathcal{K}^{reg} \ , \tag{5.3}$$

where the first, given by

$$\mathcal{K}_r^{sing}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{2\bar{g}_\mu^2 \mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \left( \frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\,2} + \vec{q}_1^{\,\prime\,2} \vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2} \right) \left\{ 1 + \bar{g}_\mu^2 \left[ \frac{11}{3\epsilon} \right.\right.$$

$$\left.\left. + \left( \frac{\vec{k}^{\,2}}{\mu^2} \right)^\epsilon \left\{ -\frac{11}{3\epsilon} + \frac{67}{9} - 2\zeta(2) + \epsilon \left( -\frac{404}{27} + 14\zeta(3) + \frac{11}{3}\zeta(2) \right) \right\} \right] \right\} \ , \tag{5.4}$$

contains all singularities and the second, putting $\epsilon = 0$ and $\bar{g}_\mu^2 = \alpha_s(\mu^2) N_c/(4\pi)$ , becomes

$$\mathcal{K}_r^{reg}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{\alpha_s^2(\mu^2) N_c^2}{16\pi^3} \left[ 2(\vec{q}_1^{\,2} + \vec{q}_2^{\,2} - \vec{q}^{\,2}) \left( \zeta(2) - \frac{50}{9} \right) - \frac{11}{3} \left( \vec{q}_1^{\,2} \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}} \right) \right.\right.$$

$$+ \vec{q}_2^{\,2} \ln\left( \frac{\vec{q}_2^{\,2}}{\vec{k}^{\,2}} \right) - \vec{q}^{\,2} \ln\left( \frac{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}{\vec{k}^{\,4}} \right) - \frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\,2} - \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}} \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \bigg) + \vec{q}^{\,2} \left( \frac{1}{2} \ln^2\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \right.$$

$$\left. + \ln\left( \frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}} \right) \ln\left( \frac{\vec{q}_2^{\,\prime 2}}{\vec{q}^{\,2}} \right) + \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}} \right) \ln\left( \frac{\vec{q}_1^{\,\prime 2}}{\vec{q}^{\,2}} \right) \right) + \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \left( \frac{\vec{q}_1^{\,\prime 2}}{2} \ln\left( \frac{\vec{q}_2^{\,2}}{\vec{k}^{\,2}} \right) \right.$$

$$- \frac{\vec{q}_2^{\,\prime 2}}{2} \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}} \right) - \frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\,2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\,2}}{2\vec{k}^{\,2}} \ln\left( \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) + \frac{\vec{q}_1^{\,\prime\,2}(\vec{q}_1^{\,2} - 3\vec{q}_2^{\,2})}{2\vec{k}^{\,2}} \ln\left( \frac{\vec{k}^{\,2}}{\vec{q}_2^{\,2}} \right)$$

$$\left. + \frac{\vec{q}_2^{\,\prime\,2}(3\vec{q}_1^{\,2} - \vec{q}_2^{\,2})}{2\vec{k}^{\,2}} \ln\left( \frac{\vec{k}^{\,2}}{\vec{q}_1^{\,2}} \right) \right) + \left( \vec{q}^{\,2}(\vec{k}^{\,2} - \vec{q}_1^{\,2} - \vec{q}_2^{\,2}) + 2\vec{q}_1^{\,2} \vec{q}_2^{\,2} + \vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2} + \vec{q}_2^{\,2} \vec{q}_2^{\,\prime\,2} \right.$$

$$\left. - \frac{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})(\vec{q}_1^{\,2} + \vec{q}_2^{\,2})(\vec{q}_1^{\,\prime\,2} - \vec{q}_2^{\,\prime\,2})}{2\vec{k}^{\,2}} - \frac{\vec{k}^{\,2}}{2}(\vec{q}_1^{\,\prime\,2} + \vec{q}_2^{\,\prime\,2}) \right) I(\vec{k}^{\,2}, \vec{q}_2^{\,2}, \vec{q}_1^{\,2})$$

$$\left. - 2J(\vec{q}_1, \vec{q}_2; \vec{q}) - 2J(-\vec{q}_2, -\vec{q}_1; -\vec{q}) \right] + \left\{ \vec{q}_i \longleftrightarrow \vec{q}_i^{\,\prime} \right\} . \tag{5.5}$$

Here the functions $I(k^2, q_2^2, q_1^2)$ and $J(q_1, q_2; q)$ are defined in Eqs. (4.23) and (4.31) correspondingly.

All singularities of $\mathcal{K}_r^{(1)}$ are contained in the first part. We remind that $\mathcal{K}_G^{(R)}$ and $\mathcal{K}_{GG}^{(R)}$ separately contain first and second order poles at $\epsilon = 0$ (see Eq. (2.12)). In the sum of these contributions the pole terms cancel, so that at fixed nonzero $\vec{k}^2$, when the term $\left(\vec{k}^{\,2}/\mu^2\right)^\epsilon$ in Eq. (5.4) can be expanded in $\epsilon$, the sum is finite at $\epsilon = 0$. But the kernel (5.4) is singular at $\vec{k}^{\,2} = 0$ so that, when it is integrated over $q_2$, the region of so small $\vec{k}^{\,2}$ values such that $\epsilon|\ln\left(\vec{k}^{\,2}/\mu^2\right)| \sim 1$ does contribute. Therefore the expansion of $\left(\vec{k}^{\,2}/\mu^2\right)^\epsilon$ is not done in Eq. (5.4). Moreover, the terms $\sim \epsilon$ are taken into account in the coefficient of the expression divergent at $\vec{k}^{\,2} = 0$ in order to save all contributions non-vanishing in the limit $\epsilon \to 0$ after the integration.

As it was already discussed, the "symmetric" part $\mathcal{K}_{GG}^{(s)}$ of the kernel (5.1) is finite in the limit $\epsilon = 0$. Moreover, it does not give singularities at $\epsilon = 0$ when the kernel is used in the equation for the Green's function. Indeed, the points $\vec{q}_2 = 0$ and $\vec{q}_2^{\,\prime} = 0$ do not give such singularities due to the "gauge invariance" properties (2.13), because these properties are valid for any representation. Its validity for $\mathcal{K}_{GG}^{(s)}$ can be checked explicitly using the properties of $b^{\alpha\beta}$. It is not difficult also to see that $\mathcal{K}_{GG}^{(s)}$ has not non-integrable singularities in the limit $\epsilon = 0$ at $\vec{k} = 0$.

For the singlet case the infrared singularities of $\mathcal{K}_r^{(1)}$ must be cancelled by the singularities of the gluon trajectory after integration of the total kernel with any nonsingular at $\vec{k} = 0$ function. The total BFKL kernel in the singlet case must be free from singularities. It is not difficult to see that it is the case, using the equality

$$\omega(t) = -2\bar{g}_\mu^2\left(\frac{1}{\epsilon} + \ln\left(\frac{-t}{\mu^2}\right)\right) - \bar{g}_\mu^4\left[\frac{11}{3}\left(\frac{1}{\epsilon^2} - \ln^2\left(\frac{-t}{\mu^2}\right)\right) + \left(\frac{67}{9} - 2\zeta(2)\right)\right.$$
$$\left.\times\left(\frac{1}{\epsilon} + 2\ln\left(\frac{-t}{\mu^2}\right)\right) - \frac{404}{27} + 2\zeta(3)\right]\ . \qquad (5.6)$$

It is convenient to represent the total kernel in such a form that the cancellation of singularities between real and virtual contributions becomes evident. For this purpose let us first switch from the dimensional regularization to the cut-off $\vec{k}^2 > \lambda^2$, $\lambda \to 0$, which is more convenient for practical purposes. With such regularization we can pass to the limit $\epsilon \to 0$ in the real part of the kernel, so that its singular part assumes the form

$$\mathcal{K}_r^{sing}(\vec{q}_1, \vec{q}_2; \vec{q}) \;\to\; \theta((\vec{q}_1 - \vec{q}_2)^2 - \lambda^2)\mathcal{K}_r^{sing}(\vec{q}_1, \vec{q}_2; \vec{q})|_{\epsilon=0}$$

$$= \frac{\alpha_s(\mu^2)N_c}{2\pi^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}}-\vec{q}^{\,2}\right)$$

$$\times\left\{1-\frac{\alpha_s(\mu)N_c}{4\pi}\left(\frac{11}{3}\ln\left(\frac{\vec{k}^{\,2}}{\mu^2}\right)-\frac{67}{9}+2\zeta(2)\right)\right\}\theta((\vec{q}_1-\vec{q}_2)^2-\lambda^2)\ . \quad (5.7)$$

The trajectory must be transformed in such a way that the cut-off regularization gives the same result as the $\epsilon$ regularization:

$$\omega(t)\ \rightarrow\ \omega_\lambda(t)=\lim_{\epsilon\to 0}\left(\omega(t)+\frac{1}{2}\int\frac{d^{2+\epsilon}q_2}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\mathcal{K}_r^{(1)}(\vec{q}_1,\vec{q}_2;\vec{q})\theta((\vec{q}_1-\vec{q}_2)^2-\lambda^2)\right)$$

$$=-\frac{\alpha_s(\mu^2)N_c}{2\pi}\left\{\ln\left(\frac{-t}{\lambda^2}\right)-\frac{\alpha_s(\mu^2)N_c}{4\pi}\left[\frac{11}{6}\left(\ln^2\left(\frac{-t}{\mu^2}\right)\right.\right.\right.$$

$$\left.\left.\left.-\ln^2\left(\frac{\lambda^2}{\mu^2}\right)\right)-\left(\frac{67}{9}-\frac{\pi^2}{3}\right)\ln\left(\frac{-t}{\lambda^2}\right)+6\zeta(3)\right]\right\}\ . \quad (5.8)$$

It is easy to check that the integral over $d^2q_2$ of the total kernel (2.2) with any function non-singular at $\vec{k}=0$ is $\lambda$ -independent in the limit $\lambda\to 0$. Moreover, it is equally easy to find a form of the kernel which does not contain $\lambda$ . It is sufficient to find a representation

$$\omega_\lambda(-\vec{q}_1^{\,2})=\int d^2q_2 f_\omega(\vec{q}_1,\vec{q}_2)\theta((\vec{q}_1-\vec{q}_2)^2-\lambda^2) \quad (5.9)$$

with such a function $f_\omega$ that the singularity non-integrable at $\vec{k}=\vec{q}_1-\vec{q}_2=\vec{q}_1^{\,\prime}-\vec{q}_2^{\,\prime}=0$ is cancelled in the “regularized virtual kernel"

$$\mathcal{K}_v^{reg}(\vec{q}_1,\vec{q}_2;\vec{q})=f_\omega(\vec{q}_1,\vec{q}_2)+f_\omega(\vec{q}_1^{\,\prime},\vec{q}_2^{\,\prime})+\frac{\mathcal{K}_r^{sing}(\vec{q}_1,\vec{q}_2;\vec{q})|_{\epsilon=0}}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\ . \quad (5.10)$$

After that we can proceed to the limit $\lambda=0$, obtaining

$$\left(\hat{\mathcal{K}}^{(1)}\Psi\right)(\vec{q}_1)=\int d^2q_2\left\{\mathcal{K}_v^{reg}(\vec{q}_1,\vec{q}_2;\vec{q})\Psi(\vec{q}_1)\right.$$

$$\left.+\frac{\mathcal{K}_r^{sing}(\vec{q}_1,\vec{q}_2;\vec{q})|_{\epsilon=0}}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\left(\Psi(\vec{q}_2)-\Psi(\vec{q}_1)\right)+\frac{\mathcal{K}_r^{reg}(\vec{q}_1,\vec{q}_2;\vec{q})}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\Psi(\vec{q}_2)\right\}\ . \quad (5.11)$$

Of course, the choice of the function $f_\omega$ contains a large arbitrariness. One simple choice is

$$f_\omega(\vec{q}_1,\vec{q}_2)=-\frac{\alpha_s(\mu^2)N_c}{2\pi^2}\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}(\vec{q}_1^{\,2}+\vec{k}^{\,2})}\left\{1-\frac{\alpha_s(\mu)N_c}{4\pi}\left(\frac{11}{3}\ln\left(\frac{\vec{k}^{\,2}}{\mu^2}\right)\right.\right.$$

$$-\frac{67}{9}+2\zeta(2)+\left(6\zeta(3)-\frac{11}{3}\zeta(2)\right)\frac{\vec{k}^{\,2}}{(\vec{q_1}^{\;2}+\vec{k}^{\,2})}\Bigg)\Bigg\}\ . \tag{5.12}$$

We have to say that the integral (4.31) for $J(q_1,q_2;q)$ entering into all kernels besides the octet ones definitely is presented not in the best form. We have decided to present it in such shape in order to give a possibility of further development to people interested in this subject. The results of our efforts on simplification of the kernel and investigation of its properties will be presented in a subsequent paper.

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# Appendix A

For the colour group $SU(N_c)$ with $N_c=3$ the possible representations $\mathcal{R}$ are $\underline{1}, \underline{8_a}, \underline{8_s}, \underline{10}, \underline{\overline{10}}, \underline{27}$. Corresponding projection operators are

$$\langle bb'|\hat{\mathcal{P}}_1|aa'\rangle=\frac{\delta_{bb'}\delta_{aa'}}{N_c^2-1}\ , \tag{A.1}$$

$$\langle bb'|\hat{\mathcal{P}}_{8_a}|aa'\rangle=\frac{f_{bb'c}f_{aa'c}}{N_c}\ , \tag{A.2}$$

$$\langle bb'|\hat{\mathcal{P}}_{8_s}|aa'\rangle=d_{bb'c}d_{aa'c}\frac{N_c}{N_c^2-4}\ , \tag{A.3}$$

$$\langle bb'|\hat{\mathcal{P}}_{10}|aa'\rangle=\frac{1}{4}\left[\delta_{ba}\delta_{b'a'}-\delta_{ba'}\delta_{b'a}-\frac{2}{N_c}f_{bb'c}f_{aa'c}+if_{ba'c}d_{b'ac}+id_{ba'c}f_{b'ac}\right]\ , \tag{A.4}$$

$$\langle bb'|\hat{\mathcal{P}}_{\overline{10}}|aa'\rangle=\frac{1}{4}\left[\delta_{ba}\delta_{b'a'}-\delta_{ba'}\delta_{b'a}-\frac{2}{N_c}f_{bb'c}f_{aa'c}-if_{ba'c}d_{b'ac}-id_{ba'c}f_{b'ac}\right]\ , \tag{A.5}$$

$$\langle bb'|\hat{\mathcal{P}}_{27}|aa'\rangle=\frac{1}{4}\left[\left(1+\frac{2}{N_c}\right)(\delta_{ba}\delta_{b'a'}+\delta_{ba'}\delta_{b'a})-\frac{2(N_c+2)}{N_c(N_c+1)}\delta_{bb'}\delta_{aa'}\right.$$

$$- \left(1 + \frac{2}{N_c + 2}\right) d_{bb'c}d_{aa'c} + d_{bac}d_{b'a'c} + d_{b'ac}d_{ba'c}\Bigg] \ . \tag{A.6}$$

Here $f_{abc}$ and $d_{abc}$ are defined by the relation

$$t^a t^b = (d_{abc} + i f_{abc})\frac{t^c}{2} + \delta_{ab}\frac{I}{2N_c}, \tag{A.7}$$

where $t^a$ are the group generators in the fundamental representation, normalized by the requirement $tr(t^a t^b) = \delta_{ab}/2$ and $I$ is the identity matrix.

For generality, we do not put here $N_c = 3$, so that above expressions are valid for the $SU(N_c)$ group with arbitrary $N_c$. Corresponding representations in this case have dimensions

$$n_1 = 1 \ , \quad n_{8_a} = n_{8_s} = N_c^2 - 1 \ , \quad n_{10} = n_{\overline{10}} = \frac{(N_c^2 - 4)(N_c^2 - 1)}{4} \ ,$$

$$n_{27} = \frac{(N_c + 3)N_c^2(N_c - 1)}{4} \ . \tag{A.8}$$

However, at $N_c > 3$ there is an additional representation with dimension

$$n_{N_c>3} = \frac{(N_c + 1)N_c^2(N_c - 3)}{4} \tag{A.9}$$

and projection operator

$$\langle bb'|\hat{\mathcal{P}}_{N_c>3}|aa'\rangle = \frac{1}{4}\Bigg[\left(1 - \frac{2}{N_c}\right)(\delta_{ba}\delta_{b'a'} + \delta_{ba'}\delta_{b'a}) + \frac{2(N_c - 2)}{N_c(N_c - 1)}\delta_{bb'}\delta_{aa'}$$

$$+ \left(1 - \frac{2}{N_c - 2}\right) d_{bb'c}d_{aa'c} - d_{bac}d_{b'a'c} - d_{ba'c}d_{b'ac}\Bigg] \ . \tag{A.10}$$

In $SU(3)$ this projection operator turns into zero due to the equality

$$d_{bb'c}d_{aa'c} + d_{ba'c}d_{b'ac} + d_{bac}d_{a'b'c} = \frac{1}{3}(\delta_{bb'}\delta_{aa'} + \delta_{ba'}\delta_{b'a} + \delta_{ba}\delta_{a'b'}) \ , \tag{A.11}$$

which holds at $N_c = 3$. The following useful identities with

$$T^a_{bc} = -i f_{abc}, \ \ D^a_{bc} = d_{abc}, \ \ \left[F^a, F^b\right] = i f_{abc}F^c, \ \ \left[F^a, D^b\right] = i f_{abc}D^c \tag{A.12}$$

are valid at arbitrary $N_c$:

$$Tr\left(T^a\right) = Tr\left(D^a\right) = Tr\left(T^a D^b\right) = 0, \ \ Tr\left(T^a T^b\right) = N_c\delta^{ab},$$

$$Tr\left(D^aD^b\right) = \frac{N_c^2-4}{N_c}\delta^{ab},\ Tr\left(T^aT^bT^c\right) = i\frac{N_c}{2}f_{abc},\ Tr\left(T^aT^bD^c\right) = \frac{N_c}{2}d_{abc},$$

$$Tr\left(D^aD^bT^c\right) = i\frac{N_c^2-4}{2N_c}f_{abc},\ Tr\left(D^aD^bD^c\right) = \frac{N_c^2-12}{2N_c}d_{abc},$$

$$Tr\left(T^aT^bT^cT^d\right) = \delta_{ad}\delta_{bc} + \frac{1}{2}\left(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}\right) + \frac{N_c}{4}\left(f_{adi}f_{bci}+d_{adi}d_{bci}\right),$$

$$Tr\left(T^aT^bT^cD^d\right) = i\frac{N_c}{4}\left(d_{adi}f_{bci}-f_{adi}d_{bci}\right),$$

$$Tr\left(T^aT^bD^cD^d\right) = \frac{1}{2}\left(\delta_{ab}\delta_{cd}-\delta_{ac}\delta_{bd}\right) + \frac{N_c^2-8}{4N_c}f_{adi}f_{bci} + \frac{N_c}{4}d_{adi}d_{bci},$$

$$Tr\left(T^aD^bT^cD^d\right) = -\frac{1}{2}\left(\delta_{ab}\delta_{cd}-\delta_{ac}\delta_{bd}\right) + \frac{N_c}{4}\left(f_{adi}f_{bci}+d_{adi}d_{bci}\right),$$

$$Tr\left(T^aD^bD^cD^d\right) = i\frac{2}{N_c}f_{adi}d_{bci} + i\frac{N_c^2-8}{4N_c}f_{abi}d_{cdi} + i\frac{N_c}{2}d_{abi}f_{cdi},$$

$$Tr\left(D^aD^bD^cD^d\right) = \frac{N_c^2-4}{N_c^2}\delta_{ad}\delta_{bc} + \frac{1}{2}\delta_{ac}\delta_{bd} + \frac{N_c^2-8}{2N_c^2}\delta_{ab}\delta_{cd}$$

$$+\frac{N_c}{4}f_{adi}f_{bci} + \frac{N_c^2-16}{4N_c}d_{adi}d_{bci} - \frac{4}{N_c}d_{abi}d_{cdi},$$

$$f_{adi}f_{bci}+d_{adi}d_{bci}-f_{abi}f_{cdi}-d_{abi}d_{cdi}+\frac{2}{N_c}\left(\delta_{ad}\delta_{bc}-\delta_{ab}\delta_{cd}\right)=0. \quad \text{(A.13)}$$

These equalities (A.13) can be derived from the relation (A.7) and the completeness of the matrices $t^a$ and $I$. The properties of the projection operators

$$\hat{\mathcal{P}}_i\hat{\mathcal{P}}_j = \delta_{ij}\hat{\mathcal{P}}_i, \quad \sum_i \hat{\mathcal{P}}_i = I \quad \text{(A.14)}$$

can be easily obtained with the help of these equalities, as well as the coefficients $c_{\mathcal{R}}$:

$$c_1 = 1\,,\ \ c_{8_a} = c_{8_a} = \frac{1}{2}\,,\ c_{10} = c_{\overline{10}} = 0\,,\ c_{27} = -c_{N_c>3} = -\frac{1}{4N_c}\ . \quad \text{(A.15)}$$

# Appendix B

Here and below all vectors are taken transverse $(D-2)$-dimensional, although the transversality sign $\perp$ is omitted. If the vector sign is not used, the Minkowski metric is assumed, so that $(ab) = -\vec{a}\vec{b}$. We use a standard representation of logarithms, i.e.

$$\ln a = \frac{d}{d\nu} a^{\nu}|_{\nu=0} \tag{B.1}$$

and the Feynman parametrization

$$\prod_{i=1}^{n} a_i^{-\alpha_i} = \frac{\Gamma\left(\sum\limits_{i=1}^{n} \alpha_i\right)}{\prod\limits_{i=1}^{n} \Gamma(\alpha_i)} \left(\prod_{i=1}^{n} \int_0^1 dx_i x_i^{\alpha_i - 1}\right) \frac{\delta(1 - \sum\limits_{i=1}^{n} x_i)}{\left(\sum\limits_{i=1}^{n} a_i x_i\right)^{\sum\limits_{i=1}^{n} \alpha_i}} . \tag{B.2}$$

Using the notations

$$R = (k_2 - y k_x)^2 - y\,(a_x - y b_x)\ , \quad k_x = k(1-x) - q_2 x = q_1(1-x) - q_2 = k - q_1 x\ ,$$
$$a_x = -\left(k^2(1-x) + q_2^2 x\right)\ , \quad b_x = -k_x^2 = a_x + q_1^2 x(1-x)\ ,$$
$$\mathcal{J}_1 = \frac{q_1'^2}{2} J_1 + (q_i \leftrightarrow q_i')\ , \tag{B.3}$$

from Eq. (4.20) we obtain

$$J_1 = \frac{\partial}{\partial \nu} \int \frac{d^{2+2\epsilon} k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{((q_1 - k_1)^2)^{\nu}}{(k_1^2)^{\nu} k_2^2} \left(\frac{q_1^2 q_2^2}{k_1^2 (q_1 - k_1)^2} - \frac{q_2^2}{(q_1 - k_1)^2}\right.$$
$$\left. - \frac{q_2^2 + 2k_1 q_2}{k_1^2}\right)|_{\nu=0} = \frac{\partial}{\partial \nu} \int_0^1 dy \int_0^1 \frac{dx (1-x)^{\nu} x^{-\nu}}{\Gamma(1+\nu)\Gamma(1-\nu)} \int \frac{d^{2+2\epsilon} k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}$$
$$\times \left(\frac{2y q_1^2 q_2^2}{R^3} - \frac{q_2^2 \nu}{(1-x)R^2} + \frac{(q_1^2 - k^2 - 2k_2 q_2)\nu}{x R^2}\right)|_{\nu=0}$$
$$= \frac{\partial}{\partial \nu} \int_0^1 \frac{dx (1-x)^{\nu} x^{-\nu}}{\Gamma(1+\nu)\Gamma(1-\nu)} \int_0^1 \frac{dy y^{\epsilon-1}}{(a_x - y b_x)^{1-\epsilon}}$$
$$\times \left[\frac{-(1-\epsilon) q_1^2 q_2^2}{(a_x - y b_x)} - \frac{\nu q_2^2}{(1-x)} + \frac{\nu}{x}(q_1^2 - k^2 - 2y k_x q_2)\right]|_{\nu=0}$$

$$= \int_0^1 dx \int_0^1 dy y^{\epsilon-1} \left[ \frac{(1-\epsilon)q_1^2 q_2^2}{(a_x - y b_x)^{2-\epsilon}} \ln\left(\frac{x}{1-x}\right) \right.$$

$$\left. - \ln(1-x)\frac{d}{dx}\frac{q_2^2}{(a_x - y b_x)^{1-\epsilon}} - \ln x \frac{d}{dx}\frac{q_1^2 - k^2 - 2y k_x q_2}{(a_x - y b_x)^{1-\epsilon}} \right] . \qquad \text{(B.4)}$$

Subsequent integrations can be performed only in the limit $\epsilon \to 0$. Note that proceeding to the limit $\epsilon \to 0$ in the integrand leads to a wrong, although convergent, integral. The singularity at $y = 0$ requires an accurate consideration. After integration over $y$ we obtain

$$J_1 = \int_0^1 dx \left[ \frac{q_1^2 q_2^2}{a_x^2} \ln\left(\frac{x}{1-x}\right) \left( \ln\left(\frac{a_x}{a_x - b_x}\right) + \frac{b_x}{a_x - b_x} \right) \right.$$

$$- \ln(1-x)\frac{d}{dx}\frac{q_2^2}{a_x} \ln\left(\frac{a_x}{a_x - b_x}\right) - \ln x \frac{d}{dx}\left(\frac{q_1^2 - k^2}{a_x} - \frac{2k_x q_2}{b_x}\right) \ln\left(\frac{a_x}{a_x - b_x}\right)$$

$$+ \frac{1}{a_x^2}\left( q_1^2 q_2^2 \ln\left(\frac{x}{1-x}\right) + q_2^2(k^2 - q_2^2)\ln(1-x) \right.$$

$$\left. \left. + (q_1^2 - k^2)(k^2 - q_2^2)\ln x \right) \ln a_x \right] . \qquad \text{(B.5)}$$

Last terms come from the singularity at $y = 0$. Performing appropriate integrations by parts, after simple though slightly tedious steps we arrive at

$$J_1 = \int_0^1 dx \left( \frac{1}{1-x}\left(\frac{q_2^2 - q_1^2}{k^2} + 1\right) \ln\left(\frac{a_x}{-q_2^2}\right) \right.$$

$$\left. + \frac{1}{k^2 b_x}\left( q_1^2(q_1^2 - q_2^2 - k^2)x + k^2(q_1^2 + q_2^2) - (q_1^2 + q_2^2)^2 \right) \ln\left(\frac{a_x}{a_x - b_x}\right) \right) . \qquad \text{(B.6)}$$

Using the equalities

$$\int_0^1 \frac{x dx}{(k(1-x) - q_2 x)^2} \ln\left(\frac{k^2(1-x) + q_2^2 x}{q_1^2 x(1-x)}\right)$$

$$= \frac{q_1^2 + k^2 - q_2^2}{2q_1^2} \int_0^1 \frac{dx}{(k(1-x) - q_2 x)^2} \ln\left(\frac{k^2(1-x) + q_2^2 x}{q_1^2 x(1-x)}\right)$$

$$- \frac{1}{2q_1^2}\left( L(1 - \frac{k^2}{q_2^2}) - L(1 - \frac{q_2^2}{k^2}) \right) - \frac{1}{4q_1^2} \ln\left(\frac{k^2}{q_2^2}\right) \ln\left(\frac{k^2 q_2^2}{q_1^4}\right) , \qquad \text{(B.7)}$$

$$L(x) = -Li_2(x) = \int_0^x \frac{dy}{y} \ln(1-y) \ , \ \ Li_2(1-x) + Li_2(1-\frac{1}{x}) = -\frac{1}{2} \ln^2 x \ , \tag{B.8}$$

we obtain

$$J_1 = \frac{4q_1^2 q_2^2 - (k^2 - q_1^2 - q_2^2)^2}{2k^2} \int_0^1 \frac{dx}{b_x} \ln\left(\frac{a_x}{a_x - b_x}\right)$$

$$+ \frac{k^2 + q_2^2 - q_1^2}{2k^2} \ln\left(\frac{k^2}{q_2^2}\right) \ln\left(\frac{q_1^2}{q_2^2}\right) \ . \tag{B.9}$$

This result can be reached by another way, using the analyticity properties of $J_1$. To do that, let us present $J_1$ at $\epsilon = 0$ as an integral in the Minkowski space:

$$J_1 = \int_0^\infty dz \int \frac{d^2 k_1}{i\pi(k_2^2 + i0)} \left(\frac{1}{z - k_1^2 - i0} - \frac{1}{z - (q_1 - k_1)^2 - i0}\right)$$

$$\times \left(\frac{q_1^2 q_2^2}{(k_1^2 + i0)((q_1 - k_1)^2 + i0)} - \frac{q_2^2}{((q_1 - k_1)^2 + i0)} - \frac{q_2^2 + 2k_1 q_2}{(k_1^2 + i0)}\right) \ . \tag{B.10}$$

Here $k_1$, $q_1$ and $q_2$ are considered as vectors in the two-dimensional Minkowski space, so that we have

$$d^2 k_1 = dk_1^{(0)} dk_1^{(1)}, \ \ k_1{}^2 = (k_1^{(0)})^2 - (k_1^{(1)})^2 \ , \tag{B.11}$$

and so on. Eq. (B.10) determines $J_1$ as a function of $q_1^2$, $q_2^2$ and $k^2$ for arbitrary values of these variables. For $q_1^2 = -\vec{q}_1^{\,2} \leq 0$, $q_2^2 = -\vec{q}_2^{\,2} \leq 0$ and $k^2 = -\vec{k}^{\,2} \leq 0$ Eq. (B.10) turns into the function given by Eq. (4.20), that can be easily seen by making the Wick rotation of the contour of integration over $k_1^{(0)}$ and performing integration over $z$. At fixed negative $q_{1,2}^2$ Eq. (B.10) determines the real analytical function of $k^2$ with the cut at $k^2 \geq 0$. According to the Cutkosky rules, one can find a discontinuity on the cut rewriting Eq. (B.10) as

$$J_1 = \int_0^\infty dz \int \frac{d^2 k_1}{i\pi(k_2^2 + i0)} \left(\left(\frac{q_1^2 q_2^2}{((q_1 - k_1)^2 + i0)} - q_2^2 - 2k_1 q_2\right)\right.$$

$$\times \left(\frac{1}{z}\left(\frac{1}{k_1^2 + i0} - \frac{1}{k_1^2 - z + i0)}\right) + \frac{1}{k_1^2 + i0} \frac{1}{(q_1 - k_1)^2 - z + i0}\right)$$

$$\left. + \frac{q_2^2}{((q_1 - k_1)^2 + i0)} \left(\frac{1}{k_1^2 - z + i0} - \frac{1}{(q_1 - k_1)^2 - z + i0}\right)\right) \ , \tag{B.12}$$

omitting the last term and making the substitutions (assuming $k^{(0)} \geq 0$)

$$\frac{1}{k_2^2+i0}\frac{1}{k_1^2+i0} \to (-2\pi i)^2\delta(k_2^2)\delta(k_1^2)\theta(k_2^{(0)})\theta(k_1^{(0)}) ,$$

$$\frac{1}{k_2^2+i0}\frac{1}{k_1^2-z+i0} \to (-2\pi i)^2\delta(k_2^2)\delta(z-k_1^2)\theta(k_2^{(0)})\theta(k_1^{(0)}) . \tag{B.13}$$

Using these rules and removing the $\delta$-functions by integration over $k_1$ (the most appropriate system for this is $k^{(1)} = 0,\ k^2 = (k^{(0)})^2$), we obtain for the imaginary part

$$\Im J_1 = \pi \int_0^\infty dz \sum_{i=\pm}\Bigg[\frac{q_1^2 q_2^2}{k^2}\left(\frac{1}{\kappa_i^0(\kappa_i^0 - z)} + \frac{1}{\kappa_i^0 z} - \frac{k^2\theta(k^2-z)}{z(k^2-z)\kappa_i}\right)$$

$$+\frac{k^2-q_1^2-q_2^2}{k^2}\left(\frac{1}{\kappa_i^0 - z} + \frac{1}{z} - \frac{k^2\theta(k^2-z)}{z(k^2-z)}\right)$$

$$+\frac{\kappa_i^0}{k^2(\kappa_i^0-z)} + \frac{\kappa_i^0}{k^2 z} + \frac{\kappa_i^0}{k^2(\kappa_i^0-z)} + \frac{\theta(k^2-z)}{(k^2-z)}\left(\frac{q_2^2}{\kappa_i} - \frac{\kappa_i}{z}\right)\Bigg] , \tag{B.14}$$

where $\kappa_\pm^0$ and $\kappa_\pm$ are given by values of $(q_1-k_1)^2$ on the mass shells $k_2^2 = 0\ ,\ \ k_1^2 = 0$ and $k_2^2 = 0\ ,\ \ k_1^2 = z$ respectively, so that

$$\kappa_\pm^0 = \frac{1}{2}\left[q_1^2 + q_2^2 - k^2 \pm \sqrt{(q_1^2+q_2^2-k^2)^2 - 4q_1^2q_2^2}\right] ,\ \ \kappa_\pm = \kappa_\pm^0 + \frac{z}{k^2}(q_2^2 - \kappa_\pm^0) . \tag{B.15}$$

The integration over $z$ is quite elementary and gives

$$\Im J_1 = -\frac{\pi}{2k^2}\Bigg[\left(k^2 - q_1^2 + q_2^2\right)\ln\frac{q_1^2}{q_2^2} + \sqrt{(k^2-q_1^2-q_2^2)^2 - 4q_1^2q_2^2}$$

$$\times \ln\left(\frac{k^2-q_1^2-q_2^2+\sqrt{(k^2-q_1^2-q_2^2)^2-4q_1^2q_2^2}}{k^2-q_1^2-q_2^2-\sqrt{(k^2-q_1^2-q_2^2)^2-4q_1^2q_2^2}}\right)\Bigg] . \tag{B.16}$$

The use of this equation and the equality (see Refs. [10] and [15] )

$$\pi\sqrt{(k^2-q_1^2-q_2^2)^2-4q_1^2q_2^2}\ \ln\left(\frac{k^2-q_1^2-q_2^2+\sqrt{(k^2-q_1^2-q_2^2)^2-4q_1^2q_2^2}}{k^2-q_1^2-q_2^2-\sqrt{(k^2-q_1^2-q_2^2)^2-4q_1^2q_2^2}}\right)$$

$$= \Im\left(\left[4q_1^2q_2^2 - (k^2-q_1^2-q_2^2)^2\right]\int_0^1 \frac{dx}{(k(1-x)-q_2x)^2}\ln\left(\frac{k^2(1-x)+q_2^2x}{q_1^2x(1-x)}\right)\right) \tag{B.17}$$

gives the result (B.9). The absence of a polynomial in $k^2$ can be easily checked by considering the integral (4.20) at large $\vec{k}^{\,2} \gg \vec{q}_1^{\,2}$. Integration regions which could contribute in this case are $|\vec{k}_1| \sim |\vec{k} - \vec{k}_1| \sim |\vec{k}|$ and $|\vec{k}_1| \sim |\vec{q}_1 - \vec{k}_1| \sim |\vec{q}_1|$. But the first region gives a vanishing contribution because of the smallness of $\ln((\vec{q}_1 - \vec{k}_1)^2/\vec{k}_1^{\,2})$ there. In the second region the integrand of Eq. (B.9) with the required accuracy is anti-symmetric with respect to the exchange $k_1 \leftrightarrow q_1 - k_1$, so that its contribution vanishes as well.

# Appendix C

We start from the integrals

$$\int_0^1 dx \left( \frac{(\tilde{t}_1 \tilde{t}_1')^{-1}}{x(1-x)} \right)_+ = \frac{L_+}{2cc'} - \frac{c+c'}{2cc'} \frac{L_-}{b-b'} \ ,$$

$$\int_0^1 dx \left( \frac{x_2 (\tilde{t}_1 \tilde{t}_1')^{-1}}{x(1-x)} \right)_+ = \frac{1}{2bb'} \left[ L_+ - \frac{b+b'}{b-b'} L_- \right] \ ,$$

$$\int_0^1 dx \left( \frac{x_2 (x_1 \tilde{t}_1 \tilde{t}_1')^{-1}}{x(1-x)} \right)_+ = \frac{L_-}{a(b-b')} \ ,$$

$$\int_0^1 dx \left( \frac{x_2^2 (x_1^2 \tilde{t}_1 \tilde{t}_1')^{-1}}{x(1-x)} \right)_+ = \frac{1}{2a^2} \left[ -L_+ - \frac{c+c'}{b-b'} L_- \right] \ ,$$

$$\int_0^1 dx \left( \frac{x_2^2 (\tilde{t}_1 \tilde{t}_1')^{-1}}{x(1-x)} \right)_+$$

$$= \frac{1}{bb'} \left[ -1 + \frac{L_+}{2bb'} (bb' + a(b+b')) - \frac{L_-}{2(b-b')} (c + c' + \frac{a(b-b')^2}{bb'}) \right] \ , \quad \text{(C.1)}$$

where

$$a = k_1^2 \ , \ \ c = (q_1 - k_1)^2 \ , \ \ c' = (q_1' - k_1)^2 \ , \ \ b = c - a, \quad b' = c' - a \ ,$$

$$L_+ = \ln \left( \frac{cc'}{a^2} \right) \ , \ L_- = \ln \left( \frac{c}{c'} \right) \ , \quad \text{(C.2)}$$

and after some algebra we arrive at Eq. (4.25). We remind that in the integrands we omit the terms giving zero after the subsequent integration. Then we use the following equalities:

$$\int \frac{d^{2+2\epsilon} k_1}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} \frac{a}{b} \ln \left( \frac{c}{a} \right) = - \left( -q_1^2 \right)^{\epsilon+1} \frac{\Gamma(1+\epsilon)\Gamma(2+\epsilon)}{\epsilon \Gamma(4+2\epsilon)} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{b}\ln\left(\frac{c}{a}\right) = \left(-q_1^2\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(2+2\epsilon)} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{(q_1k_1)}{ab}\ln\left(\frac{c}{a}\right) = \left(-q_1^2\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)}(\psi(1+\epsilon)-\psi(1+2\epsilon)) \ ,$$

$$q_1^2\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{(q_1-k_1)(q_1'-k_1)}{ac}\ln\left(\frac{c}{a}\right) = -(q_1q_1')\left(-q_1^2\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\epsilon^2\Gamma(1+2\epsilon)} \ ,$$

$$q^2\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{L_-}{b-b'} = -\left(-q^2\right)^{\epsilon+1}\frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(2+2\epsilon)} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{c+c'}{b-b'}L_- = -\left(-q^2\right)^{\epsilon+1}\frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(2+2\epsilon)\Gamma(3+2\epsilon)} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{(q_1-k_1)(q_1'-k_1)}{c}\frac{L_-}{b-b'}$$

$$= -\left(-q^2\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)}(\psi(1+\epsilon)-\psi(2+2\epsilon)) \ . \qquad \text{(C.3)}$$

The first three of these integrals can be easily calculated with the help of the representation

$$\frac{1}{b}\ln\left(\frac{a+b}{a}\right) = \int_0^1 \frac{dx}{a+bx} \ , \qquad \text{(C.4)}$$

the forth with the help of Eq. (B.1) and the last three using the representation

$$\frac{L_-}{b-b'} = \int_0^1 \frac{dx}{cx+c'(1-x)} \ . \qquad \text{(C.5)}$$

Using these integrals we arrive at Eq. (4.26). As for the integral $I_+(q_1,q_1')$ of Eq. (4.27), it can be written as

$$I_+(q_1,q_1') = -\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left(\frac{(q_1-k_1)(q_1'-k_1)}{cc'}\ln\left(\frac{cc'}{(q^2)^2}\right)\right.$$

$$\left.-\frac{c+c'}{cc'}\ln\left(\frac{k_1^2}{q^2}\right)\right) - \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{q^2}{(q_1-k_1)^2(q_1'-k_1)^2}\ln\left(\frac{k_1^2}{q^2}\right) \ . \quad \text{(C.6)}$$

The first integral in Eq. (C.6) can be easily calculated at arbitrary$\epsilon$; we find

$$-\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left(\frac{(q_1-k_1)(q_1'-k_1)}{cc'}\ln\left(\frac{cc'}{(q^2)^2}\right) - \frac{c+c'}{cc'}\ln\left(\frac{k_1^2}{q^2}\right)\right)$$

$$= \frac{\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)} \left( 2\left(-q^2\right)^\epsilon \left( \frac{1}{\epsilon} - \psi(1) + \psi(1-\epsilon) - \psi(1+\epsilon) + \psi(1+2\epsilon) \right) \right.$$

$$\left. - \frac{1}{\epsilon} \left( \left(-q_1^2\right)^\epsilon + \left(-q_1'^2\right)^\epsilon \right) \right) \ . \tag{C.7}$$

The second integral in Eq. (C.6) was analyzed in Ref. [16]. In the limit $\epsilon \to 0$ we have

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{q^2}{(q_1-k_1)^2(q_1'-k_1)^2} \ln\left(\frac{k_1^2}{q^2}\right)$$

$$= \frac{1}{\epsilon}\left(-q^2\right)^\epsilon \ln\left(\frac{(q^2)^2}{q_1^2 q_1'^2}\right) - \frac{1}{2}\ln^2\left(\frac{q_1^2}{q_1'^2}\right) \ . \tag{C.8}$$

# Appendix D

We use Eqs. (4.15), (4.13) and after a simple algebra we obtain

$$\mathcal{J}_3 = \int_0^1 dx \left( \frac{1}{x(1-x)} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{\tilde{t}_1 \tilde{t}_2'} \left[ \frac{(D-2)}{4} x_1 x_2 (q_1^2 - 2q_1k_1) \right.\right.$$

$$\times(q_1'^2 - 2q_1'\, k_2) - \frac{2x_2}{x_1}(q_1q_1')(k_1(q_1'-k_2)) + q_1'^2\left(2\frac{q_1k_2}{x_1} - 2q_1q_1' + q_1k_1\right)$$

$$+4x_1q_1^2(q_1'k_2) + q_1'q_1(q_1'q_1 - q_1'k_1 - q_1k_2) + 2(q_1'k_1)(q_1k_2) - 2(q_1'k_2)(q_1k_1)$$

$$+\frac{q_1'^2}{k_2^2}\left(2\frac{x_2}{x_1}(q_1k_2)(q_1'k_1 - kk_2) + x_2(q_1'k_2)(q_2^2 - k^2 + 2q_1k_2)\right.$$

$$\left.\left.\left. +2(q_2k_2)(q_1q + q_1k_2)\right) + q_1'^2 q_1^2 \frac{(q_2k_2)(q_2'k_1)}{k_1^2k_2^2}\right]\right)_+ + (q_i \leftrightarrow q_i') \ . \tag{D.1}$$

Terms in Eq. (D.1) with $x_1$ in the denominators require subtraction, so that the prescription (3.41) is important for them (and only for them). Note that neither the integrand in Eq. (D.1) itself, nor the subtraction terms contain non-integrable infrared singularities. Nevertheless, the integral $\mathcal{J}_3$ has a pole at $\epsilon = 0$. The pole comes from the ultraviolet region. Evidently, the ultraviolet divergency is artificial and appears as a result of the separation shown in Eq. (4.9). It is easy to see from Eqs. (4.12) and (4.13) that the terms leading to such divergencies cancel in the sum $A_2 + A_3$.

The integration over $k_1$ is performed using a standard Feynman parameterization. The basic integrals are

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{\tilde{t}_1\tilde{t}'_2} = x_1 x_2 \int_0^1 \frac{dz}{Q^{2(1-\epsilon)}} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{k_i^\mu}{\tilde{t}_1\tilde{t}'_2} = x_1 x_2 \int_0^1 \frac{dz r_i^\mu}{Q^{2(1-\epsilon)}} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{k_1^\mu k_2^\nu}{\tilde{t}_1\tilde{t}'_2} = x_1 x_2 \int_0^1 \frac{dz}{Q^{2(1-\epsilon)}} \left( r_1^\mu r_2^\nu - \frac{g^{\mu\nu}}{2\epsilon} Q^2 \right) \ , \qquad \text{(D.2)}$$

where

$$Q^2 = -x(1-x)(q_1^2 z + q_1'^2(1-z)) - z(1-z)(q_2^2 x + q_2'^2(1-x) - q^2 x(1-x)) \ ,$$

$$r_1 = zxq_1 + (1-z)(xk - (1-x)q'_2), \ r_2 = z((1-x)k - xq_2) + (1-z)(1-x)q'_1. \qquad \text{(D.3)}$$

The integrals with $\tilde{t}_1\tilde{t}'_2 k_i^2$ can be calculated joining first $\tilde{t}_1$ and $\tilde{t}'_2$ and then $k_i^2$. We have

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{\tilde{t}_1\tilde{t}'_2 k_i^2} = -x_1 x_2 (1-\epsilon) \int_0^1 dz \int_0^1 \frac{y dy}{\left(y\left(\mu_i^2 + y r_i^2\right)\right)^{2-\epsilon}} \ ,$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{k_i^\mu}{\tilde{t}_1\tilde{t}'_2 k_i^2} = -x_1 x_2 (1-\epsilon) \int_0^1 dz \int_0^1 \frac{dy y^2 r_i^\mu}{\left(y\left(\mu_i^2 + y r_i^2\right)\right)^{2-\epsilon}}$$

$$= -x_1 x_2 \int_0^1 \frac{dz r_i^\mu}{\mu_i^2 Q^2} + \mathcal{O}(\epsilon) \ , \quad \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{k_i^\mu k_i^\nu}{\tilde{t}_1\tilde{t}'_2 k_i^2}$$

$$= -x_1 x_2 (1-\epsilon) \int_0^1 dz \int_0^1 \frac{y dy}{\left(y\left(\mu_i^2 + y r_i^2\right)\right)^{2-\epsilon}} \left( y^2 r_i^\mu r_i^\nu - \frac{g^{\mu\nu} y \left(\mu_i^2 + y r_i^2\right)}{2(1-\epsilon)} \right)$$

$$= -x_1 x_2 \int_0^1 dz \left[ \frac{r_i^\mu r_i^\nu}{r_i^2} \left( \frac{1}{r_i^2} \ln\left(\frac{Q^2}{\mu_i^2}\right) - \frac{1}{Q^2} \right) - \frac{g^{\mu\nu}}{2r_i^2} \ln\left(\frac{Q^2}{\mu_i^2}\right) \right] + \mathcal{O}(\epsilon) \ , \qquad \text{(D.4)}$$

where

$$\mu_i^2 = Q^2 - r_i^2 \ , \quad \mu_1^2 = -zxq_1^2 - (1-z)(xk^2 + (1-x)q_2'^2) \ ,$$

$$\mu_2^2 = -z((1-x)k^2 + xq_2^2) - (1-z)(1-x)q_1'^2 \ . \qquad \text{(D.5)}$$

Finally, the integral with $\tilde{t}_1\tilde{t}'_2k_1^2k_2^2$ can be calculated joining $\tilde{t}_1$ and $\tilde{t}'_2$, then the result with $k_1^2$ and subsequently with $k_2^2$. We obtain

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{k_2^\mu k_1^\nu}{\tilde{t}_1\tilde{t}'_2k_1^2k_2^2}$$

$$= x_1x_2(2-\epsilon)(1-\epsilon)\int_0^1\int_0^1\int_0^1 \frac{dzydyt^3dt}{\left(t\left(y\left(\mu_1^2+2kr_1\right)-k^2+t(k-yr_1)^2\right)\right)^{3-\epsilon}}$$

$$\times\left[(k-yr_1)^\mu(k-t(k-yr_1))^\nu+\frac{g^{\mu\nu}}{2(2-\epsilon)}\left(y\left(\mu_1^2+2kr_1\right)-k^2+t(k-yr_1)^2\right)\right]$$

$$= x_1x_2\int_0^1\frac{dz}{d}\left[-k^\mu k^\nu\left(\frac{1}{k^2}+\frac{Q^2}{d}\mathcal{L}\right)-r_2^\mu k^\nu\left(\frac{1}{\mu_2^2}-\frac{\mu_1^2}{d}\mathcal{L}\right)\right.$$

$$\left.-k^\mu r_1^\nu\left(\frac{1}{\mu_1^2}-\frac{\mu_2^2}{d}\mathcal{L}\right)+r_2^\mu r_1^\nu\left(\frac{1}{Q^2}+\frac{k^2}{d}\mathcal{L}\right)+\frac{g^{\mu\nu}}{2}\mathcal{L}\right]+\mathcal{O}(\epsilon), \qquad \text{(D.6)}$$

where

$$d=\mu_1^2\mu_2^2+k^2Q^2=z(1-z)x(1-x)\left((k^2-q_1^2-q_2'^2)(k^2-q_1'^2-q_2^2)+k^2q^2\right)$$

$$+q_1^2q_2^2xz(x+z-1)+q_1'^2q_2'^2(1-x)(1-z)(1-x-z), \quad \mathcal{L}=\ln\left(\frac{\mu_1^2\mu_2^2}{-k^2Q^2}\right). \qquad \text{(D.7)}$$

At arbitrary $D$ we have

$$\mathcal{J}_3=\int_0^1 dx\int_0^1 dz\Bigg\{-\frac{1+\epsilon}{\epsilon}q_1q_1'\left(x_1x_2Q^{2\epsilon}+\frac{2}{x_1}\left(x_2Q^{2\epsilon}-Q_0^{2\epsilon}\right)\right)$$

$$+\frac{1+\epsilon}{2Q^{2(1-\epsilon)}}x_1x_2(q_1^2-2q_1r_1)(q_1'^2-2q_1'\,r_2)-\frac{2}{x_1}\Bigg[\left(x_2q_1q_1'\left(r_1(q_1'-r_2)\right)\right.$$

$$\left.-q_1'^2q_1r_2\right)\frac{1}{Q^{2(1-\epsilon)}}+\left(z(1-z)q_2'^2q_1q_1'+q_1'^2(zq_1k+(1-z)q_1q_1')\right)\frac{1}{Q_0^{2(1-\epsilon)}}\Bigg]$$

$$+\frac{1}{Q^{2(1-\epsilon)}}\Big(q_1'^2q_1\left(r_1-2q_1'\right)+4x_1q_1^2(q_1'r_2)+q_1'q_1(q_1'q_1-q_1'r_1-q_1r_2)$$

$$+2(q_1'r_1)(q_1r_2)-2(q_1'r_2)(q_1r_1)\Big)+q_1'^2\int_0^1 ydy\Bigg[\frac{1}{\left(y(\mu_2^2+yr_2^2)\right)^{1-\epsilon}}$$

$$\times\left(-\frac{x_2}{x_1}q_1(q_1'+k)+x_2q_1q_1'+q_1q_2\right)-\frac{y(1-\epsilon)}{\left(y(\mu_2^2+yr_2^2)\right)^{2-\epsilon}}$$

$$\times\left(2\frac{x_2}{x_1}(q_1r_2)(q_1'k-y(q_1'+k)r_2)+x_2(q_1'r_2)(q_2^2-k^2+2yq_1r_2)\right.$$

$$\left.+2(q_2r_2)(q_1(q+yr_2))\right)+\frac{1}{\left(y(\mu_0^2+yr_0^2)\right)^{1-\epsilon}}\left(\frac{1}{x_1}q_1(q_1'+k)\right)+\frac{y(1-\epsilon)}{\left(y(\mu_0^2+yr_0^2)\right)^{2-\epsilon}}$$

$$\times\left(\frac{2}{x_1}(q_1r_0)(q_1'k-y(q_1'+k)r_0)\right)+(2-\epsilon)(1-\epsilon)q_1^2$$

$$\times\int_0^1\frac{t^3dt}{\left(t\left(y\left(\mu_1^2+2kr_1\right)-k^2+t(k-yr_1)^2\right)\right)^{3-\epsilon}}\Big((q_2(k-yr_1))((k-t(k-yr_1))q_2')$$

$$+\frac{q_2q_2'}{2(2-\epsilon)}\left(y\left(\mu_1^2+2kr_1\right)-k^2+t(k-yr_1)^2\right)\Big)\Bigg]\Bigg\}+(q_i\leftrightarrow q_i')\;, \qquad \text{(D.8)}$$

where

$$Q_0^2=-z(1-z)q_2'^2,\quad \mu_0^2=-zk^2-(1-z)q_1'^2,\quad r_0=zk+(1-z)q_1',\quad r_0^2=Q_0^2-\mu_0^2. \qquad \text{(D.9)}$$

In the limit $\epsilon\to 0$ we arrive at Eq. (4.31).

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*V.S. Fadin and R. Fiore*

**Non-forward NLO BFKL kernel**

*В.С. Фадин, Р. Фиоре*

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