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CALCULATION OF REGGEON VERTICES IN QCD

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Аннотация

The method of calculation of effective vertices of interaction of the Reggeized gluon and quark with particles in QCD in the next-to-leading order is developed. The method is demonstrated in the case of already known vertices of both gluon-gluon and quark-quark transitions in the scattering of gluons and quarks on the Reggeized gluon. It is used for the calculation of the gluon-quark transition in the scattering on the Reggeized quark.

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1 Introduction

Investigation of a possibility of Reggeization of elementary particles in field theories was started in Ref. [1], where it was shown that the electron in Quantum Electrodynamics (QED) does Reggeizes in perturbation theory. Soon after it was proved [2] that, contrary to the electron, the photon in QED remains elementary. The problem of Reggeization of elementary particles in non-Abelian gauge theories (NAGT's) naturally appeared with the development of these theories. In Ref. [3] it was found that the criteria of Reggeization formulated by Mandelstam [2] are fulfilled in NAGT's for all particles. The notion "Reggeization" was used in Refs. [1, 3] for the disappearance, because of radiative corrections, of the non-analytic terms in the complex angular momentum plane, related to elementary particle exchanges in the Born approximation. In Refs. [4, 5] it was shown by direct calculations in the leading logarithmic approximation (LLA) that, for the gauge bosons of NAGT's, this term can be understood in a much stronger sense. That means not only the existence of the Reggeon with the quantum numbers of the gauge boson, negative signature and trajectory $j(t) = 1+\omega(t)$ passing through 1 at $t = m_V^2$, m_V being a gauge boson mass, but also that in each order of perturbation theory at large c.m.s. energies $\sqrt{s}$ this Reggeon gives the leading contribution to the scattering amplitudes with the quantum numbers of the gauge boson and the negative signature in the t-channel. Below we use the term "Reggeization" in such strong sense. In Ref. [6] it was demonstrated, also by direct calculations in the LLA, the Reggeization of fermions in NAGT's. Therefore, in Quantum Chrodynamics (QCD), which is a particular case of NAGT, all elementary particles, i.e. quarks and gluons, do Reggeize.

The Reggeization of elementary particles plays a very important role for the description of high energy processes. The gluon Reggeization is the basis of the famous BFKL equation [5]. The Pomeron, which determines the high energy behaviour of cross sections, in QCD is a compound state of two Reggeized gluons. The Odderon, responsible for the difference of particle and antiparticle cross sections, can be constructed as a compound state of three Reggeized gluons [7]. One could also construct colorless objects from Reggeized quarks and antiquarks which should be relevant to phenomenological Reggeon trajectories successfully used for the description of processes with exchange of quantum numbers.

For phenomenological applications the LLA is not satisfactory, since neither the scale of energy, nor the scale of momentum transfers in the argument of the coupling constant are fixed in this approximation. The calculation of radiative corrections to the kernel of the BFKL equation has taken many years of a hard work [8-13]. Three years ago the kernel of the BFKL equation was obtained at the next-to-leading order (NLO) [14] for the case of the forward scattering, i.e. the momentum transfer $t = 0$ and the vacuum quantum numbers in the t-channel. Although in the $\overline{MS}$ renormalization scheme with a "reasonable" scale setting for the running QCD coupling radiative corrections appear to be very large, use of non-abelian renormalization schemes and the BLM procedure for the scale setting opens a way for applications of the NLO BFKL in the high energy phenomenology [15]. Good possibilities for applications to deep inelastic processes are given by the renormalization group improvement of the BFKL equation [16].

Due to the Reggeization of quarks and gluons, an important role in high energy QCD belongs to the vertices of Reggeon-particle interactions. In particular, these vertices are necessary for the determination of the BFKL kernel. For their calculation powerful methods based on analyticity and unitarity were developed starting from the LLA [4, 5]. They were intensively used for the calculation of the Reggeon-particle interaction vertices in the NLO. But these methods are not adjusted for the direct calculation of the vertices, which are obtained from the comparison of appropriate scattering amplitudes with their Reggeized form, so that to obtain a vertex one must calculate a whole amplitude. This seems to be too complicated, and a method of calculation of Reggeon vertices themselves is desirable. In principle, it could be based on the effective action for the interactions of the Reggeized quarks and gluons with the usual QCD partons [17, 18]. However, a straightforward application of the effective action leads to vertices depending on the auxiliary parameter η in the rapidity space [17], serving by a cut-off in the relative longitudinal momenta of the produced particles. In order to find the correspondence between these vertices and the conventional ones we need again to know the whole amplitudes. Therefore we adopt the approach based on the properties of the integrals corresponding to the Feynman diagrams with two particles in the t-channel, noticed in Ref. [19]. It was already used in Ref. [19] for the determination of the vertex for the quark-antiquark production in the interaction of the virtual photon with the Reggeized gluon.

In this paper we develop the method of the direct calculation of the Reggeon-particle vertices. In the next Section we explain the essence of the method both in the case of the Reggeized gluon (Subsection 2.1) and the Reggeized quark (Subsection 2.2). In Section 3 we show applications of the method. In Subsection 3.1 the quark-quark-Reggeon (QQR) vertex is considered. Subsection 3.2 is devoted to the more complicated case of the gluon-gluon-Reggeon (GGR) vertex. In Subsection 3.3 we calculate the vertices for the case in which the Reggeon is the Reggeized quark. Finally, Section 4 contains a discussion of the method and the obtained results.

2 Method of calculation

2.1 Reggeized gluon vertices

Let us consider the amplitude of the process $A + B \to A^{'} + B^{'}$ at large c.m.s. energy squared $s = (p_A + p_B)^2$ and fixed momentum transfer squared $t = (p_{A'} - p_A)^2 = (p_B - p_{B'})^2$ supposing that the one-gluon state is possible in the t-channel. Then the projection of this amplitude on the color octet state in the t-channel taken with the negative signature (i.e. antisymmetrized with respect to $s \leftrightarrow u \approx -s$) has the form

$$A_{8^-} = \Gamma^i_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^i_{B'B} \ , \tag{1}$$

where $\Gamma^i_{A'A}$ are the Reggeon vertices for the $A \to A^{'}$ transitions and $j(t) = 1 + \omega(t)$ is the gluon trajectory. In the leading order

$$\omega(t) = \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}}\frac{N}{2}\int\frac{d^{D-2}k_\perp}{k_\perp^2(q-k)_\perp^2} = -\frac{g^2 N\Gamma(1-\epsilon)(\vec{q}^{\;2})^\epsilon}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \ . \tag{2}$$

Here $t = q^2 \approx q_\perp^2 = -\vec{q}^{\;2}$, the subscript $\perp$ denotes components transverse to the plane of initial momenta (we use also the vector sign for these components), N is the number of colours (N=3 in QCD) and $D = 4 + 2\epsilon$ is the space-time dimension taken different from 4 for regularization. The Reggeon vertices describing the quark-quark and gluon-gluon transitions in the leading order and in the helicity basis are quite simple:

$$\Gamma^{(0)i}_{A'A} = g\delta_{\lambda_{A'}\lambda_A}\langle A'|T^i|A\rangle \ , \tag{3}$$

where $\langle A'|T^i|A\rangle$ stands for the matrix element of the colour group generator in the corresponding representation. It is necessary to note that the form (1) has a general nature and is valid not only for such elementary transitions; moreover, $A^{'}$ and $B^{'}$ (as well as A and B) can be groups of particles with fixed (not growing with s) invariant masses.

In the leading order the Reggeon vertices can be easily obtained from the Feynman diagrams for the process $A + B \rightarrow A^{'} + B^{'}$ in the Born approximation. Evaluating the diagrams we use the light-cone momenta p_1 and p_2, so that in the general case of massive particles or clusters of particles A and B their momenta p_A and p_B are presented as

$$p_A = p_1 + \frac{m_A^2}{s} p_2 \ , \quad p_B = p_2 + \frac{m_B^2}{s} p_1 \ , \quad s = 2p_1p_2 \ . \tag{4}$$

We use the Feynman gauge for virtual gluons; for external gluons we use physical polarizations with different gauge-fixing conditions for gluons moving along p_A and p_B, so that if the gluon momentum k has large component along p_1 (p_2), its polarization vector $e(k)$ satisfies equations $e(k)k = e(k)p_2 = 0$ ($e(k)k = e(k)p_1 = 0$). For the gluon propagator connecting the vertices μ and ν with momenta predominantly along p_1 and p_2 respectively we do the usual trick of retaining only the first term in the decomposition of the metric tensor

$$g^{\mu\nu} = \frac{2p_2^\mu p_1^\nu}{s} + \frac{2p_1^\mu p_2^\nu}{s} + g_\perp^{\mu\nu} \rightarrow \frac{2p_2^\mu p_1^\nu}{s} \tag{5}$$

in the numerator of the propagator. Using this trick one obtains that in the leading order the Reggeon vertex $\Gamma_{A'A}^{(0)i}$ is equal to the $Ag \rightarrow A'$ amplitude, where the gluon g has colour index i and polarization vector equal to $-p_2/s$. But in the next orders of perturbation theory this relation is evidently broken. Moreover, contrary to usual QCD vertices (such as, for example, the quark-quark-gluon vertex) for which we can draw a definite set of Feynman diagrams with perfectly defined rules for the calculation of their contributions, we have not such rules for the Reggeon vertices. These vertices are extracted from the comparison of radiative corrections to the $A + B \rightarrow A^{'} + B^{'}$ scattering amplitudes with the Reggeized form expressed in Eq. (1). At the NLO the Feynman diagrams for the process $A+B \rightarrow A'+B'$ can be divided into four classes. The first class includes corrections to the t-channel gluon propagator, the second and third are related to corrections to the interaction of the t-channel gluon with the particles A, A' and B, B' correspondingly, and the last one contains the diagrams with the two-gluon exchange in the t-channel. The contributions of the diagrams of the first three classes have the same dependence on s as the Born amplitudes; moreover, the contributions of the first and second (first and third) classes depend on properties of the particles B, B' (A, A') in the same way as the Born amplitudes. It is evident therefore that the contribution of the diagrams of the second (third) class must be attributed to the vertex $\Gamma^{i}_{A'A}$ $(\Gamma^{i}_{B'B})$, whereas the contribution of the first class must be divided in equal parts between these vertices. Consequently, only the two-gluon exchange diagrams create a problem. In their contributions corrections to both vertices and the trajectory are mixed, so that the problem is to separate them. Note that there is a well known uncertainty in the vertices evident from Eq. (1): we can change the vertices changing simultaneously the energy scale. The scale which we have chosen is $-t$, as it is fixed in Eq. (1).

Let us analyze the contributions of the two gluon exchange diagrams. They are shown schematically in Fig.1. Using the Sudakov decomposition for the gluon momenta:

$$k = \beta p_1 + \alpha p_2 + k_\perp \ , \qquad q = p_{A'} - p_A = p_B - p_{B'} = \beta_q p_1 + \alpha_q p_2 + q_\perp \ , \tag{6}$$

we get for the Sudakov variables

$$\alpha_q = \frac{\vec{q}^{\,2} + m_{A'}^2 - m_A^2}{s} \ , \quad \beta_q = -\frac{\vec{q}^{\,2} + m_{B'}^2 - m_B^2}{s} \ , \quad d^D k = \frac{s}{2} d\alpha d\beta d^{D-2} k_\perp \ . \tag{7}$$

Evaluating the diagrams of Fig.1 we retain, as usually, only the first term in the decomposition of the metric tensor (5).

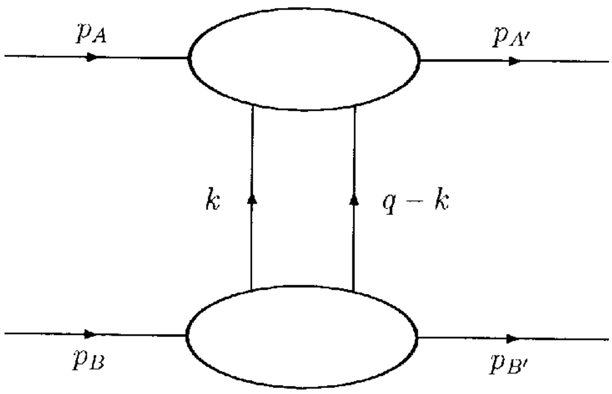

Рис. 1: Schematic representation of diagrams with two-particle state in the t-channel for the process $A + B \to A' + B'$ at NLO.

The first important observation is that in the region $|\alpha| \ll 1 \quad (|\beta| \ll 1)$ we can factor out the vertex $\Gamma^{(0)i}_{B'B} \quad (\Gamma^{(0)i}_{A'A})$ from the diagrams of Fig.1. This is evident for the colour structure, since we consider the colour octet and negative signature in the t-channel, so that the virtual gluons in the t-channel must be taken in the antisymmetric colour octet. The operator $\hat{\mathcal{P}}_{8^-}$ for the projection of the two-gluon colour states on the antisymmetric colour octet is given by

$$\langle c_1 c_1' | \hat{\mathcal{P}}_{8^-} | c_2 c_2' \rangle = \frac{f_{c_1 c_1' c} f_{c_2 c_2' c}}{N} \ , \tag{8}$$

where f_{abc} are the structure constants of the colour group. The relation

$$\frac{f_{c_2 c_2' c}}{N} \left(T^{c_2'} T^{c_2} \right)_{B'B} = -\frac{i}{2} T^c_{B'B} \tag{9}$$

defines the relevant colour coefficients. For the ordinary spin structure this observation is almost evident as well in the case when the particles A and A' (B and B') are quarks, since the numerators of the quark propagators at the lower (upper) parts of the diagram Fig.1 are surrounded by $\not{p}_1$ ($\not{p}_2$) due to Eq. (5). In the gluon case it is not difficult to see too, though it is less evident. Let us consider, for definiteness, the lower part of the diagram Fig.1, assuming that the particles B and B' are gluons. Remind that we use the Feynman gauge for virtual gluons and physical polarizations e_B and $e_{B'}$ ($e_B p_B = 0 = e_{B'} p_{B'}$) in the gauge $e_B p_1 = e_{B'} p_1 = 0$. Taking the polarization vectors of both t-channel gluons equal to p_1/s according to Eq. (5), we obtain the following expression for the lower part of the diagram of Fig.1 projected by the operator defined in Eq. (8) on the antisymmetric colour octet state:

$$\begin{aligned}
\mathcal{M}^{c_1 c_1'}_{BB'} &= \frac{g^2}{2} f_{c_1 c_1' c} T^c_{B'B} e^{\beta}_B \frac{p_1^\mu}{s} \frac{p_1^\nu}{s} e^{\beta'}_{B'} \\
&\times \left[\frac{\gamma_{\beta\mu}{}^{\rho}(p_B, -k, k - p_B) \gamma_{\rho\nu\beta'}(p_B - k, k - q, -p_{B'})}{(p_B - k)^2} \right. \\
&\left. - \frac{\gamma_{\beta\nu}{}^{\rho}(p_B, k - q, -k - p_{B'}) \gamma_{\rho\mu\beta'}(p_{B'} + k, -k, -p_{B'})}{(p_{B'} + k)^2} \right] \ .
\end{aligned} \tag{10}$$

Here c_1 and c_1' are the colour indices of the gluons with momenta k and $q - k$ correspondingly and

$$\gamma^{\mu\nu\rho}(k_1, k_2, k_3) = [g^{\mu\nu}(k_1 - k_2)^\rho + g^{\mu\rho}(k_3 - k_1)^\nu + g^{\nu\rho}(k_2 - k_3)^\mu] \tag{11}$$

is the three-gluon vertex. The convolution in Eq. (10) gives us

$$\mathcal{M}^{c_1 c_1'}_{BB'} = \frac{g^2}{2} f_{c_1 c_1' c} T^c_{B'B} \times$$

$$\times \left(\frac{[-e_B^\rho(1-\alpha/2) - 2(e_B k)p_1^\rho/s]\,[-e_{B'\rho}(1-\alpha/2) - 2(e_{B'}(k-q))p_{1\rho}/s]}{(p_B-k)^2} \right.$$

$$\left. - \frac{[-e_B^\rho(1+\alpha/2) - 2(e_B(q-k))p_1^\rho/s]\,[-e_{B'\rho}(1+\alpha/2) + 2(e_{B'}(k))p_{1\rho}/s]}{(p_{B'}+k)^2} \right) =$$

$$= \frac{g^2}{2} f_{c_1 c_1' c} T^c_{B'B} (e_B e_{B'}) \left(\frac{(1-\alpha/2)^2}{(p_B-k)^2} - \frac{(1+\alpha/2)^2}{(p_{B'}+k)^2} \right) . \qquad (12)$$

Since $(e_B e_{B'}) = -\delta_{\lambda_B \lambda_{B'}}$ and in the case under consideration the particles B and B' are massless gluons, so that $m_B = m_{B'} = 0$, the result at small α can be written as

$$\mathcal{M}^{c_1 c_1'}_{BB'} = -\frac{g}{2} f_{c_1 c_1' c} \left(\frac{1}{(p_B-k)^2 - m_B^2} - \frac{1}{(p_{B'}+k)^2 - m_{B'}^2} \right) \Gamma^{(0)c}_{B'B} . \qquad (13)$$

It is easy to see that in this form the obtained equation is valid also for the case when the particles B and B' are quarks. It completes the proof of the factorization of the vertex $\Gamma^{(0)i}_{B'B}$ at $|\alpha| \ll 1$.

Let us consider now the basic integrals I and I' for the diagrams of Fig.1. The first of them is

$$I = \int \frac{d^D k}{(2\pi)^D i} \times \qquad (14)$$

$$\times \frac{1}{(k^2+i0)((q-k)^2+i0)((p_A+k)^2 - m_A^2 + i0)((p_B-k)^2 - m_B^2 + i0)} ,$$

and the second can be obtained from I by the substitution $p_B \leftrightarrow -p_{B'}$. Following Ref. [19], we introduce three regions of the α and β variables:

$$\begin{array}{lll} \text{the central region:} & |\alpha| < \alpha_0 , & |\beta| < \beta_0 , \\ \text{the A-region:} & |\beta| \geq \beta_0 , & |\alpha| < \alpha_0 , \\ \text{the B-region:} & |\alpha| \geq \alpha_0 , & |\beta| < \beta_0 , \end{array} \qquad (15)$$

α_0 and β_0 being chosen so that

$$\alpha_0 << 1 , \qquad \beta_0 >> 1 , \qquad s\alpha_0\beta_0 >> |t| . \qquad (16)$$

As well as in Ref. [19] we will take the limit $s\alpha_0\beta_0 \to \infty$, while $\alpha_0 \to 0$ and $\beta_0 \to 0$, when $s \to \infty$. Our definition of the regions differs from that used in Ref. [19], but this difference concerns only the regions where $|\alpha|$ and $|\beta|$ are both "large" ($|\beta| > \beta_0$, $(|\alpha| > \alpha_0)$) and is therefore nonessential, since these regions give a contribution of relative order $|t|/(s\alpha_0\beta_0)$ which vanishes in the limit $s \to \infty$ [19]. Consequently, it is sufficient to calculate only the contributions of the three regions defined by the relations (15) and (16). Moreover, calculating the contribution of the region A (B) we can remove the restriction on α (β) that can simplify calculations. It is clear that the contribution of the region A (B) after extracting the factor $2s\Gamma^{(0)i}_{B'B}/t$ $(2s\Gamma^{(0)i}_{A'A}/t)$ must be attributed to the vertex $\Gamma^i_{A'A}$ ($\Gamma^i_{B'B}$). Since in the A-region

$$(p_B - k)^2 - m_B^2 \simeq -(p_{B'} + k)^2 + m_{B'}^2 \simeq -2p_2k = -s\beta \ , \tag{17}$$

we obtain for the contribution of this region, using Eqs. (8), (9) and (13) and restoring all relevant coefficients,

$$\Gamma^{i(\mathrm{A})}_{A'A} = g\frac{t}{2s}T^i_{c_1c_1'}\int\frac{d^Dk}{(2\pi)^Di}\frac{p_2^\mu p_2^\nu A^{c_1c_1'}_{\mu\nu}(p_A,k;p_{A'},k-q)}{(k^2+i0)((q-k)^2+i0)(p_2k)}\theta(2|p_2k|-\beta_0 s) \ . \tag{18}$$

Here $T^i_{ab} = -if_{iab}$ are the matrix elements of the colour group generators in the adjoint representation, $A^{c_1c_1'}_{\mu\nu}(p_A,k;p_{A'},k-q)$ is the one-particle irreducible in the t-channel part of the amplitude of the process $A + g(k) \to A' + g(k-q)$. Moreover we used the possibility, which we discussed above, to remove the restriction $|\alpha| \ll \alpha_0$ in the definition (15) of the region A.

In the central region the integrand in Eq. (14) can be considerably simplified. First of all, evidently we can neglect the longitudinal components of the momentum transfer q. Remind that the Reggeon vertices are calculated taking the limit $s \to \infty$ before the limit $\epsilon \to 0$, so that performing the analysis we can think of the transverse momenta of the t-channel gluons being fixed (not depending on s). Since $\beta_0 \to 0$ when $s \to \infty$, it is easy to see that, whereas the pole in the complex plane of α from the third denominator in Eq. (14) is found at fixed values of $s|\alpha|$, all other poles are in the region $s|\alpha| \to \infty$. Therefore the contour of the integration over α can be shifted to the region $s|\alpha| \to \infty$. An analogous conclusion is valid for the integration over β. It means that we can use the relations (17) in the central region too; moreover, along with these relations we can put also

$$(p_A + k)^2 - m_A^2 \simeq -(p_{A'} - k)^2 + m_{A'}^2 \simeq 2p_1k = s\alpha \ , \tag{19}$$

so that we obtain

$$I^{\text{central}} = \frac{s}{2}\int \frac{d^{D-2}k_\perp}{(2\pi)^D i}\times \tag{20}$$

$$\int\limits_{-\alpha_0}^{\alpha_0}\int\limits_{-\beta_0}^{\beta_0} \frac{d\alpha d\beta}{(s\alpha\beta + k_\perp^2 + i0)(s\alpha\beta + (q-k)_\perp^2 + i0)(s\alpha + i0)(-s\beta + i0)} \simeq$$

$$\frac{1}{2s}\int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}(k_\perp^2 - (q-k)_\perp^2)}\left[\frac{1}{k_\perp^2}\ln\!\left(\frac{-s\alpha_0\beta_0}{-k_\perp^2}\right) - \frac{1}{(q-k)_\perp^2}\ln\left(\frac{-s\alpha_0\beta_0}{-(q-k)_\perp^2}\right)\right].$$

The integrals over $k_\perp$ can be easily calculated and we get [19]

$$I^{\text{central}} = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\frac{(\vec{q}^{\;2})^\epsilon}{st}\left[\ln\left(\frac{-s\alpha_0\beta_0}{\vec{q}^{\;2}}\right) - \psi(1) + \psi(1-\epsilon)\right.$$
$$\left. - 2\psi(\epsilon) + 2\psi(2\epsilon)\right] - \frac{\omega^{(1)}(t)}{g^2 N st}\left[\ln\left(\frac{-s}{\vec{q}^{\;2}}\right) + \phi(\alpha_0) + \phi(\beta_0)\right] , \tag{21}$$

where

$$\phi(z) = \ln z + \frac{1}{2}\left(\frac{1}{\epsilon} - \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon) + 2\psi(1+2\epsilon)\right). \tag{22}$$

Let us emphasize that this contribution does not depend on masses of the particles A and B. In the central region both vertices $\Gamma^{(0)i}_{A'A}$ and $\Gamma^{(0)i}_{B'B}$ are factored out, so that we have

$$A_{8^-}^{\text{central}} = -g^2 N s^2 \Gamma^{(0)i}_{A'A}\left[I^{\text{central}} - I'^{\text{ central}}\right]\Gamma^{(0)i}_{B'B} =$$
$$\Gamma^{(0)i}_{A'A}\frac{2s}{t}\omega^{(1)}(t)\left[\frac{1}{2}\ln\left(\frac{-s}{\vec{q}^{\;2}}\right) + \frac{1}{2}\ln\left(\frac{s}{\vec{q}^{\;2}}\right) + \phi(\alpha_0) + \phi(\beta_0)\right]\Gamma^{(0)i}_{B'B} . \tag{23}$$

The above relation was proved in Ref. [19] for the case of the process $\gamma^* q \to (q\bar{q})q$, where the $q\bar{q}$ pair is produced in the fragmentation region of the photon. But, since we have the gluon Reggeization (which was proved [20] in the LLA), and since the s-dependence can come only from the two-gluon exchange diagrams, it is clear that this relation is valid for any process. It can be proved for any particular process applying the trick (5) to propagators of both t-channel gluons. Clearly, the logarithmic terms in Eq. (23) correspond to the expansion of the power terms in Eq. (1), whereas all other contributions must be distributed between the corrections to the vertices $\Gamma^i_{A'A}$ and $\Gamma^i_{B'B}$. The way to do it is evident, so that

$$\Gamma^{i(\text{central})}_{A'A} = \Gamma^{(0)i}_{A'A}\omega^{(1)}(t)\phi(\beta_0) . \tag{24}$$

The intermediate parameter β_0 in Eq. (24) cancels when we combine this contribution with the contribution to $\Gamma^{i}_{A'A}$ from the A-region (18).

Let us summarize what we have obtained. To the one loop accuracy the Reggeon vertex $\Gamma^{(i)}_{A'A}$ can be presented as a sum of the following contributions:

-the $Ag \to A'$ amplitude, where the virtual gluon g has the colour index i and the polarization vector $-p_2/s$, with the gluon self-energy taken with the coefficient 1/2, just as for external particles;

-the sum of the contributions given by Eqs. (18) and (24). Remind that $A^{c_1c_1'}_{\mu\nu}(p_A, k; p_{A'}, k-q)$ in Eq. (18) is the one-particle irreducible in the t-channel part of the amplitude of the process $A + g(k) \to A' + g(k-q)$ in the leading order.

The essential point is that in this approach one can use known results for the gluon propagator and vertices, and the only new piece which must be calculated is $\Gamma^{i(\mathrm{A})}_{A'A}$, given by Eq. (18), which can be easily found (see below).

2.2 Reggeized quark vertices

The same approach can be used for the calculation of effective vertices of the Reggeon-particle interaction in the case when the Reggeon is the Reggeized quark. Due to the quark Reggeization the amplitudes with the quark quantum numbers in the t-channel and the positive signature can be presented [6] in a way analogous to the form (1):

$$A_{3^+} = \Gamma_{A'A}\frac{1}{m-\not{q}_\perp}\frac{1}{2}\left[\left(\frac{-s}{-t}\right)^{\delta(\not{q}_\perp)} + \left(\frac{s}{-t}\right)^{\delta(\not{q}_\perp)}\right]\Gamma_{B'B}\ , \qquad (25)$$

where m is the quark mass, $\Gamma_{A'A}$ are the vertices for the $A \to A'$ transitions and $\delta(\not{q}_\perp)$ determines the quark Regge trajectory. In the leading order [6]

$$\delta(\not{q}_\perp) = \delta^{(1)}(\not{q}_\perp) = \frac{g^2}{(2\pi)^{D-1}}C_F(\not{q}_\perp - m)\int\frac{d^{D-2}k_\perp}{(\not{k}_\perp - m)(q-k)^2_\perp} =$$

$$\frac{g^2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}2C_F(\not{q}_\perp - m)\int_0^1\frac{dx(x\ \not{q}_\perp + m)}{((1-x)(m^2+\vec{q}^{\ 2}))^{1-\epsilon}}\ , \qquad (26)$$

where $C_F = (N^2-1)/(2N)$. Elementary transitions due the to coupling with the Reggeized quark in the leading order are:

- the gluon $\to$ quark transition with the vertex

$$\Gamma^{(0)}_{QG} = -g\bar{u}(p_Q)\ \not{e}(p_G)t^G\ , \qquad (27)$$

where p_Q and p_G are the quark and gluon momenta, $e(p_G)$ is the gluon polarization vector and t^G is the colour group generator in the fundamental representation corresponding to the gluon colour state,

- the gluon $\rightarrow$ antiquark transition with the vertex

$$\Gamma^{(0)}_{\bar{Q}G} = -g \not{e}(p_G) t^G v(p_{\bar{Q}}) \ , \tag{28}$$

- the corresponding vertices for inverse transitions which are defined by

$$\Gamma^{(0)}_{GQ} = \bar{\Gamma}^{(0)}_{QG} \equiv \gamma^0 \left(\Gamma^{(0)}_{QG}\right)^\dagger \ , \ \Gamma_{G\bar{Q}} = \bar{\Gamma}^{(0)}_{\bar{Q}G} \equiv \left(\Gamma^{(0)}_{\bar{Q}G}\right)^\dagger \gamma^0 \ . \tag{29}$$

Remind that for gluons moving along p_1 (p_2) their polarization vectors must be taken in the gauge $e(p_G)p_2 = 0$ ($e(p_G)p_1 = 0$).

The above vertices can be easily obtained from the Feynman diagrams for the process $A + B \rightarrow A^{'} + B^{'}$ with the quark exchange in the t-channel in the Born approximation. Note, for definiteness, that $q = p_{A\prime} - p_A = p_B - p_{B\prime}$ is the momentum flowing along the quark line, and that the form (25) does not take into account the factor (-1) prescribed by the Feynman rules in the case when both the beginning and the end of the quark line correspond to the antiquark. So, in the leading order the vertices for the Reggeized quark simply coincide with the vertices for the ordinary quark, with properly chosen gluon polarizations. Of course, it is not the case in the next orders of perturbation theory. At the NLO, quite analogously to the case of the Reggeized gluon, the corresponding Feynman diagrams are divided into four classes. The first class corresponds to corrections to the t-channel quark propagator, the second and third are related to corrections to the interaction of the t-channel quark with the particles A, A' and B, B' respectively, and the last one contains the diagrams with the quark-gluon exchange in the t-channel. The contribution of the diagrams of the second (third) class must be attributed to the vertex $\Gamma^i_{A'A}$ ($\Gamma^i_{B'B}$), whereas the contribution of the first class must be divided in equal parts between these vertices. Again the only uncertainty comes from the two-particle exchange diagrams, schematically presented in Fig.1.

Again we use the Sudakov decomposition (6), retain only the first term in the decomposition of the metric tensor in the t-channel gluon propagator (5) and consider the three regions (15) of the α and β variables. In the region $|\alpha| < \alpha_0 \quad (|\beta| < \beta_0)$ we can factor out the vertex $\Gamma^{(0)}_{B'B} \quad (\Gamma^{(0)}_{A'A})$ from the diagrams of Fig.1. The relevant coefficients in the colour space can be calculated using the operator $\hat{\mathcal{P}}_3$ for the projection of the quark-gluon colour states in the $t-$channel on the fundamental representation:

$$\langle c\alpha|\hat{\mathcal{P}}_3|c'\alpha'\rangle = \langle\alpha|\frac{t^c t^{c'}}{C_F}|\alpha'\rangle \tag{30}$$

and the relations

$$t^c t^b t^c = \left(C_F - \frac{N}{2}\right) t^b \ , \qquad t^c t^i T^c_{ib} = \frac{N}{2} t^b \ . \tag{31}$$

Unlike the case of the two-gluon exchange, where use of the operator $\hat{\mathcal{P}}_{8^-}$ automatically gives us the negative signature amplitude, here we need to perform "signaturization"in an explicit way.

Let us consider the lower part of the diagram of Fig.1 in the case when the particle B is the quark and B' is the gluon. Without loss of generality we can take the exchanged particle with momentum k for the quark. Then the particle with momentum $q-k$ is the gluon; let its colour index be c. Denoting $\mathcal{M}^{c\beta'}_{B'B}$ the matrix element of the lower part of the diagram of Fig.1, with p_1/s instead of its polarization vector (see Eq. (5)) and having omitted the wave functions $u(p_B)$ and $e^{\beta'}_{B'}$ of the external particles, we obtain

$$\mathcal{M}^{c\beta'}_{B'B} = ig^2 \Bigg[\frac{p_{1\nu}}{s} \gamma^{\rho\nu\beta'}(p_B - k, k-q, -p_{B'})\gamma_\rho \frac{[t^{B'}, t^c]}{(p_B - k)^2}$$

$$- \gamma^{\beta'} \frac{(\not{p}_{B'} + \not{k} + m)}{(p_{B'} + k)^2 - m^2} \frac{\not{p}_1}{s} t^{B'} t^c \Bigg] \simeq$$

$$-ig^2 \gamma^{\beta'} \left[\frac{[t^{B'}, t^c]}{(p_B - k)^2}(1 - \frac{\alpha}{2}) + \frac{t^{B'} t^c}{(p_{B'} + k)^2 - m^2}(1+\alpha) \right] \ . \tag{32}$$

Here $\gamma^{\rho\nu\beta'}$ is the three-gluon vertex defined in Eq. (11). We have taken into account that $\mathcal{M}^{c\beta'}_{B'B}$ must be convoluted with the polarization vector $e^{\beta'}_{B'}$ of the gluon B' satisfying $e_{B'}p_1 = 0$, so that we have omitted the terms with $p_1^{\beta'}$. We have omitted also the terms with $\not{p}_1/s$ standing on the most left position. This can be done because in such a case we can move, with the help of the anticommutation relations for γ matrices, $\not{p}_1/s$ to act on the spinor from the upper blob of Fig.1 and then use the Dirac equation, getting only terms vanishing at large s.

Since we include in Eq. (32) an extra dependence on s through the polarization vector of the t-channel gluon, in order to take the amplitude with the positive signature we must antisymmetrize the last expression with respect to the substitution $p_B \leftrightarrow -p_{B'}$. In the A-region ($|\alpha| < \alpha_0, \ \ \beta \geq \beta_0$) it gives us

$$e^{\beta'}_{B'} \mathcal{M}^{(+)c\beta'}_{B'B} u(p_B) = -i\frac{g^2}{2} t^c \left[\frac{1}{2p_2 k} - \frac{1}{-2p_2 k} \right] t^{B'} \ \not{e}_{B'} u(p_B) \ , \tag{33}$$

that proves the factorization of the vertex $\Gamma^{(0)}_{B'B} = -gt^{B'}\not{e}_{B'}u(p_B)$ in the case when the particle B is the quark and B' is the gluon. Pay attention that the matrix t^c in Eq. (33) performs the projection of the quark-gluon state in the t-channel on the fundamental representation (see Eq. (30)), so that only this representation does survive in the positive signature at small $|\alpha|$. The case when the particle B is the gluon and B' is the antiquark can be considered quite similarly. As a result one obtains the factorization of the vertex with $\Gamma^{(0)}_{B'B}$ with the same coefficient as in Eq. (33).

Note that although the two terms in the square brackets in Eq. (33) are equal in the A-region, they are written separately. It is done in order to have a possibility to use this expression in the central region as well. The basic integrals for the considered case of the quark-gluon state in the t-channel differ in the central region from the corresponding integrals for the two-gluon state because of the non zero quark mass and the numerator $\not{k}_\perp + m$ of the quark propagator; however it does not change the properties of the integrals which were discussed after Eq. (18). The contour of integration over β in the central region can be shifted to the region $s|\beta| \to \infty$, so that the approximations used in the derivation of Eq. (33) can be used in the central region too; the only difference is that in the central region we have to take into account the positions of the poles in the complex β-plane, which are fixed by the $+i0$ prescription in the denominators of Eq. (33) and are different for the two terms there.

As well as in the Reggeized gluon case, the contribution of the region A must be attributed to the vertex $\Gamma_{A'A}$. Extracting the factor $(m - \not{q}_\perp)^{-1}\Gamma^{(0)}_{B'B}$ from the total contribution of this region, we obtain

$$\Gamma^{(\mathrm{A})}_{A'A} = -g\int \frac{d^Dk}{(2\pi)^D i}\frac{p_2^\nu A^c_\nu(p_A, k; p_{A'}, k-q)t^c}{(k^2 - m^2 + i0)((q-k)^2 + i0)(p_2k)}$$
$$\times\ \theta(2|p_2k| - \beta_0 s)(\not{k} + m)(m - \not{q}_\perp)\ . \tag{34}$$

Here $A^c_\nu(p_A, k; p_{A'}, k-q)$ is the one-particle irreducible in the t-channel part of the amplitude of the process $A + q(k) \to A' + g(k-q)$, c is the colour index of the gluon with momenta $k - q$ and ν is its polarization index. As well as in Eq. (18) we removed here the restriction $|\alpha| \ll \alpha_0$ in the definition of the region A (see Eq. (15).

The contribution of the central region can be presented as

$$A^{\mathrm{central}}_{3+} = g^2 C_F s \Gamma^{(0)}_{A'A}\left[J^{\mathrm{central}} - J'^{\ \mathrm{central}}\right]\Gamma^{(0)}_{B'B}\ , \tag{35}$$

where, as it was already mentioned, the integrals J^{central} and $J'^{\ central}$ differ only by the quark mass and the numerator $\not{k}_\perp + m$ in the integrands from the integrals $I^{\rm central}$ (see Eq. (20)) and $I'^{\rm \ central}$, so that

$$J^{\rm central}\frac{s}{2}\int\frac{d^{D-2}k_\perp}{(2\pi)^D i}$$

$$\times\int\limits_{-\alpha_0}^{\alpha_0}\int\limits_{-\beta_0}^{\beta_0}\frac{d\alpha d\beta(\not{k}_\perp+m)}{(s\alpha\beta+k_\perp^2-m^2+i0)(s\alpha\beta+(q-k)_\perp^2+i0)(s\alpha+i0)(-s\beta+i0)}$$

$$\simeq\frac{1}{2s}\int\frac{d^{D-2}k_\perp(\not{k}_\perp+m)}{(2\pi)^{D-1}(k_\perp^2-m^2-(q-k)_\perp^2)}\left[\frac{1}{k_\perp^2-m^2}\ln\left(\frac{-s\alpha_0\beta_0}{m^2-k_\perp^2}\right)\right.$$
$$\left.-\frac{1}{(q-k)_\perp^2}\ln\left(\frac{-s\alpha_0\beta_0}{-(q-k)_\perp^2}\right)\right]\ . \qquad (36)$$

Of course, this contribution does not depend on masses of the particles A and B. The integral $J'^{\rm \ central}$ differs from $J^{\rm central}$ only by the sign of s.

Integration over $k_\perp$ yields

$$J^{\rm central}=\frac{\delta^{(1)}(\not{q}_\perp)}{g^2C_F2s(m-\not{q}_\perp)}\ln\left(\frac{-s}{\vec{q}^{\ 2}}\right)+\frac{1}{2s}(\Delta(\not{q}_\perp,\alpha_0)+\Delta(\not{q}_\perp,\beta_0))\ , \quad (37)$$

where $\delta(\not{q}_\perp)$ is defined in Eq. (26) and

$$\Delta(\not{q}_\perp,z)=\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\int\limits_0^1\frac{dx(x\not{q}_\perp+m)}{((1-x)(m^2+x\vec{q}^{\ 2}))^{1-\epsilon}}\left(\psi(1)-\psi(1-\epsilon)\right.$$
$$\left.+\ln\left(\frac{(1-x)(m^2+x\vec{q}^{\ 2})}{\vec{q}^{\ 2}z^2}\right)\right)\ . \qquad (38)$$

Consequently, for the contribution of the central region (35) we find

$$A_{3^+}^{\rm central}=\Gamma^{(0)}_{A'A}\left[\frac{\delta(\not{q}_\perp)}{2(m-\not{q}_\perp)}\left(\ln\left(\frac{-s}{\vec{q}^{\ 2}}\right)+\ln\left(\frac{s}{\vec{q}^{\ 2}}\right)\right)\right.$$
$$\left.+g^2C_F(\Delta(\not{q}_\perp,\alpha_0)+\Delta(\not{q}_\perp,\beta_0)\right]\Gamma^{(0)}_{B'B}\ . \qquad (39)$$

This relation is valid for any process. The terms proportional to $\delta(\not{q}_\perp)$ in Eq. (39) correspond to the expansion of the Regge factors in Eq. (25); the remaining terms must be distributed between the corrections to the vertices $\Gamma_{A'A}$ and $\Gamma_{B'B}$. Therefore we obtain

$$\Gamma^{\rm (central)}_{A'A}=\Gamma^{(0)}_{A'A}g^2C_F(m-\not{q}_\perp)\Delta(\not{q}_\perp,\beta_0)\ , \qquad (40)$$

where $\Delta(/q_\perp, \beta_0)$ is defined in Eq. (38). The parameter β_0 in the contribution (40) cancels when we add to it the contribution (34) from the A-region.

As a result, we obtain the prescription for the calculation of the Reggeon vertices to the one loop accuracy. The vertex $\Gamma_{A'A}$ turns out to be the sum of the following contributions:

-the $Aq \to A'$ amplitude, with the quark self-energy taken with the coefficient 1/2, just as for external particles;

-the contributions given by Eqs. (34) and 40). Remind that $A^c_\nu(p_A, k; p_{A'}, k-q)$ is the one-particle irreducible in t-channel part of the amplitude of the process $A + q(k) \to A' + g(k-q)$, c is the colour index of the gluon with momentum $k-q$ and ν is its polarization index. The parameter β_0 cancels in the sum of these contributions.

It is important that in this approach the bulk of the vertex is expressed in terms of the quark self energy and vertices which are known. The only piece which must be calculated is $\Gamma^{(\mathrm{A})}_{A'A}$, defined by Eq. (40).

Note that for massless quarks we get

$$\delta^{(1)}(\not{q}_\perp) = -\frac{g^2 C_F \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left(\vec{q}^{\,2}\right)^\epsilon \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} = \frac{C_F}{N}\omega^{(1)}(t) \; , \tag{41}$$

$$\Delta(\not{q}_\perp, z) = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \not{q}_\perp \left(\vec{q}^{\,2}\right)^{\epsilon-1} \frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)} \Big(-2\ln z + \psi(1) - \psi(1-\epsilon)$$
$$+2\psi(\epsilon) - 2\psi(2\epsilon)\Big) = \frac{\not{q}_\perp}{\vec{q}^{\,2}} \frac{\omega^{(1)}(t)}{g^2 N} \phi(z) \; , \tag{42}$$

where $\omega^{(1)}(t)$ is defined in Eq. (2) and $\phi(z)$ in Eq. (22). For the corrections to the vertex from the central region in the massless case we obtain from Eq. (40)

$$\Gamma^{(\mathrm{central})}_{A'A} = \Gamma^{(0)}_{A'A} \frac{C_F \omega^{(1)}(t)}{N} \phi(\beta_0) \; , \tag{43}$$

with $\phi(z)$ defined in Eq. (22).

3 Calculation of the Vertices

Let us demonstrate how the method can be applied calculating the one-loop corrections to the vertices of interaction of the Reggeized gluon and quark with particles A and A', which are ordinary quarks and gluons. In the following we consider massless quarks, so that the momenta of the particles A and A' are equal to p_1 and $p_{1'}$ respectively and the Reggeon momentum is

$$q = p_{1'} - p_1 = -\frac{\vec{q}^{\,2}}{s} p_1 + \frac{\vec{q}^{\,2}}{s} p_2 + q_\perp \simeq q_\perp \ . \tag{44}$$

Considering the Reggeized gluon we denote its colour index by c.

3.1 Quark-Quark-Reggeized gluon Vertex

According to our prescription, the first contribution comes from the amplitude $q(p_1) + g(q) \to q(p_{1'})$, with the gluon polarization vector equal to $-p_2/s$. The one-loop corrections to this amplitude are well known. Putting the Reggeon vertex in the form

$$\Gamma^c_{Q'Q} = \Gamma^{(0)c}_{Q'Q}(1 + \delta_Q) \ , \tag{45}$$

where the Born vertex $\Gamma^{(0)c}_{Q'Q}$ is defined in Eq. (3), and taking into account that

$$\bar{u}(p_{1'}) \frac{\not{p}_2}{s} u(p_1) = \delta_{\lambda_{1'}\lambda_1} \ , \tag{46}$$

we obtain from the gluon self-energy (taken with the coefficient 1/2)

$$\delta_Q^{s.e.} = \omega^{(1)}(t) \frac{(5 + 3\epsilon)N - 2(1 + \epsilon)n_f}{4(1 + 2\epsilon)(3 + 2\epsilon)N} \tag{47}$$

and from the irreducible quark-quark-gluon vertex

$$\delta_Q^v = \omega^{(1)}(t) \left[\frac{-1}{4(1 + 2\epsilon)} - \frac{1}{4N^2} \left(1 + \frac{2}{\epsilon(1 + 2\epsilon)} \right) \right] \ . \tag{48}$$

Here n_f is the number of quark flavors and $\omega^{(1)}(t)$ is defined in Eq. (2). The contribution of the quark self-energy is zero for massless quarks.

The only contribution which we need to calculate is defined by Eq. (18):

$$\Gamma^{c(A)}_{Q'Q} = g^3 N \frac{t}{s} u(p_{1'}) \int \frac{d^D k}{(2\pi)^D i} \frac{\not{p}_2(\not{p}_1 + \not{k}) \ \not{p}_2 t^c}{(k^2 + i0)((q - k)^2 + i0)((p_1 + k)^2 + i0)(2p_2 k)}$$
$$\times \theta(2|p_2 k| - \beta_0 s) u(p_1) \ . \tag{49}$$

Using the Sudakov decomposition (6) and neglecting β_q with respect to β we obtain

$$\delta_Q^A = g^2 N \frac{t}{2} \int \frac{d\beta}{\beta}(1+\beta)\theta(|\beta|-\beta_0) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \times \tag{50}$$

$$\int \frac{d(s\alpha)}{2\pi i} \frac{1}{(s\alpha\beta + k_\perp^2 + i0)(s(\alpha-\alpha_q)\beta + (q-k)_\perp^2 + i0)(s\alpha(1+\beta) + k_\perp^2 + i0)} .$$

The integral over α can be easily calculated with the residue method. Then, changing β with $-\beta$ we have

$$\delta_Q^A = g^2 N \frac{t}{2} \int_{\beta_0}^{1} \frac{d\beta}{\beta}(1-\beta)^2 \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - q_\perp(1-\beta))^2} =$$

$$-g^2 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left(\vec{q}^{\,2}\right)^{\epsilon} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \int_{\beta_0}^{1} \frac{d\beta}{\beta}(1-\beta)^{2\epsilon}$$

$$= \omega^{(1)}(t)\left(-\ln\beta_0 + \psi(1) - \psi(1+2\epsilon)\right) . \tag{51}$$

The total correction is the sum of the pieces given by Eqs. (47), (48), (51) and the contribution (24) from the central region, with $\phi(z)$ defined in Eq. (22). The result we arrive at is

$$\delta_Q = \omega^{(1)}(t)\frac{1}{2}\left[\frac{1}{\epsilon} + \psi(1-\epsilon) + \psi(1) - 2\psi(1+\epsilon) + \frac{2+\epsilon}{2(1+2\epsilon)(3+2\epsilon)}\right.$$

$$\left. - \frac{1}{2N^2}\left(1 + \frac{2}{\epsilon(1+2\epsilon)}\right) - \frac{n_f}{N}\frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)}\right] , \tag{52}$$

in accordance with Ref. [11].

3.2 Gluon-Gluon-Reggeized gluon Vertex

Let us denote with e_1 and $e_{1'}$ the polarization vectors of the gluons with momenta $p_A = p_1$ and $p_{A'} = p_{1'}$ correspondingly; they satisfy the conditions $e_1 p_1 = e_{1'} p_{1'} = 0$, $e_1 p_2 = e_{1'} p_2 = 0$.

Since the helicity conservation in the Born vertices (see Eq. (4)) is violated by radiative corrections [9], let us write the Reggeon vertex in a general form:

$$\Gamma_{G'G}^c = g T_{G'G}^c \left[\delta_{\lambda_{1'}\lambda_1}(1 + \delta_G^{(+)}) + \delta_{\lambda_{1'},-\lambda_1}\delta_G^{(-)}\right] , \tag{53}$$

where $T_{G'G}^c$ are the matrix elements of the colour group generator in the adjoint representation and

$$\delta_{\lambda_{1'}\lambda_1} = -e_{1'}^* e_1 , \qquad \delta_{\lambda_{1'},-\lambda_1} = -e_{1'}^* e_1 + (D-2)\frac{(e_{1'}^* q)(e_1 q)}{q^2} . \tag{54}$$

The first contribution to the vertex comes from the amplitude $g(p_1) + g(q) \to g(p_{1'})$, where the polarization vector of the gluon with momentum $q = p_{1'} - p_1$ is taken equal to $-p_2/s$. The one-loop corrections to this amplitude are well known. From the gluon self-energy, taken with the coefficient $1/2$, we have a contribution only to $\delta_G^{(+)}$, the same as to δ_Q of Eq. (47):

$$\delta_G^{(+)(s.e.)}) = \omega^{(1)}(t) \frac{(5+3\epsilon)N - 2(1+\epsilon)n_f}{4(1+2\epsilon)(3+2\epsilon)N} \ . \tag{55}$$

From the irreducible three-gluon vertex we have contributions to both helicity conserving and non-conserving parts; moreover, for the helicity non-conserving part there are no other contributions, so that we have

$$\delta_G^{(+)v} = \omega^{(1)}(t) \left[\frac{3}{8\epsilon} + \frac{1}{2(1+\epsilon)} - \frac{5}{4(1+2\epsilon)} \right.$$

$$\left. - \frac{1}{3+2\epsilon} + \frac{n_f}{2N} \frac{2(1+\epsilon)^3 + \epsilon^2}{(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)} \right] \ , \tag{56}$$

$$\delta_G^{(-)} = \frac{\epsilon \omega^{(1)}(t)}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \left(-1 + \frac{n_f}{N(1+\epsilon)} \right) \ , \tag{57}$$

Remind that n_f is the number of quark flavors and $\omega^{(1)}(t)$ is defined in Eq. (2). Contributions from the self-energy of the on-mass shell gluons are zero.

The only contribution which we need to calculate is defined by Eq. (18). We get for it:

$$\Gamma_{G'G}^{c(A)} = -g^3 N T_{G'G}^c \frac{t}{2s} \int \frac{d^D k}{(2\pi)^D i} \frac{p_2^\mu p_2^\nu}{(k^2 + i0)((q-k)^2 + i0)(p_2 k)} \theta(2|p_2 k| - \beta_0 s)$$

$$\times \frac{e_{1'}^{*\alpha'} e_1^{\alpha} \gamma_{\alpha\mu}{}^{\rho}(p_1, k, -p_1 - k) \gamma_{\rho\nu\alpha'}(p_1 + k, k - q, -p_{1'})}{(p_1 + k)^2} \ , \tag{58}$$

where the vertices $\gamma_{\mu\nu\rho}$ are defined in Eq. (11). The calculation of this contribution can be done in the same way as in the $\Gamma_{Q'Q}^{c(A)}$ case. Using the Sudakov decomposition (6) and neglecting β_q with respect to β we obtain

$$\Gamma_{G'G}^{c(A)} = -g^3 N T_{G'G}^c e_1^* e_1 \frac{t}{2} \int \frac{d\beta}{\beta} (1 + \beta/2)^2 \theta(|\beta| - \beta_0) \int \frac{d^{D-2} k_\perp}{(2\pi)^{D-1}} \ \times \tag{59}$$

$$\int \frac{d(s\alpha)}{2\pi i} \frac{1}{(s\alpha\beta + k_\perp^2 + i0)(s(\alpha - \alpha_q)\beta + (q-k)_\perp^2 + i0)(s\alpha(1+\beta) + k_\perp^2 + i0)},$$

so that we find

$$\delta^{(+)A} = g^2 N \frac{t}{2} \int_{\beta_0}^{1} \frac{d\beta}{\beta} (1-\beta/2)^2 (1-\beta) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - q_\perp(1-\beta))^2}$$

$$= -g^2 N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left(\vec{q}^{\,2}\right)^{\epsilon} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \int_{\beta_0}^{1} \frac{d\beta}{\beta} (1-\beta)^{2\epsilon-1} (1-\beta/2)^2$$

$$= \omega^{(1)}(t) \left(-\ln \beta_0 + \psi(1) - \psi(1+2\epsilon) + \frac{1}{8\epsilon(1+2\epsilon)} \right) . \tag{60}$$

The total correction to the helicity conserving vertex is given by the sum of the contributions (55), (56), (60) and (24), the last from the central region, with $\phi(z)$ defined in Eq. (22). We obtain

$$\delta_G^{(+)} = \omega^{(1)}(t) \frac{1}{2} \left[\frac{2}{\epsilon} + \psi(1-\epsilon) + \psi(1) - 2\psi(1+\epsilon) - \frac{9(1+\epsilon)^2 + 2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} \right.$$
$$\left. + \frac{n_f}{N} \frac{(1+\epsilon)^3 + \epsilon^2}{(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)} \right] , \tag{61}$$

in accordance with Ref. [9].

3.3 Gluon-Quark-Reggeized quark Vertex

Calculation of one-loop corrections to the vertices of the Reggeized quark is performed in a similar way. Let us consider for definiteness the vertex for the gluon $\to$ quark transition. The gluon and quark momenta are $p_A = p_1$ and $p_{A'} = p_{1'}$ correspondingly, the Reggeized quark momentum is $q = p_{1'} - p_1$ and the gluon colour index is G. The gluon polarization vector e satisfy the conditions $ep_1 = ep_2 = 0$. Then the vertex can be presented as

$$\Gamma_{QG} = -g \bar{u}(p_{1'}) t^G \left[\not{e}(1+\delta_e) + \frac{(eq)\ \not{q}_\perp}{q_\perp^2} \delta_q \right] . \tag{62}$$

The first contribution to the vertex comes from the amplitude $g(p_1) + q(q) \to q(p_{1'})$ where the quark with momentum q is off-mass shell . Calculating this amplitude we can omit the terms with $\not{p}_2/s$ standing on the most right position, since in such case we can move $\not{p}_2/s$ in the Reggeized amplitude (25) with the help of the anticommutation relations for γ matrices to act on the spinor wave function of the quark with momentum along p_2 and then use the Dirac equation, getting only terms vanishing at large s.

The off-mass shell quark self-energy contributes only to δ_e. Taking into account the coefficient 1/2, we obtain

$$\delta_e^{s.e.} = \delta^{(1)}(\not{q}_\perp)\frac{1}{2}\left(\frac{-1-\epsilon}{1+2\epsilon}\right) . \tag{63}$$

Notice that for the massless quark $\delta^{(1)}(\not{q}_\perp) = (C_F/N)\omega^{(1)}(t)$, being $\omega^{(1)}(t)$ defined in Eq. (2). The contributions of the self-energy of the on-mass shell quark and gluon are zero. The irreducible gluon-quark-quark vertex contributes to both spin structures:

$$\delta_e^v = \omega^{(1)}(t)\left[\frac{1}{2\epsilon} - \frac{1}{4N^2}\left(1 - \frac{2}{1+2\epsilon}\right)\right] , \tag{64}$$

$$\delta_q^v = \omega^{(1)}(t)\left[\frac{-1}{2\epsilon} + \frac{\epsilon}{2(1+2\epsilon)} - \frac{1}{2N^2}\left(\frac{2-\epsilon}{1+2\epsilon}\right)\right] . \tag{65}$$

The unknown contribution is defined by Eq. (34):

$$\Gamma_{QG}^{(\mathrm{A})} = g^3\bar{u}(p_{1'})t^G \int \frac{d^Dk}{(2\pi)^D i}\frac{\theta(2|p_2k| - \beta_0 s)}{(k^2+i0)((q-k)^2+i0)(p_2k)}\left[\frac{1}{2N}\frac{\not{p}_2(\not{p}_1 + \not{k})\not{e}}{(p_1+k)^2+i0}\right.$$
$$\left. + \frac{N}{2}\frac{p_2^\nu e^\alpha\gamma^\rho\gamma_{\alpha\rho\nu}(p_1, -p_{1'}+k, q-k)}{(p_{1'}-k)^2+i0}\right]\not{k}\,\not{q}_\perp , \tag{66}$$

where the vertex $\gamma_{\mu\nu\rho}$ is defined in Eq. (11).

The calculation of this contribution differs from the calculation of $\Gamma_{Q'Q}^{c(\mathrm{A})}$ and $\Gamma_{G'G}^{c(\mathrm{A})}$ only by the spinor and tensor algebra. The result is

$$\delta_e^A = \omega^{(1)}(t)\left[\frac{c_F}{N}\left(-\ln\beta_0 + \psi(1) - \psi(1+2\epsilon)\right) - \frac{1}{4(1+2\epsilon)}\right] , \tag{67}$$

$$\delta_q^A = \omega^{(1)}(t)\left[\frac{1}{2\epsilon} + \frac{1}{N^2(1+2\epsilon)}\right] . \tag{68}$$

The contribution to the vertex from the central region is shown in Eq. (43). The total correction δ_e, given by the sum of the contributions (63), (64), (67) and the correction (43) from the central region, is

$$\delta_e = \omega^{(1)}(t)\left[\frac{c_F}{2N}\left(\frac{1}{\epsilon}-\frac{2-\epsilon}{1+2\epsilon}+\psi(1)+\psi(1-\epsilon)-2\psi(1+\epsilon)\right)\right.$$
$$\left.+\frac{1}{2\epsilon}-\frac{\epsilon}{2(1+2\epsilon)}\right]\ , \tag{69}$$

$$\delta_q = \omega^{(1)}(t)\frac{\epsilon}{2(1+2\epsilon)}\left(1+\frac{1}{N^2}\right)\ . \tag{70}$$

Note that δ_q is not singular at $\epsilon \to 0$, since the corresponding spin structure in Eq. (62) is absent at the leading order.

4 Discussion

We developed the method which permits both for the case of the Reggeized gluon and the Reggeized quark to calculate the Reggeon-particle vertices at the next-to-leading order directly, without calculation of scattering amplitudes. In this order the only uncertainty in the definition of the vertices is related to the Feynman diagrams with two particles in the t-channel. The remarkable property of the integrals, corresponding to these diagrams, is that they can be presented as the sum of the contributions of three integration regions: the central region, where the t-channel particles have small components of momenta along the momentum of any of the colliding particles A and B, and two other regions, which can be called regions of fragmentation of the particles A and B respectively. It is possible to separate precisely these regions and to define unambiguously their contributions. The contributions of the fragmentation regions must be attributed to the corresponding vertices. As for the "central" region, it occurs that its contribution does not depend on the colliding particles. We want to stress that this property is crucial for the Reggeization, so that we could conclude about its existence from the fact of the Reggeization. It permits to determine the contributions to the Reggeon-particle vertices from the central region, which are universal (i.e. independent of the colliding particles), and to formulate the rules of calculation of the vertices. For the vertices of the Reggeized gluon these rules are given at the end of Subsection 2.1, for the vertices of the Reggeized quark at the end of Subsection 2.2.

[1] We were informed by the authors of Ref. [21], who are considering this vertex for the massive quark case in the $t-$channel unitarity approach [9], that in the massless limit their result is in complete agreement with our one.

Using these rules we calculated at the next-to-leading order the elementary vertices for the interaction of the Reggeized gluon and quark. The vertices of the Reggeized gluon are already known, so that in this case the calculations we performed serve for the demonstration of the validity of the method and the check of the results. As far as we know, the vertices of the Reggeized quark were not yet calculated in the NLO, so that these results are new.

Note once more that the developed method is valid not only for the calculation of the elementary transition vertices considered here. Evidently, it can be applied in the general case, including transitions between groups of particles with fixed (not growing with s) invariant masses.

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