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PARALLEL COUPLED ACCELERATING STRUCTURE CALCULATION

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Abstract

The conventional TW-mode accelerating structures are usually used for high gradient linacs. But these structures grow old quickly during running. It is very serious problem for creation next linear colliders [1].

One way to overcoming it is using the parallel coupled accelerating structure with rectangular waveguide feeder [2].

In this article the calculation of parallel coupled accelerating structure is presented.

Расчет структуры с параллельной связью

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Аннотация

Для линейных ускорителей с большим градиентом обычно используются ускоряющие структуры на бегущей волне. Однако эти структуры быстро стареют во время тренировок. Это сейчас является очень серьезной проблемой при создании линейных коллайдеров [1].

Одним из путей преодоления этого является использование структур с параллельной связью и прямоугольным волноводным фидером [2].

В этой статье приводится расчет ускоряющей структуры с параллельной связью.

Introduction

The conventional TW-mode accelerating structures are usually used for high gradient linacs. But these structures grow old quickly during running. It is very serious problem for creation next linear colliders [1].

One way to overcoming it is using the parallel coupled accelerating structure with rectangular waveguide feeder [2]. Such kind of structure has a lot of advantages:

The RF breakdown takes place only into single cavity and does not provoke a breakdown in the other cavities. Only $1/N$ fraction of full RF stored energy is involved in process of damage (N – is the number of cavities).

The coupling cavity slot is placed on the flank edge of cavity. It is not a place with strong electric field. But the damage of an aperture is not so catastrophic.

In the parallel coupled cavity structure products of destroy surface are removing out quickly from the cavities in waveguide feeder, which have a large cross size (when waveguide feeder is standard rectangular form).

There is very simple HOM problem solution: it is possible to make the damping HOM slot with a higher mode load along a waveguide feeder or on the end of waveguide.

The most attractive variant of parallel coupled accelerating structure with rectangular waveguide feeder is shown on Fig. 1. It has four feed waveguides. For operate mode $\theta = \pi$ waveguides with $\Lambda = 2\lambda$ must be used. This waveguide has not so large wave resistance and group velocity equals to $0.5{\cdot}c$. Such accelerator can be fed by two RF klystron with double RF output.

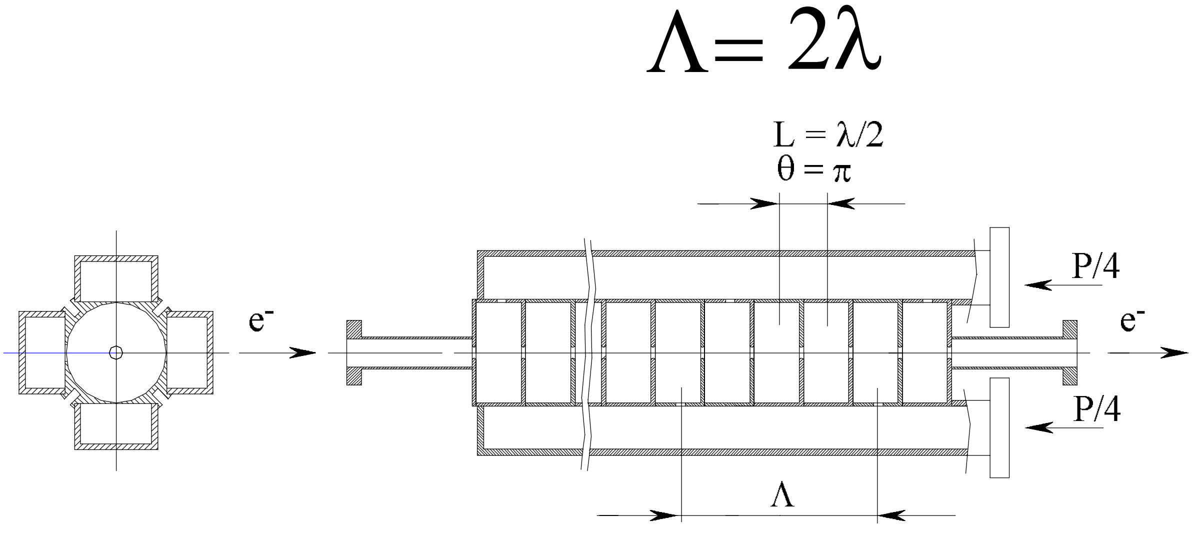

Figure 1. Parallel coupled cavities accelerating structures with four rectangular feeding waveguides.

Single waveguide-cavity aperture coupling

Fig. 2 is shown rectangular waveguide coupled in common wide wall with cylindrycal cavity.

Assume that coupling aperture is infinitely thin. The slot tangential longitudinal electric field is equal to

$$\vec{E}_\tau = e \cdot \vec{\Im} = e \cdot \Im(z,x) \cdot \vec{z}^{\,0}\,,\ ([\vec{E}] = V/m\,, [e] = V\,, [\Im] = 1/m\,). \quad (1)$$

There are: e is amplitude and $\Im$ is suitable normalized distribution function of coupling slot: $\int_S \Im \cdot \Im^* dS = 1$.

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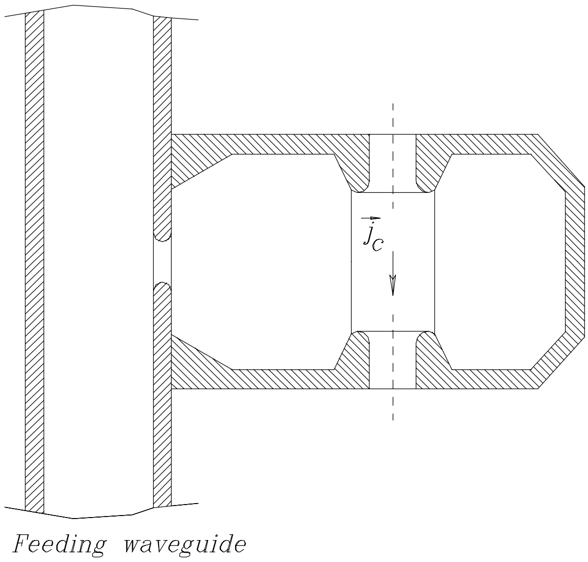

Figure 2. Waveguide-cavity aperture coupling.

Tangential electric field of aperture excites in feed rectangular waveguide the electric and magnetic fields that one can present as a sum of eigenvectors:

$$\vec{E} = \sum_{(ze)} U_{ze} \cdot \vec{e}_z + \sum_{(qe)} U_{qe} \cdot \vec{e}_{qe} + \sum_{(qh)} U_{qh} \cdot \vec{e}_{qh} \equiv \sum_{(a)} U_a \cdot \vec{e}_a \ , \tag{2a}$$

$$\vec{H} = \sum_{(zh)} I_{ze} \cdot \vec{h}_z + \sum_{(qh)} I_{qh} \cdot \vec{h}_{qh} + \sum_{(qe)} I_{qe} \cdot \vec{h}_{qe} \equiv \sum_{(b)} I_b \cdot \vec{h}_b \ . \tag{2b}$$

There are: U_a and I_b the amplitude coefficients, z is the longitudinal class, q – the transverse class of waves, a and b are encompassing index. $\vec{e}_a$ and $\vec{h}_b$ are electric and magnetic normalized eigenvectors distribution functions:

$$\int_S (\vec{e}_a \cdot \vec{e}_b)\, dS = \int_S (\vec{h}_b \cdot \vec{h}_a)\, dS = \delta_{ab} = \begin{cases} 1 & \text{for } b = a, \\ 0 & \text{for } b \neq a. \end{cases}$$

Dimensionality: $[U_a] = V$, $[I_b] = A$, $[\vec{e}_a] = [\vec{h}_b] = 1/m$.

Tangential electric field of coupling aperture excites in cavity the electric and magnetic fields can be expanded in terms of the complete set of eigenmodes as well:

$$\vec{E} = \sum_{\nu} u_\nu \cdot \vec{\varepsilon}_\nu \ , \tag{3a}$$

$$\vec{H} = \sum_{\nu} i_\nu \cdot \vec{h}_\nu \ . \tag{3b}$$

There are: u_ν and i_ν – the amplitude coefficients, $\vec{\varepsilon}_\nu$ and $\vec{h}_\nu$ are electric and magnetic normalized eigenvectors distribution functions:

$$\int_V \vec{\varepsilon}_\nu \cdot \vec{\varepsilon}_\mu \, dV = \int_V \vec{h}_\nu \cdot \vec{h}_\mu \, dV = \delta_{\nu\mu}$$

Dimensionality: $[u_\nu] = V \cdot m^{1/2}$, $[i_\nu] = A \cdot m^{1/2}$, $[\vec{\varepsilon}_\nu] = [\vec{h}_\nu] = 1/m^{3/2}$.

According to [3] for slot with tangential longitudinal electric field the amplitudes of electric field excited in rectangular waveguide are equal to

$$U_a^+ \{\vec{E}_\tau\} = -\frac{e}{2} \cdot m_a \cdot e^{-K_a z} \ , \ \ U_a^- \{\vec{E}_\tau\} = \frac{e}{2} \cdot m_a \cdot e^{K_a z} \ , \tag{4}$$

where sign (+) according to forward and (–) backward exciting wave,

$$m_a = \int_{-h/2}^{h/2} f_a(z) \operatorname{ch} K_a z \, dz \,, \tag{5}$$

$$K_a \equiv K_{e,h} = \sqrt{\chi_{e,h}^2 - k^2} \,, \; k = \frac{\omega}{c} \,, \tag{6}$$

$$\chi_a \equiv \chi_{e,h} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \,, \tag{7}$$

there are in (7): a – wide of waveguide, b – high of waveguide,

$$f_a(z) = \oint_L \left[\vec{n}\,\mathcal{E}\right] \cdot \vec{h}_a dl \,, \qquad [f_a] = 1/m \tag{8}$$

$\vec{n}$ - external normal vector to surface of slot and L – is the transverse cross-section contour of waveguide.

In our case of excitation (coupling slot is in wide wall of rectangular waveguide) f_a are equal to

$$f_a(z) = \oint_L \left[\vec{n}\vec{E}_\tau\right] \cdot \vec{h}_a dl = \int_{-l/2}^{l/2} \left[\vec{n}\vec{E}_\tau(z,x)\right] \cdot \vec{h}_a(x)\, dx = -\int_{-l/2}^{l/2} \mathcal{E}(z,x) \cdot h_{ax}(x)\, dx \,. \tag{9}$$

For slot with tangential longitudinal electric field the amplitudes of electric field in cavity given by

$$u_\nu = \frac{-k_{0\nu} M_{S\nu} \cdot e - \left(j\omega\mu_0 + R_{S\nu} H_{\nu\nu}\right) \cdot j_{0\nu}}{k_{0\nu}^2 - k^2 + j\omega\varepsilon_0 R_{S\nu} H_{\nu\nu}} \,, \tag{10a}$$

$$i_\nu = \frac{-j\omega\varepsilon_0 M_{S\nu} \cdot e + k_{0\nu} \cdot j_{0\nu}}{k_{0\nu}^2 - k^2 + j\omega\varepsilon_0 R_{S\nu} H_{\nu\nu}} \,. \tag{10b}$$

There are:

$k = \frac{\omega}{c}$, ω – operate frequency, $k_{0\nu} = \frac{\omega_{0\nu}}{c}$, $\omega_{0\nu}$ is ν resonant frequency,

$$M_{S\nu} = \int_S \left[\vec{n}\,\mathcal{E}'\right] \cdot \vec{h}_\nu dS \,, \; [M_{S\nu}] = 1/m^{1/2} \,, \tag{11}$$

$$j_{0\nu} = \int_V \left(\vec{j}_c \cdot \vec{\varepsilon}_\nu\right) dV \,, \; [\vec{j}_c] = A/m^2 \,, \; [j_{0\nu}] = A/m^{1/2} \,, \tag{12}$$

$\vec{j}_c$ – density of off-site current. In our case it is accelerating current,

$R_{S\nu}$ – real part of total surface resistance:

$$Z_{S\nu} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0 + \frac{\sigma}{j\omega_{0\nu}}}} \approx \sqrt{\frac{\mu_0\mu_r\omega_{0\nu}}{2\sigma}} \cdot (1+j) = R_{S\nu} \cdot (1+j) \,, \tag{13}$$

$H_{\nu\nu} = \int_S \vec{h}_\nu \vec{h}_\nu^{\,*} dS$ (S - is the cavity wall surface).

The main quality factor on ν -resonant frequency is equal to

$$Q_{0\nu} = \frac{\omega_{0\nu}\mu_0}{R_{S\nu}} \cdot \frac{\int_V \vec{h}_\nu \vec{h}_\nu^{\,*} dV}{\int_S \vec{h}_\nu \vec{h}_\nu^{\,*} dS} = \frac{\omega_{0\nu}\mu_0}{R_{S\nu}} \cdot \frac{1}{H_{\nu\nu}} \,.$$

Then (10) can be written

$$u_\nu = \frac{-k_{0\nu} M_{S\nu} \cdot e - \left(j\omega\mu_0 + \frac{\omega_{0\nu}\mu_0}{Q_{0\nu}}\right) \cdot j_{0\nu}}{k_{0\nu}^2 - k^2 + j \cdot \frac{k_{0\nu} k}{Q_{0\nu}}} \,, \tag{14a}$$

$$i_\nu = \frac{-j\omega\varepsilon_0 M_{S\nu}\cdot e + k_{0\nu}\cdot j_{0\nu}}{k_{0\nu}^2 - k^2 + j\cdot\dfrac{k_{0\nu}k}{Q_{0\nu}}}. \tag{14b}$$

In order to find unknown exciting coefficient *e* in (4) and (10) it is necessary to equate the tangential magnet fields of waveguide and cavity at the coupling aperture. As a result we can find

$$e\cdot(Y_W + Y_C) \equiv e\cdot Y_T = I_W + I_C. \tag{15}$$

There are: waveguide conductivity Y_W

$$Y_W = \frac{1}{2}\cdot\sum_a Y_a\cdot m_a^2 = \frac{1}{2}\cdot Y_0\cdot m_W^2 + j\cdot B, \tag{16}$$

where index "0" and "*W*" on the right corresponds to the basic propagating H_{10}-mode, but conductivity *B* (real) equals to

$$B = \frac{1}{2}\cdot \mathrm{Im}(\sum_{a\neq 0} Y_a\cdot m_a^2), \tag{17}$$

the waveguide conductivity Y_a for both *e* and *h* waves see appendix 2,
the cavity conductivity Y_C

$$Y_C = \sum_\nu \frac{j\omega\varepsilon_0 M_{S\nu}^2}{k_{0\nu}^2 - k^2 + j\cdot\dfrac{k_{0\nu}k}{Q_{0\nu}}}, \tag{18}$$

waveguide exciting current I_W

$$I_W = \sum_a Y_a\cdot(U_a^+ - U_a^-)\cdot m_a, \tag{19}$$

where U_a^+ and U_a^- – the amplitudes incident waves on the right side and on the left side accordingly, cavity exciting urrent I_C

$$I_C = \sum_\nu \frac{k_{0\nu} M_{S\nu} j_{0\nu}}{k_{0\nu}^2 - k^2 + j\cdot\dfrac{k_{0\nu}k}{Q_{0\nu}}}. \tag{20}$$

One note: in general case if we don't know a suitable distribution function $\vec{\mathcal{E}}$ we must to represent slot tangential longitudinal electric field $\vec{E}_\tau$ as a complete set of normalized orthogonal vectors:

$$\vec{E}_\tau = \sum_p e_p\cdot\vec{\mathcal{E}}_p,\quad \int_S \mathcal{E}_p\cdot\mathcal{E}_q^* dS = \delta_{pq}.$$

In this case the equations for solutions will be a little extended [3].

We assume that non propagating waves in waveguide exist only near the aperture and didn't affect to the other slots. In other words only basic propagating wave H_{10}-mode exist in waveguide. In this case the amplitudes of electric field excited in rectangular waveguide are equal to (see (4))

$$U_0^\pm = \mp\frac{e}{2}\cdot m_W, \tag{21}$$

there m_W - the coefficient (5) that according to propagating H_{10}-mode.

Only propagating waves H_{10}-mode exist in waveguide with amplitudes $U^\pm$ on the left of coupling slot and $U_{+Z}^\pm$ on the right of coupling slot. So the waveguide exciting currents I_W (19) is then

$$I_W = Y_0\cdot(U^+ - U_{+Z}^-)\cdot m_W, \tag{22}$$

In addition we take into account only one exciting cavity mode (like E_{010}-mode cylindrical cavity).

Then the cavity exciting current I_C (20) and cavity conductivity Y_C (18)

$$I_C = -j\cdot\frac{j_0\cdot Q_0 M_S}{k(1+2jQ_0\cdot\delta k)} = -j\cdot\frac{k_0}{k}\cdot\frac{j_0 Z_0 Y_C}{M_S}, \tag{23}$$

$$Y_C = \frac{Q_0 M_S^2}{Z_0 k_0(1+2jQ_0\cdot\delta k)}.$$

The amplitude u of cavity (see (14a))

$$u = j\frac{M_S\,Q_0}{k(1+2jQ_0\cdot\delta\,k)}\cdot e - \frac{Q_0 Z_0\left(1-j\dfrac{k_0}{kQ_0}\right)}{k_0(1+2jQ_0\cdot\delta\,k)}\cdot j_0\,, \tag{24}$$

where off-site accelerating current j_0 is (see 12))

$$j_0 = \int_V \left(\vec{j}_c\cdot\vec{\varepsilon}\,\right)dV = 2I\cdot\int_{-L/2}^{L/2}\varepsilon_z(z)\cdot\cos(\omega\,t)\,dz \equiv 2I\cdot Int\,, \tag{25}$$

I – average accelerating beam current (amplitude of the first current harmonic), $[I]=$ A, $[Int]=1/m^{1/2}$,

$\omega\,t = 2\pi\cdot\dfrac{z}{\beta_e\lambda}$, β_e – relative velocity of accelerating electrons. It is expected that the center of current bunch pass a cavity at $t=0$ time.

$$\delta\,k = \frac{1}{2}\cdot\frac{k^2-k_0^2}{k_0 k} = \frac{1}{2}\cdot\frac{(k-k_0)\cdot(k+k_0)}{k_0 k} \approx \frac{1}{2}\cdot\frac{(k-k_0)\cdot 2k_0}{k_0 k} \approx \frac{k-k_0}{k_0}\,.$$

The resonant frequency ω_{res} of coupled cavity defined by setting equal to zero the imaginary part of input conductivity Y that equals to

$$Y = \frac{(Y_C + j\cdot B)}{m_w^2} = \frac{1}{m_w^2}\cdot\left[\frac{Q_0 M_S^2}{Z_0 k_0\left(1+4Q_0^2\cdot\delta\,k^2\right)} + j\cdot\left(B - 2Q_0\cdot\delta\,k\frac{Q_0 M_S^2}{Z_0 k_0\left(1+4Q_0^2\cdot\delta\,k^2\right)}\right)\right].$$

The equation to find the resonance frequency is

$$\delta\,k_{res}^2 - \frac{M_S^2}{2Z_0 k_0 B}\cdot\delta\,k_{res} + \frac{1}{4Q_0^2} = 0\,. \tag{26}$$

From which one can find

$$\delta\,k_{res} = \frac{M_S^2}{4Z_0 k_0 B}\cdot\left[1+\sqrt{1-\left(\frac{2Z_0 k_0 B}{M_S^2 Q_0}\right)^2}\,\right]. \tag{27}$$

The coupling coefficient β between source waveguide and accelerating cavity is defined by

$$\beta \overset{df}{=} \left.\frac{P_{ex}}{P_c}\right|_{\omega=\omega_{res}}. \tag{28}$$

The average power dissipated in the cavity walls

$$P_c = \frac{\omega_0 W}{Q_0} = \frac{\omega_0}{Q_0}\cdot\frac{\varepsilon_0}{2}\int_V\left|\vec{E}\right|^2 dV = \frac{\omega_0\varepsilon_0\cdot|u|^2}{2Q_0}\int_V|\vec{\varepsilon}|^2\,dV = \frac{k_0}{2Z_0 Q_0}\cdot|u|^2\,. \tag{29}$$

The power dissipated in external load without external sources

$$P_{ex} = \frac{\left|U_0^+\right|^2\cdot Y_0}{2} + \frac{\left|U_0^-\right|^2\cdot Y_0}{2} = \frac{|e|^2 m_W^2 Y_0}{4} = \frac{m_W^2 Y_0 k_0^2\left(1+4Q_0^2\cdot\delta\,k^2\right)}{4M_S^2 Q_0^2}\cdot|u|^2\,. \tag{30}$$

Then β is written

$$\beta = \frac{m_W^2\cdot Y_0 Z_0 k_0\left(1+4Q_0^2\cdot\delta\,k_{res}{}^2\right)}{2M_S^2 Q_0} = \frac{Y_0 Q_0 \delta\,k_{res} m_W^2}{B}\,. \tag{31}$$

Using (31) one can write cavity conductivity Y as

$$Y = \frac{jB+Y_C}{m_W^2} = \frac{Y_0}{2\beta}\cdot\left[1-\frac{1-2jQ_0\delta\,k_{res}}{1+2jQ_0\delta\,k}\cdot 2jQ_0\Delta k\right] \equiv \frac{Y_0}{2\beta}\cdot F(k)\,, \tag{32}$$

there $\Delta k = \dfrac{k-k_{res}}{k_0} \approx \dfrac{k-k_{res}}{k_{res}}$.

The cavity effective shunt impedance defined by

$$Z_e \overset{df}{=} \frac{\left(\frac{1}{L}\int\limits_{-L/2}^{L/2} E_z(z)\cdot\cos(\omega t)dz\right)^2}{P_{Loss}/L} = \frac{|u|^2\left(\int\limits_{-L/2}^{L/2} \varepsilon_z(z)\cdot\cos(\omega t)dz\right)^2}{P_{Loss}L} = \frac{2Z_0Q_0}{kL}\cdot Int^2 . \tag{33}$$

where L is cavity length, $Int \equiv \int\limits_{-L/2}^{L/2} \varepsilon_z(z)\cdot\cos(\omega t)\,dz$ (see (25)).

Then the power dissipated in external load without external sources one can rewrite as

$$P_{ex} = \frac{\beta\cdot|u|^2 k_0}{2Z_0Q_0} = \frac{\beta\cdot|u|^2 Int^2}{Z_e L} . \tag{34}$$

Parallel coupled structure

Parallel coupled accelerating structure (see Fig. 1) consists of four rectangular waveguide feeders and a few identical accelerating cavities. The coupling apertures of cavities powered from one waveguide are located on equal distance. The operate accelerating mode is assumed $\theta = \pi$. The equivalent layout of one (n th) period such structure is shown on Fig. 3.

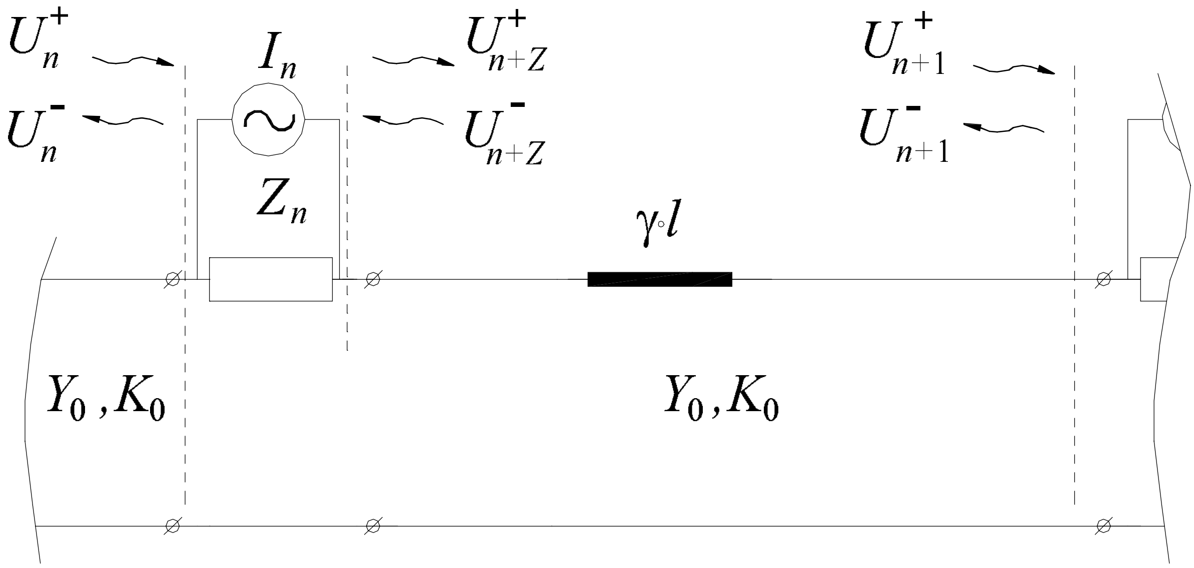

Figure 3. Equivalent layout of one period.

There are on Fig. 3:
$n = 1,2...,N$ (N is the number of accelerating cavities per one waveguide),
equivalent resistance Z_n

$$Z_n = \frac{1}{Y_n} = \frac{m_{W(n)}^2}{jB_{(n)} + Y_{C(n)}} \equiv \frac{m_{W(n)}^2}{\left[Y_{T(n)} - \frac{Y_0 m_{W(n)}^2}{2}\right]} \equiv \frac{m_{W(n)}^2}{Y_{T(n)}}\cdot\frac{1}{\left[1-\frac{Y_0 m_{W(n)}^2}{2Y_{T(n)}}\right]} . \tag{35}$$

The amplitudes of exciting waves $U_0^{\pm}$ (see (21))

$$U_0^{\pm}\{I_{C(n)}\} = \mp\frac{e\{I_{C(n)}\}}{2}\cdot m_{W(n)} = \mp\frac{I_{C(n)}\cdot m_{W(n)}}{2Y_{T(n)}} = \mp\frac{I_n}{2}\cdot\frac{Z_n\cdot\frac{2}{Y_0}}{\left(Z_n+\frac{2}{Y_0}\right)} = \mp\frac{I_n}{2}\cdot\frac{m_{W(n)}^2}{Y_{T(n)}} . \tag{36}$$

Then the equivalent current I_n on Fig. 3 is equal to:

$$I_n = \frac{I_{C(n)}}{m_{W(n)}} = -j \cdot \frac{j_{0(n)} \cdot Q_{0(n)}}{k \cdot \left(1 + 2jQ_{0(n)} \cdot \delta\, k_{(n)}\right)} \cdot \frac{M_{S(n)}}{m_{W(n)}} = -j \cdot \frac{k_0}{k} \cdot \frac{i_{0(n)} Z_0 Y_{C(n)}}{m_{W(n)} m_{S(n)}} . \tag{37}$$

l is the length of waveguide between coupling slots (between accelerating cavities), $\gamma = (\alpha + K_0)$. The attenuation constant α and propagation constant for rectangular H_{10}-mode waveguide $a \times b$ are

$$\alpha^{H_{10}} = \frac{R_S}{Z_0} \cdot \frac{\Lambda^{H_{10}}}{\lambda} \cdot \left[\frac{1}{b} + \frac{2}{a} \cdot \left(\frac{\lambda}{2a} \right)^2 \right], \quad K_0 = j \cdot \frac{2\pi}{\Lambda^{H_{10}}},$$

where $\Lambda^{H_{10}} = \dfrac{\lambda}{\sqrt{1 - \left(\dfrac{\lambda}{2a} \right)^2}}$ is the waveguide length of rectangular H_{10}-mode.

The amplitudes U^+_{n+Z} and U^-_n

$$U^+_{n+Z} = U^+_n + U^+_{0(n)} \{\vec{E}_\tau\} = U^+_n - \frac{e_n}{2} m_{W(n)} = U^+_n - \frac{Y_0 \cdot m^2_{W(n)}}{2Y_{T(n)}} \cdot \left(U^+_n - U^-_{n+Z}\right) - \frac{I_n \cdot m^2_{W(n)}}{2Y_{T(n)}},$$

$$U^-_n = U^-_{n+Z} + U^-_{0(n)} \{\vec{E}_\tau\} = U^-_{n+Z} + \frac{e_n}{2} m_{W(n)} = U^+_n - \frac{Y_0 \cdot m^2_{W(n)}}{2Y_{T(n)}} \cdot \left(U^+_n - U^-_{n+Z}\right) + \frac{I_n \cdot m^2_{W(n)}}{2Y_{T(n)}}.$$

These equations can be rewritten as

$$\begin{pmatrix} U^+ \\ U^- \end{pmatrix}_n = T(Z_n) \cdot \begin{pmatrix} U^+ \\ U^- \end{pmatrix}_{n+Z} + \left(1 + \frac{Z_n Y_0}{2}\right) \cdot \frac{I_n \cdot m_{W(n)}}{2Y_{T(n)}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \equiv T(Z_n) \cdot \begin{pmatrix} U^+ \\ U^- \end{pmatrix}_{n+Z} + \frac{I_n Z_n}{2} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

where $T(Z_n)$ is standard nonnormalized transform matrix for resistance Z_n:

$$T(Z_n) = \begin{pmatrix} 1 + \dfrac{Z_n Y_0}{2} & -\dfrac{Z_n Y_0}{2} \\ \dfrac{Z_n Y_0}{2} & 1 - \dfrac{Z_n Y_0}{2} \end{pmatrix}. \tag{38}$$

$$\begin{pmatrix} U^+ \\ U^- \end{pmatrix}_n = T(Z_n) \cdot T(\gamma\, l) \cdot \begin{pmatrix} U^+ \\ U^- \end{pmatrix}_{n+1} + \frac{I_n Z_n}{2} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{39}$$

where $T(\theta_\Lambda)$ is standard transform matrix for l long waveguide:

$$T(\gamma\, l) = \begin{pmatrix} e^{\gamma \cdot l} & 0 \\ 0 & e^{-\gamma \cdot l} \end{pmatrix}.$$

In our case of accelerating structure (Fig. 1) length of waveguide between slots equals to $\Lambda = 2\lambda$. Assume that the attenuation constant of waveguide $\alpha \approx 0$. Then transform matrix $T(\theta_\Lambda) = E \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Assume that on the right end of waveguide located short-wall: $U^-_{N+1} = -U^+_{N+1}$. Then we can write

$$\begin{pmatrix} U^+ \\ U^- \end{pmatrix}_n = \left[\prod_{k=n}^{N} T(Z_k) \right] \cdot \begin{pmatrix} U^+ \\ -U^+ \end{pmatrix}_{N+1} + \left[\sum_{k=n}^{N} \frac{I_k Z_k}{2} \right] \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{40}$$

For initial and reflect input amplitudes

$$\begin{pmatrix} U^+ \\ U^- \end{pmatrix}_{Inp} = \left[\prod_{k=1}^{N} T(Z_k) \right] \cdot \begin{pmatrix} U^+ \\ -U^+ \end{pmatrix}_{N+1} + \left[\sum_{k=1}^{N} \frac{I_k Z_k}{2} \right] \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{41}$$

The product $\prod_{k=1}^{N} T(Z_k) = T(Z_{Tot})$, where $Z_{Tot} = \sum_{k=1}^{N} Z_k$.

In other words our case is the simple case of resistance Z_k placed in successively.

The amplitude of reflect wave from (41) is

$$U^-_{Inp} = \frac{Z_{Tot}Y_0 - 1}{Z_{Tot}Y_0 + 1} \cdot U^+_{Inp} + \frac{2}{Z_{Tot}Y_0 + 1} \cdot \sum_{k=`1}^{N} \frac{I_k Z_k}{2} . \tag{42}$$

Or simpler: $U_{Inp} = Z_{Tot} \cdot I_{Inp} + V$, where $U_{Inp} = U^+_{Inp} + U^-_{Inp}$,

$$I_{Inp} = Y_0 \cdot \left(U^+_{Inp} - U^-_{Inp}\right) = \frac{2}{Z_{Tot}Y_0 + 1} \cdot \left(U^+_{Inp} - \sum_{k=`1}^{N} \frac{I_k Z_k}{2} \right) ,$$

and $V = \sum_{k=`1}^{N} \left(I_k Z_k\right)$.

Multiply (40) by $T^{-1}(Z_{Tot}) \equiv T(-Z_{Tot})$ to obtain:

$$\begin{pmatrix} U^+ \\ U^- \end{pmatrix}_n = T^{-1}\left(\sum_{k=n-1}^{N} Z_k\right) \cdot \begin{pmatrix} U^+ \\ U^- \end{pmatrix}_{Inp} - \left[\sum_{k=n-1}^{N} \frac{I_k Z_k}{2} \right] \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} . \tag{43}$$

$$U_n = \left(U^+_n + U^-_n\right) = U_{Inp} - I_{Inp} \cdot \sum_{k=n-1}^{N} Z_k - \sum_{k=n-1}^{N} I_k Z_k , \tag{44}$$

$$\begin{aligned} \left(U^+_n - U^-_{n+1}\right) &= \left(1 + \frac{Y_0 Z_n}{2}\right) \cdot U^+_{Inp} - \left(1 + \frac{Y_0 Z_n}{2}\right) \cdot U^-_{Inp} + \frac{I_n Z_n}{2} = \\ &= \left(1 + \frac{Y_0 Z_n}{2}\right) \cdot \left(U^+_{Inp} - U^-_{Inp}\right) + \frac{I_n Z_n}{2} \end{aligned} \tag{45}$$

From (15) take into account (45) we have

$$e_n = \frac{m_{W(n)}}{Y_T(n)} \cdot \left[Y_0 \left(U^+_n - U^-_{n+1}\right) + I_n \right] = \frac{1}{m_{W(n)}} \cdot \frac{Z_n}{Z_{Tot}Y_0 + 1} \cdot \left[2Y_0 U^+_{Inp} + \widetilde{I}_n \right], \tag{46}$$

where

$$\widetilde{I}_n = I_n + Y_0 \cdot \left(I_n \cdot \sum_{k=`1}^{N} Z_k - \sum_{k=`1}^{N} I_k Z_k \right) = I_n \cdot \left(Z_{Tot}Y_0 + 1\right) - Y_0 \cdot \sum_{k=`1}^{N} I_k Z_k .$$

If all cavities are equally ($Z_k = Z$ and $\widetilde{I}_n = I_n = I_0$) then

$$e_n = e = \frac{1}{m_W} \cdot \frac{ZY_0}{NZY_0 + 1} \cdot \left[2U^+_{Inp} + \frac{I_0}{Y_0} \right]. \tag{47}$$

In this case the amplitude of reflect wave

$$\begin{aligned} U^-_{Inp} &= \frac{NZY_0 - 1}{NZY_0 + 1} \cdot U^+_{Inp} + \frac{NZY_0}{NZY_0 + 1} \cdot \frac{I_0}{Y_0} \approx \\ &\approx \frac{1}{1 + 2jQ_{LN}\Delta k} \cdot \left\{ \left(\frac{2N\beta - 1}{2N\beta + 1} - 2jQ_{LN}\Delta k \right) \cdot U^+_{Inp} - j \cdot e^{j\varphi_{res}} \cdot \frac{N\sqrt{2\beta \cdot Z_e L}}{2N\beta + 1} \cdot \sqrt{\frac{2}{Y_0}} \cdot I \right\}, \end{aligned} \tag{48}$$

but the amplitude u_n of nth cavity from (24) is equal

$$u_n = u = j\frac{M_S\,Q_0}{k\left(1+2jQ_0\cdot\delta\,k\right)}\cdot\frac{2}{m_W}\cdot\frac{ZY_0}{NZY_0+1}\cdot U^{+}_{Inp} +$$

to $$+\left\{\left[\frac{M_S\,Q_0}{k\left(1+2jQ_0\cdot\delta\,k\right)}\cdot\frac{1}{m_W}\right]^2\cdot\frac{ZY_0}{NZY_0+1}-\frac{Q_0Z_0Y_0\left(1-j\dfrac{k_0}{kQ_0}\right)}{k_0\left(1+2jQ_0\cdot\delta\,k\right)}\right\}\cdot\frac{2I\cdot Int}{Y_0}\approx \quad (49)$$

$$\approx\frac{j\cdot e^{j\varphi_{res}}}{1+2jQ_{LN}\Delta\,k}\cdot\frac{2\cdot\sqrt{2\beta\cdot Z_eL\cdot P_{Inp}}}{\left(2N\beta+1\right)\cdot Int}\cdot e^{\varphi_{Gen}}-\frac{1}{1+2jQ_{LN}\Delta\,k}\cdot\frac{Z_eL\cdot I}{Int\cdot\left(2N\beta+1\right)}.$$

There are on (48) and (49):

the detuning angle φ_{res} defined as: $\mathrm{tg}\left(\varphi_{res}\right)=-2Q_0\cdot\delta\,k_{res}\equiv-2Q_0\cdot\dfrac{k_{res}-k_0}{k_0}$,

$Q_{LN}=\dfrac{Q_0}{2N\beta+1}$ - loaded cavity quality factor of system,

$P_{Inp}=\dfrac{Y_0\left|U^{+}_{Inp}\right|^2}{2}$ - input power, φ_{Gen} - the phase of generator incident wave. Usually $\varphi_{res}\approx\dfrac{\pi}{2}$ ($\delta\,k_{res}<0$) and then

$$U^{-}_{Inp}=-U^{+}_{Inp}+\frac{e^{j\psi(k)}}{1+4Q^2_{LN}\left(\Delta\,k\right)^2}\left\{\frac{2\cdot2N\beta}{2N\beta+1}\cdot U^{+}_{Inp}+\frac{N\sqrt{2\beta\cdot Z_eL}}{2N\beta+1}\cdot\sqrt{\frac{2}{Y_0}}\cdot I\right\}, \quad (50)$$

$$u_n=\frac{e^{j\psi(k)}}{1+4Q^2_{LN}\left(\Delta\,k\right)^2}\cdot\left(\frac{2\cdot\sqrt{2\beta\cdot Z_eL\cdot P_{Inp}}}{2N\beta+1}\cdot e^{j\varphi_g}-\frac{Z_eL\cdot I}{2N\beta+1}\right)\cdot\frac{1}{Int}. \quad (51)$$

$\mathrm{tg}\left[\psi(k)\right]=-2Q_{LN}\cdot\Delta\,k\equiv-2Q_{LN}\cdot\dfrac{k-k_{res}}{k_{res}}$ and $\varphi_g\equiv\varphi_{Gen}+\pi$.

The real u_n equals to $\mathrm{Re}(u_n\cdot e^{j\omega t})$. In order to obtain an energy gate of accelerating particle per single cavity it is necessary to integrate real u_n over z:

$$W_n=\int_{-L/2}^{L/2}\mathrm{Re}(u_n\cdot e^{j\omega t})\cdot\varepsilon_z(z)dz=$$
$$=\frac{1}{1+4Q^2_{LN}\left(\Delta\,k\right)^2}\cdot\left\{\frac{2\cdot\sqrt{2\beta\cdot Z_eL\cdot P_{Inp}}}{\left(2N\beta+1\right)}\cdot\cos\left[\psi(k)+\varphi_g\right]-\frac{Z_eL\cdot I}{\left(2N\beta+1\right)}\cdot\cos\left[\psi(k)\right]\right\}. \quad (52)$$

On the resonance frequency ($\Delta\,k=0$) we obtain:

$$W_n=\frac{2\cdot\sqrt{2\beta\cdot Z_eL\cdot P_{Inp}}}{2N\beta+1}\cdot\cos\varphi_g-\frac{Z_eL\cdot I}{2N\beta+1}, \quad (53)$$

$$U^{-}_{Inp}=\frac{2N\beta-1}{2N\beta+1}\cdot U^{+}_{Inp}+\frac{N\sqrt{2\beta\cdot Z_eL}}{2N\beta+1}\cdot\sqrt{\frac{2}{Y_0}}\cdot I. \quad (54)$$

The charm of such kind cavities feeding and operate on π-mode is the equally increment of accelerating particles energy in all cavities (Constant Gradient accelerating regime) with the equally sizes of all cavities and coupling slots.

The total energy gain is equal to

$$W_{Tot}=N\cdot W_n=N\cdot\frac{2\cdot\sqrt{2\beta\cdot Z_eL\cdot P_{Inp}}}{2N\beta+1}\cdot\cos\varphi_g-N\cdot\frac{Z_eL\cdot I}{2N\beta+1}. \quad (55)$$

For matched regime on the resonance frequency without current (reflect power is equal to zero): $2\beta = \frac{1}{N}$ and

$$P_{ref} = \left. \frac{(2\beta N - 1)^2}{(2\beta N + 1)^2} \cdot P_{Inp} \right|_{2\beta = 1/N} = 0 ,$$

$$u_n = \left(\sqrt{\frac{Z_e L \cdot P_{Inp}}{N}} \cdot e^{j\varphi_g} - \frac{Z_e L \cdot I}{2} \right) \cdot \frac{1}{Int} , \tag{56}$$

$$W_n = \sqrt{\frac{Z_e L \cdot P_{Inp}}{N}} \cdot \cos \varphi_g - \frac{I \cdot Z_e L}{2} , \tag{57}$$

$$W_{Tot} = N \cdot W_n = \sqrt{N} \cdot \sqrt{P_{Inp} Z_e L} \cdot \cos \varphi_g - \frac{N \cdot I \cdot Z_e L}{2} . \tag{58}$$

The simple case of $I = 0$ and $\varphi_g = 0$. The reflect power is equal to zero $P_{ref} = 0$, the amplitude in n cavity

$$u_n = \sqrt{\frac{Z_e L \cdot P_{Inp}}{N}} \cdot \frac{1}{Int} = u_0 , \tag{59}$$

the energy gain per one cavity

$$W_n = \sqrt{\frac{P_{Inp} Z_e L}{N}} = W_0 , \tag{60}$$

and the total energy gain

$$W_{Tot} = \sqrt{N} \cdot \sqrt{P_{Inp} Z_e L} = W_{Tot.0} . \tag{61}$$

If m of N cavities are shorted (size of slot equal zero) for example in case of breakdown, then we must to replace N by N-m.
In this case the reflect power will be nonzero and increased up to

$$P_{ref} = \left. \left| \frac{2\beta N - 1}{2\beta N + 1} \right|^2 \cdot P_{Inp} \right|_{\substack{2\beta = 1/N, \\ N \Rightarrow N - m}} = \frac{m^2}{(2N\text{-}m)^2} \cdot P_{Inp} , \tag{62}$$

there is m maybe $0,1,\dots,N$,

$$u_n = \begin{cases} \left. \dfrac{2 \cdot \sqrt{2\beta \cdot Z_e L \cdot P_{Inp}}}{(2\beta N + 1) \cdot Int} \right|_{\substack{2\beta = 1/N, \\ N \Rightarrow N - m}} = \dfrac{2N}{2N\text{-}m} \cdot u_0 & \text{for } n \neq m, \\ = 0 & \text{for } n = m, \end{cases} \tag{63}$$

$$W_n = \begin{cases} \left. \dfrac{2 \cdot \sqrt{2\beta \cdot Z_e L \cdot P_{Inp}}}{(2\beta N + 1) \cdot Int} \right|_{\substack{2\beta = 1/N, \\ N \Rightarrow N - m}} = \dfrac{2N}{2N\text{-}m} \cdot W_0 & \text{for } n \neq m, \\ = 0 & \text{for } n = m, \end{cases} \tag{64}$$

$$W_{Tot} = (N - m) \cdot W_n = \frac{2(N - m)}{2N - m} \cdot W_{Tot.0} . \tag{65}$$

All written above is related only to single waveguide feeder. One can see that as distinct from travelling wave mode accelerator the increase amplitude of electric field in parallel coupled cavity accelerator is small, when one cavity is breakdown.

Example:

Operate frequency	$11424\ \text{MHz}$,
Input power	$P_{Inp} = 2 \times 75\ \text{MW} = 150\ \text{MW}$,
Nb. of waveguides	4 ,
Nb. of cavities per single feeding waveguide	45,
Effective shunt impedance	$Z_e = 90\ \text{M}\Omega/\text{m}$,
Energy gain	$E_{acc} = 75\ \text{MV/m}$,
Input power for single cavity	$P_1 \cong 0.83\ \text{MW}$,
Total length of accelerating section	$L \cong 2.37\ \text{m}$,
Energy gain per section	$U \cong 178\ \text{MeV}$.

If only 1 from 45 cavities of single feeding waveguide is shorted in case of breakdown, the amplitude of electric field in the other cavities concerned to this waveguide is increased only up to

$$u_n = \frac{2N}{2N\text{-}m} \cdot u_0 = \frac{90}{89} \cdot u_0 = 1.011 \cdot u_0 .$$

REFERENCES

[1] *J. W. Wang and G. A. Loew* Field Emission and RF Breakdown in High-Gradient Room-Tempera-ture Linac Structures", SLAC-PUB-7684, October 1997.

[2] *O. N. Brezhnev, V. M. Pavlov, O. V. Pirogov, Ju. D. Chernousov.* Accelerating Structure for high Gradient Accelerator., Тезисы докладов XVII Международного семинара по уско-рителям заряженных частиц, стр. 71, 17-23 сентября 2001 г., Алушта, Харьков 2001 г.

[3] *Б. М. Машковцев, К. Н. Цибизов. Б. Ф. Емелин.* Теория волноводов", Изд. "Наука", Москва-Ленинград, 1966 г., 350 с.

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Appendix

1. Scalar functions ψ_e (E-waves) and ψ_h (H-waves)

Rectangular waveguide $a \times b$ (width a, high b):

$$\psi_h = \frac{2}{\sqrt{ab}} \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b},$$

$$\psi_e = \sqrt{\frac{2-\delta_{0m}}{a}} \cdot \sqrt{\frac{2-\delta_{0n}}{b}} \cdot \cos\frac{m\pi x}{a} \cdot \cos\frac{n\pi y}{b},$$

$$\delta_{oj} = \begin{cases} 1 & j = 0 \\ 0 & j \neq 0 \end{cases}, \quad \int_S \psi_{h,e}^2 dS = 1.$$

Circular waveguide:

$$\psi_h = \sqrt{\frac{2-\delta_{0m}}{\pi}} \cdot \frac{\mu_{mn}}{a \cdot \sqrt{\mu_{mn}^2 - m^2}} \cdot \frac{J_m\left(\mu_{mn}\frac{r}{R}\right)}{J_m(\mu_{mn})} \cdot \begin{cases} \cos m\varphi \\ \sin m\varphi \end{cases}, \qquad J'_m(\mu_{mn}) = 0,$$

$$\psi_e = \sqrt{\frac{2-\delta_{0m}}{\pi}} \cdot \frac{1}{a} \cdot \frac{J_m\left(\varepsilon_{mn}\frac{r}{R}\right)}{J_{m-1}(\varepsilon_{mn})} \cdot \begin{cases} \cos m\varphi \\ \sin m\varphi \end{cases}, \qquad J_m(\varepsilon_{mn}) = 0,$$

there R is the radius of waveguide.

2. Main normalized functions of cylinder wave guides

H-waves:

$$h_{zh} = \psi_h \cdot \vec{z}^0,$$

$$\vec{h}_{\perp h} = \frac{1}{\chi_h} \cdot \vec{\nabla}_\perp \psi_h,$$

$$\vec{\varepsilon}_{\perp h} = \left[\vec{h}_{\perp h} \times \vec{z}^0\right] = \frac{1}{\chi_h} \cdot \left[\vec{\nabla}_\perp \psi_h \times \vec{z}^0\right],$$

E-waves:

$$e_{ze} = \psi_e \cdot \vec{z}^0,$$

$$\vec{e}_{\perp e} = \frac{1}{\chi_e} \cdot \vec{\nabla}_\perp \psi_e,$$

$$\vec{h}_{\perp e} = \left[\vec{z}^0 \times \vec{e}_{\perp e}\right] = \frac{1}{\chi_e} \cdot \left[\vec{z}^0 \times \vec{\nabla}_\perp \psi_e\right].$$

For both H- and E-waves:

$$\vec{h}_{\perp h,e} = \left[\vec{z}^0 \times \vec{e}_{\perp h,e}\right], \; \vec{e}_{\perp h,e} = \left[\vec{h}_{\perp h,e} \times \vec{z}^0\right],$$

$$\int_S (h_{zh} \cdot h_{zh}) dS = \int_S \left(\vec{h}_{\perp h,e} \cdot \vec{h}_{\perp h,e}\right) dS = 1,$$

$$\int_S (e_{ze} \cdot e_{ze}) dS = \int_S \left(\vec{e}_{\perp h,e} \cdot \vec{e}_{\perp h,e}\right) dS = 1,$$

$$\chi_{e,h} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2},$$

$$K_{e,h} = \sqrt{\chi_{e,h}^2 - k^2}, \; k = \frac{\omega}{c},$$

Waveguide conductivity

$$Y_a = \frac{1}{Z_0} \cdot \begin{cases} \dfrac{K_h}{jk} & \text{for } a = h, \\ \dfrac{jk}{K_e} & \text{for } a = e, \end{cases} \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \text{ Ohm}.$$

3. Main normalized functions of cylinder cavities

H-modes:

$$\vec{\varepsilon}_h = \sqrt{\frac{2}{L}} \cdot \frac{1}{\chi_h} \cdot \left[\vec{\nabla}_\perp \psi_h \times \vec{z}^0\right] \cdot \sin\frac{l\pi}{L} z \,,$$

$$\vec{h}_h = \sqrt{\frac{2}{L}} \cdot \frac{1}{k_h} \cdot \left(\frac{1}{\chi_h} \cdot \vec{\nabla}_\perp \psi_h \cdot \frac{l\pi}{L} \cdot \cos\frac{l\pi}{L} z + \chi_h \psi_h \cdot \vec{z}^0 \cdot \sin\frac{l\pi}{L} z \right),$$

E-modes:

$$\vec{e}_e = \sqrt{\frac{2-\delta_{0l}}{L}} \cdot \frac{1}{k_e} \cdot \left(\frac{1}{\chi_e} \cdot \vec{\nabla}_\perp \psi_e \cdot \frac{l\pi}{L} \cdot \sin\frac{l\pi}{L} z - \chi_e \psi_e \cdot \vec{z}^0 \cdot \cos\frac{l\pi}{L} z \right),$$

$$\vec{h}_e = \sqrt{\frac{2-\delta_{0l}}{L}} \cdot \frac{1}{\chi_e} \cdot \left[\vec{z}^0 \times \vec{\nabla}_\perp \psi_e\right] \cdot \cos\frac{l\pi}{L} z \,.$$

There are: L is the high of cavity, $l = 0,1,2\ldots$,

$$k_{h,e} = \sqrt{\chi_{h,e}^2 + \left(\frac{l\pi}{L}\right)^2} = \omega_{h,e}\sqrt{\varepsilon_0 \mu_0} \,.$$

For both H- and E-waves:

$$\int_V \vec{e}_{h,e}^{\,2}\, dV = 1\,, \quad \int_V \vec{h}_{h,e}^{\,2}\, dV = 1\,.$$

4. Simple case of exciting coefficients

The simple example (see Fig. 4):

for waveguide slot $\qquad \Im(z,x) = \sqrt{\frac{2}{hl}} \cdot \cos\frac{\pi}{l} x \,,$

where x is the coordinate along slot, l is the length of slot and h is the high of slot (z size), but the same distribution function for cavity slot

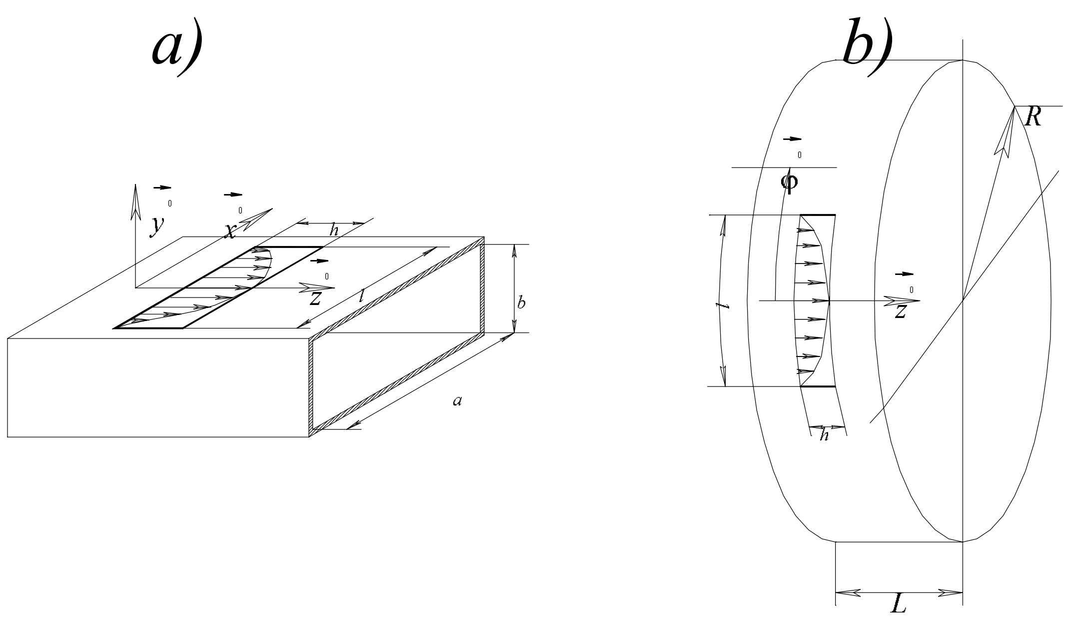

Figure 4. a) rectangular waveguide with coupling aperture in wide wall,
b) cylindrical cavity with coupling aperture in side-wall.

$$\vec{E}_\tau = e\cdot\vec{\mathcal{E}}'(z,\varphi) \equiv e\cdot \mathcal{E}'(z,\varphi)\cdot \vec{z}^{\,0},$$

where $\quad \mathcal{E}'(z,\varphi) = \sqrt{\dfrac{2}{h\cdot l}}\cdot \cos\dfrac{\pi R}{l}\varphi, \qquad -\dfrac{l}{2R} \le \varphi \le \dfrac{l}{2R}.$

In this case coefficients f_a (8) and m_a (5) are equal to

$$f_a = \frac{\pi}{\chi_a\sqrt{a\cdot b}}\cdot\sqrt{\frac{l}{h}}\cdot\sin\frac{m\pi}{2}\cdot\cos\frac{n\pi}{2}\cdot\left\{\frac{\sin\left[\frac{\pi}{2}\cdot\left(1-\frac{ml}{a}\right)\right]}{\frac{\pi}{2}\cdot\left(1-\frac{ml}{a}\right)} + \frac{\sin\left[\frac{\pi}{2}\cdot\left(1+\frac{ml}{a}\right)\right]}{\frac{\pi}{2}\cdot\left(1+\frac{ml}{a}\right)}\right\}\cdot\begin{cases}\frac{m}{a}\cdot\sqrt{2-\delta_{0n}} & for\, H_{mn}\ waves\ (idex\ a=h),\\ \frac{n}{b}\cdot\sqrt{2} & for\, E_{mn}\ waves\ (idex\ a=e).\end{cases}$$

$$m_a = f_a\cdot h\cdot\frac{\mathrm{sh}\frac{K_a h}{2}}{\frac{K_a h}{2}}.$$

The simple E_{010} -mode cylindrical cavity example (R - the radius of cavity, L - the length of cavity):

$$\varepsilon_z(r) = \sqrt{\frac{2}{L}}\cdot\sqrt{\frac{2}{\pi}}\cdot\frac{1}{R}\cdot\frac{J_0\left(\chi_{01}\frac{r}{R}\right)}{J_1(\chi_{01})}, \quad \chi_{01} = 2.405.$$

$$h_\varphi(r=R) = \frac{1}{\sqrt{\pi\cdot L\cdot R}},$$

The coefficient M_S (11) and integral Int (25) are equal to

$$M_S = \int_{-\frac{l}{2R}}^{\frac{l}{2R}}\int_{-h/2}^{h/2}\mathcal{E}'(z,\varphi)\cdot h_\varphi(r=R)\cdot R\,d\varphi\,dz = \frac{2}{\pi\cdot R}\cdot\sqrt{\frac{2hl}{\pi L}},$$

$$Int = \int_z \varepsilon_z(z)\cdot\cos(\omega t)dz \equiv \int_{-L/2}^{L/2}\varepsilon_z(z)\cdot\cos\left(\frac{2\pi\cdot z}{\beta_e\lambda}\right)dz = \frac{\sin\frac{\theta\cdot L}{2p}}{\frac{\theta\cdot L}{2p}}\cdot\frac{1}{R\cdot J_1(\chi_{01})}\cdot\sqrt{\frac{L}{\pi}},$$

where θ – operate mode,

$p = \dfrac{\theta}{2\pi}\cdot\lambda\cdot\beta_e$ – period of system (must be more then L),

$\beta_e = v_e/c$ – relative velocity of accelerating electrons.

5. Equivalent circuit for waveguide-cavity coupling

We assumed that nonpropagating waves in waveguide exist only near the aperture and didn't affect to the other slots. In other words only basic propagating wave H_{10}-mode exist in waveguide. In this case we can draw the equivalent circuit for rectangular waveguide with coupling slot (Fig. 5a). The power supply at e Volt is loading on conductivity Y_W (see (16)). Y_0 is the waveguide conductivity of H_{10}-mode. This voltage source excites in waveguide only H_{10}-mode with amplitudes (see (4))

$$U_0^+ = -\frac{e}{2}\cdot m_W, \qquad U_0^- = \frac{e}{2}\cdot m_W. \tag{A1}$$

The cavity stored energy (see Fig. 5b)

$$W = \frac{CU_C^2}{2} = \frac{\varepsilon_0}{2}\cdot\int_V u^2(\vec{\varepsilon}\cdot\vec{\varepsilon})dV = \frac{\varepsilon_0\cdot u^2}{2} = \frac{L\cdot I_L^2}{2} = \frac{\mu_0}{2}\cdot\int_V i^2(\vec{h}\cdot\vec{h})dV = \frac{\mu_0\cdot i^2}{2}.$$

Assume that $U_C = u\cdot\sqrt{k_0}$ and $I_L = i\cdot\sqrt{k_0}$. Then parameters of equivalent circuit will be: $C = \varepsilon_0/k_0$,

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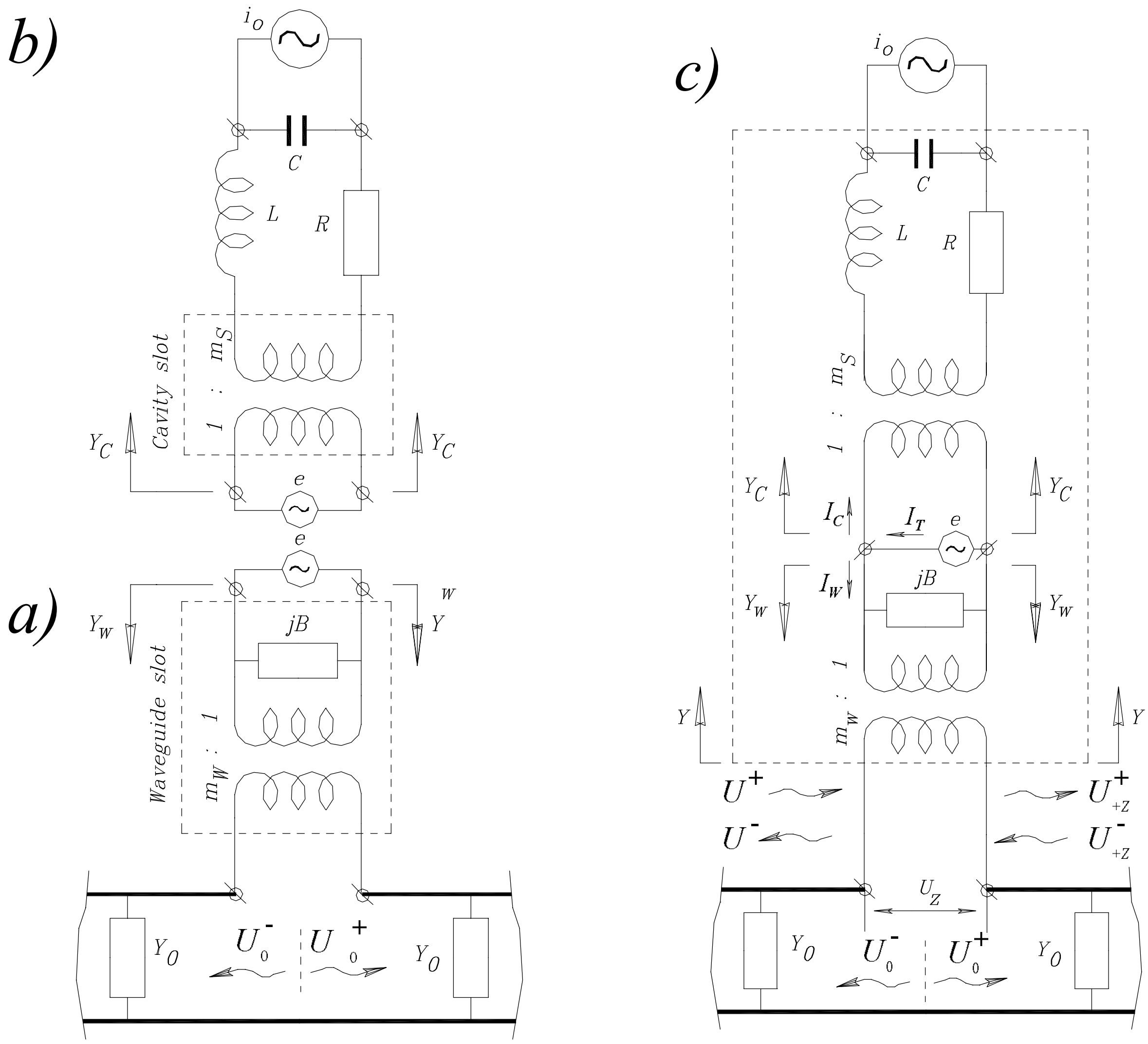

Figure 5. Equivalent circuit: a) waveguide with coupling slot in wide wall;
b) cavity with coupling slot in flank; c) total circuit for waveguide-cavity coupling.

$$L = \mu_0 / k_0, \qquad R = \frac{\omega_0 L}{Q_0} = \frac{\omega_0 \mu_0}{Q_0 k_0} = \frac{Z_0}{Q_0}. \tag{A2}$$

The amplitude u of cavity (see (24))

$$u = j\frac{M_S\, Q_0}{k\left(1+2jQ_0\cdot\delta\, k\right)}\cdot e - \frac{Q_0 Z_0\left(1 - j\dfrac{k_0}{kQ_0}\right)}{k_0\left(1+2jQ_0\cdot\delta\, k\right)}\cdot j_0. \tag{A3}$$

But capacity voltage U_C from equivalent circuit

$$U_C = -\frac{1}{j\omega C}\cdot\frac{m_S}{R + j\omega L + \dfrac{1}{j\omega C}}\cdot e - \frac{\dfrac{1}{j\omega C}\cdot\left(R + j\omega L\right)}{\left(R + j\omega L\right) + \dfrac{1}{j\omega C}}\cdot i_0.$$

Or using parameters of equivalent circuit (A2):

$$U_C = \sqrt{k_0}\cdot\left\{ j\frac{\left(\sqrt{k_0}\cdot m_S\right)\cdot Q_0}{k\left(1+2jQ_0\cdot\delta k\right)}\cdot e - \frac{Q_0 Z_0\left(1-j\dfrac{k_0}{kQ_0}\right)}{k_0\left(1+2jQ_0\cdot\delta k\right)}\cdot\left(i_0\sqrt{k_0}\right)\right\}. \tag{A4}$$

Compare (A3) and (A4) we obtain the transformation ratio m_S and current i_0:

$$m_S = \frac{M_S}{\sqrt{k_0}},\ i_0 = \frac{j_0}{\sqrt{k_0}}. \tag{A5}$$

The cavity conductivity Y_C (see (18))

$$Y_C = \frac{Q_0 M_S^2}{Z_0 k_0\left(1+2jQ_0\cdot\delta k\right)} \equiv \frac{m_S^2}{R + j\omega L + \dfrac{1}{j\omega C}}, \tag{A6}$$

Fig. 4c shows total circuit for waveguide-cavity coupling. Voltage *e* according to (15) is equal to

$$e\cdot\left(Y_W + Y_C\right) \equiv e\cdot Y_T = I_T = I_W + I_C. \tag{A7}$$

There Y_T is total conductivity for "*e*"- point (see Fig. 5c):

$$Y_T = Y_W + Y_C = \frac{Y_0 m_W^2}{2} + jB + Y_C. \tag{A8}$$

Only propagating waves H_{10}-mode exist in waveguide with amplitudes $U^{\pm}$ on the left of coupling slot and $U^{\pm}_{+Z}$ on the right of coupling slot. So the waveguide exciting currents I_W and I_C in (A7) or (15) are equal to

$$I_W = Y_0\cdot\left(U^+ - U^-_{+Z}\right)\cdot m_W, \tag{A9}$$

$$I_C = -j\cdot\frac{j_0\cdot Q_0 M_S}{k\left(1+2jQ_0\cdot\delta k\right)} \equiv \frac{1}{j\omega C}\cdot\frac{m_S\cdot i_0}{R + j\omega L + \dfrac{1}{j\omega C}}. \tag{A10}$$