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# COULOMB BLOCKADE IN QUANTUM DOTS WITH OVERLAPPING RESONANCES:

Towards an Explanation of the Phase Behaviour in the Mesoscopic Double-Slit Experiment

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## Coulomb blockade in quantum dots with overlapping resonances:

Towards an explanation of the phase behaviour in the mesoscopic double-slit experiment

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#### Abstract

Coulomb blockade (CB) in a quantum dot (QD) with one anomalously broad level is considered. In this case many consecutive pronounced CB peaks correspond to occupation of one and the same broad level. Between the peaks the electron jumps from this level to one of the narrow levels and the transmission through the dot at the next resonance essentially repeats that at the previous one. This offers a natural explanation to the recently observed behavior of the transmission phase in an interferometer with a QD. Single particle resonances of very different width are natural if the dot is not fully chaotic. This idea is illustrated by the numerical simulations for a non-integrable QD whose classical dynamics is intermediate between integrable and chaotic. Possible manifestations for the Kondo experiments in the QD are discussed.

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#### 1 Introduction. The Double-Slit Experiment

Much progress has recently been achieved in the fabrication and experimental investigation of ultrasmall few-electron devices - such as Quantum dots [1]. A useful theoretical tool for the investigation of chaotic QD-s is the Random Matrix theory (see reviews [2, 3]). Nevertheless, many experimentally observed features of these artificial multi-electron systems still have not found a reasonable theoretical explanation.

A challenging problem which has resisted adequate theoretical interpretation arises from the experiments [4, 5] which determine the phase of the wave transmitted through the QD. The goal of this talk will be to present a mechanism which may lead to a satisfactory explanation of these results. The QD in the experiment was imbedded into one arm of the Aharonov–Bohm (AB) interferometer, which allowed to measure not only the magnitude of the transmission, but also the phase acquired by the electrons traversing the dot. The total current through the interferometer is [5]

$$J_{total} \sim |t_{QD} + e^{i\phi_{AB}}t_{sl}|^2 , \qquad (1)$$

where  $t_{QD}$  and  $t_{sl}$  are the transmission amplitudes through the QD and the second reference arm of the interferometer. The AB phase is proportional to the magnetic flux  $\Phi$  threading the interferometer  $\phi_{AB}=2\pi\Phi/\Phi_0$ . The complex amplitude  $t_{QD}$  (as well as the number of electrons in the dot) is slowly changed by the plunger gate voltage  $V_g$  and does not depend on the weak magnetic field. Parametrising the amplitudes as  $t=|t|\exp(i\theta)$  the interference term in eq. (1) becomes, in obvious notation:

$$|t_{QD}||t_{sl}|\cos(\phi_{AB} + \theta_{sl} - \theta_{QD}). \tag{2}$$

Thus by independently changing  $V_g$  and  $\phi_{AB}$  one may find both  $|t_{QD}|$  and the variation of the phase  $\Delta\theta_{QD}$  as a function of the gate voltage.

In the experiments [4, 5] the coherent component of the electron transport through the QD was measured directly for the first time. Later experiments

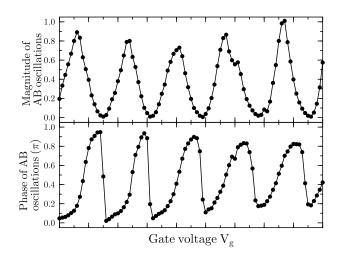


Figure 1: The magnitude of AB oscillations  $\sim |t_{QD}|$  (arbitrary units) and the variation of the phase  $\Delta\theta_{QD}$  (in units of  $\pi$ ) as a function of the gate voltage.

allowed also an investigation of the dephasing rate of the electron state in the QD [6]. Here we discuss the unexpected and not understood yet observed behavior of the phase  $\theta_{QD}$  of the transmitted electron.

This unusual behavior of the phase is illustrated in fig. 1 (an approximate drawing, very close to the real experimental figure of ref. [5]). As one can see from the figure, in accordance with the Breit-Wigner picture, the measured phase increased by  $\pi$  around each CB peak. Absolutely unexpected, was a fast jump of the phase by  $-\pi$  between the resonances just at the minimum of the transmitted current. Such a behavior is in evident contradiction with the theoretical expection for transmission via the consecutive levels in 1-dimensional quantum well. The phase shift between the resonances in this case simply accounts for the number of nodes of the (almost real) wave function inside the dot, which is changed by one in 1d at the consecutive resonances.

A number of theoretical papers attempted to find an explanation of the  $-\pi$  jumps. It is relatively easy to find a mechanism which leads to the fast drops of phase in some part of the CB valleys. It is more complicated to explain why for the long series of peaks the increase of the phase by  $\pi$  at the resonance will be accompanied by the  $-\pi$  drop at the valley. In two-dimensional

QD the phase drops associated with the nodes of the transmission amplitude arise already within the single-particle picture (the Fano mechanism). However, in order to have a sequence of such events one should consider a QD of a very special form [7, 8]. For electrons with spin and only one level in the QD, the phase drops between the resonances corresponding to adding of the first and second electron to the dot [9], but again this effect is not easy to generalize for a series of peaks. The mechanism of refs. [10, 11] makes non-trivial assumptions on the geometry of the QD and the way it changes under the change of plunger gate voltage. An interesting generic mechanism, which indeed may lead to the sequence of  $\sim U_{CB}/\Delta$  drops between the resonances was suggested in ref. [12]. However, within this approach the extra phase jumps do not occur in the same fashion in consecutive resonances.

In this paper we propose a mechanism according to which the transmission at many CB peaks proceeds through one and the same level in the QD. This means that the phases at the wings of different resonances should coincide and therefore the increase by  $\pi$  at the resonance must be compensated. This compensation occurs via narrow jumps between the resonances and is accompanied by a fast rearrangement of the electrons in the dot.

The experiment [5] was clearly done in the CB regime. However, in order to be able to measure the AB oscillations in the CB valleys the dot was significantly "opened". As a result, the widths of the resonances turn out to be rather large, being only few times smaller than the charging energy (see fig. 1). Also the widths and heights of all observed resonances are quite similar. These features of the experimental results, which have not attracted so wide an attention as the phase jumps, also naturally follow from our mechanism.

It is generally believed that the CB is observed only if the widths of resonances are small compared to the single-particle level spacing in the dot  $\Delta$ . This condition assumes that couplings of all levels to the leads are of the same order of magnitude. However, as we will show, even for nonintegrable ballistic QD-s the widths of the resonances may vary by orders of magnitude. In this case it does not make sense to compare the width of few broad resonances with the level spacing, determined by the majority of narrow, practically decoupled, levels. Our goal in this paper is twofold. First, in the following section we will consider a simple numerical example which shows that coexistence of narrow and broad resonances is quite natural in ballistic QD-s with relatively strong coupling to the leads. Second, we will consider how the CB and filling of the levels in the dot are changed in the presence of a dominant single broad resonance.

A useful theoretical model for the description of charging effects in QD-s

is the tunneling Hamiltonian (see e.g. [13])

$$H = \sum_{i} \varepsilon_{i} a_{i}^{+} a_{i} + U_{CB} \left( \sum_{i} a_{i}^{+} a_{i} - N_{0} \right)^{2} / 2$$

$$+ \sum_{k} \varepsilon_{k}^{L} b_{k}^{L+} b_{k}^{L} + \sum_{k,j} [t_{j}^{L} a_{j}^{+} b_{k}^{L} + h.c.] + L \leftrightarrow R .$$
(3)

Here  $a(a^+)$  and  $b(b^+)$  are the annihilation(creation) operators for electron in the dot and in the lead, L and R stand for the "left" and "right" lead and summation over spin orientations is included everywhere.

Refined theoretical treatment of the charging effect within the Hamiltonian (3) includes modern theoretical tools such as bosonisation and mapping onto the Kondo problem [14]. For our purposes, it will be possible to simplify further eq. (3). We will consider the case of only one level N in the dot significantly coupled to the leads (only  $t_N^{L,R} \neq 0$ ). If in addition the width of this level is larger than the single-particle level spacing  $\Delta$ , a very nontrivial regime of CB may be described by means of simple second order of perturbation theory estimates. Surprisingly this simple limit of CB in the QD with broad level have not been considered yet.

### 2 Semi-Chaotic Quantum Dots

In this section we will show by explicit numerical example how the anomalously broad levels may appear even in sufficiently large and irregular QD.

A simple example of a system for which the widths differ drastically is the integrable QD [10, 11]. Integrable QD-s having only few  $(2 \div 4)$  electrons have been produced recently [15]. However, for large numbers of electrons  $N_e \sim 100 \div 1000$  it will be much more difficult to produce an integrable QD<sup>1</sup>. Nevertheless, at least in classical mechanics, one finds a considerable gap left between integrable and fully chaotic systems. Even in the nonintegrable dot there may coexist two kinds of trajectories – quasiperiodic and chaotic. In this case, in 2-dimensions any trajectory (even the chaotic one!) does not cover all the phase space formally allowed by energy conservation. Consequently, the corresponding wave functions do not cover all the area of the QD. If such a regime is or may be realized in QD-s, it easily

 $<sup>^1</sup>$ It is not clear how close to integrable was the QD used in the experiment [5]. However, the QD containing  $\sim 200$  electrons was  $\sim 50$  times smaller than the nominal elastic mean free path. Thus, disorder should not be essential for the dynamics of the electrons.

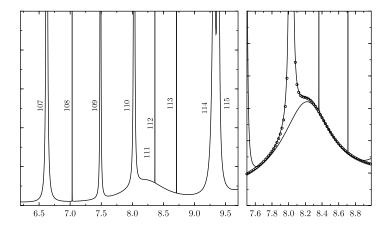


Figure 2: The energy dependence of the probability to find the electron in the QD for fixed incoming(outgoing) current. The numbers of the corresponding levels in the dot are shown. Right panel: A part of figure close to the resonance #111 together with the fit by two Breit-Wigner peaks (circles) and single broad peak with  $\Gamma=0.7$ .

explains, why the widths of the resonances may vary by orders of magnitude. Moreover, many other features of such a QD may differ strongly from those of the chaotic QD (CB intervals, energy level statistics [21]). To illustrate this idea we have performed numerical simulations for a model QD of the size l coupled smoothly to the leads with the potential

$$V = -x^{2} (1 - x/l)^{2} + (y + 0.2x^{2}/l)^{2} (1 + 2(2x/l - 1)^{2}).$$
 (4)

The dot is strongly nonintegrable, but, similarly to the experimental geometry [5], sufficiently symmetric. For numerical simulations we considered the QD on the lattice having a lattice spacing equal to unity and l=40. The kinetic term is given by the standard nearest-neighbor hopping Hamiltonian. With the hopping matrix element  $\tau \approx 40$  our dot has about 100 electrons (~ 200 with spin). The calculations were performed for the right lead closed by a hard wall at x=l (see ref. [22] for some more details). Only one mode may propagate along the left lead within the energy interval  $5.8 < \varepsilon < 10.3$ 

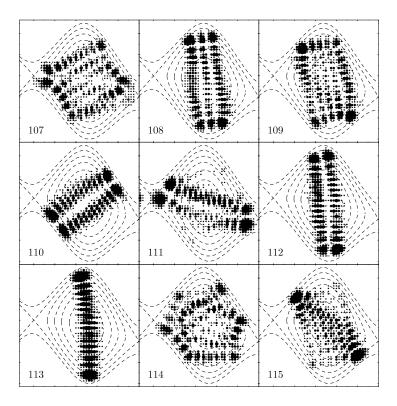


Figure 3: The density ( $\sim$ dark area) of electrons in the dot for the nine levels shown in fig. 1. Dashed lines are the lines of constant potential  $V=20,\ 0,\ -20,\ -40,\ -60$  and -80. The single-channel wire is attached from the left. About 95% of the norm of the wave function in the dot is shown.

(the energy band is  $V < \varepsilon < V + 8\tau$ ). The phase of the electron in the lead was not fixed and we were able to find a solution of the Schrödinger equation at any energy. The probability to find an electron in the dot as a function of energy for fixed in- and out-going current in the lead is shown in fig. 2. Above-barrier resonances with very different widths are seen. Moreover, the width of the resonance #111 turns out to be few times larger than the level spacing. The origin of this hierarchy of widths becomes clear from fig. 3, where we have plotted  $|\psi|^2$  in the QD at these resonances. The quantized version of different variants of classical motion may be found on this figure. The most narrow level #113 corresponds to short transverse periodic orbit. Other broader levels, such as #109, may be considered as the projections of the invariant tori corresponding to quasi-periodic classical motion. This classical trajectory reaches the line  $V(x,y) = \varepsilon$  only at few points. The presence of different types of (quasi)periodic motion is natural for the nonintegrable dot (see the wave functions #113, #110, #111 and less pronounced #115). The candidates for chaotic classical motion (e.g. #114) also correspond to relatively broad resonances. Even in this case only a part of QD is covered by the trajectory.

Most interesting is the level #111, which is well coupled to the leads. The corresponding classical trajectory covers some invariant tori in the dot starting from the left contact. The Breit-Wigner width of this resonance is  $\Gamma \approx 0.7$  (same units as in fig. 2), which is sufficiently larger than  $\Delta$ .

Essentially the same pictures with "regular" and "chaotic" wave functions may be found in ref. [21], where also the potential of the QD was treated selfconsistently. Other mechanisms which may lead to the "stability" of the broad level will be considered in the next section.

#### 3 Coulomb Blockade for a Single Broad Level

With our numerical example we have investigated only the single particle properties of the ballistic nonintegrable QD. Now let us turn to the many-particle effects. First of all, the parameters of the dot are slowly changed by the change of the plunger gate voltage  $V_g$ . We describe this effect by letting the levels in the dot flow as

$$\varepsilon_i = \varepsilon_i (V_q = 0) - V_q \quad , \quad i = 1, 2, 3...$$
 (5)

The occupied levels are now those with  $i \leq 0$ . The energies of the electrons in the wire are given by

$$\varepsilon(k) = k^2/2m - E_F. \tag{6}$$

Here  $k = n\pi/L$  with large integer n and L is the length of the wire. As it was mentioned in the introduction, we suppose that only one (N-th) level in the dot is coupled strongly to the leads. It is enough to consider coupling with only one lead [18]. Let the corresponding matrix element be  $t_N$  and

$$\Gamma_N = 2\pi |t_N|^2 dn/d\varepsilon \gg \Delta . \tag{7}$$

In our example (after the doubling of number of levels due to spin) the values were  $\Gamma_N/\Delta \sim 5$ . The charging energy is assumed even larger,  $U_{CB} \gg \Gamma$ . The widths of other levels are much smaller than  $\Delta$  and may be neglected. Our aim is to show, that transmission of a current at about  $\Gamma/\Delta$  consecutive CB peaks will proceed through one and the same level  $\varepsilon_N$  in the dot.

Let us start with spinless electrons. As long as we assume  $t_i=0$  for all  $i\neq N$ , the spectrum of the tunneling Hamiltonian (3) consists of completely decoupled branches corresponding to different occupation numbers of narrow levels. Let at first only the levels with  $i\leq 0$  be occupied. It is easy to find the second order correction to the total energy of the wire and QD, when the N-th level lies far above the Fermi energy  $(\varepsilon_N(V_q)\gg\Gamma)$ 

$$\Delta E_{tot}^{(0)} = \int_0^{k_F} \frac{|t_N|^2}{\varepsilon(k) - \varepsilon_N} \frac{L}{\pi} dk = \frac{-\Gamma}{2\pi} \ln\left(\frac{4E_F}{\varepsilon_N(V_g)}\right). \tag{8}$$

The levels in the wire are lowered due to the repulsion from the unoccupied level  $\varepsilon_N$ . We will not include into  $E_{tot}$  the trivial constant which arose due to occupation of the decoupled levels with  $i \leq 0$  and unperturbed electron energies in the lead. Generalization of eq. (8) for the case of negative  $\varepsilon_N$ ,  $\varepsilon_N \ll -\Gamma$  (the broad level being below the Fermi energy) is straightforward (note that the level  $\varepsilon_N$  is occupied, not the level  $\varepsilon_1$  as one might expect):

$$E_{tot}^{(0)} = \varepsilon_N - \frac{\Gamma}{2\pi} \ln \left( \frac{4E_F}{|\varepsilon_N|} \right). \tag{9}$$

Here the first term,  $\varepsilon_N < 0$ , accounts for the energy loss due to replacement of one electron from lead to the dot. The correction due to the second order of perturbation theory now includes both lowering of levels with  $\varepsilon(k) < \varepsilon_N$  and rising of those with  $\varepsilon(k) > \varepsilon_N$ . The perturbative treatment fails for  $|\varepsilon(k) - \varepsilon_N| \le \Gamma$ , but the corresponding shifts of levels below and above  $\varepsilon_N$  evidently compensate each other, which is equivalent to taking the principal value of the integral in Eq.(8).

Finally, the exact solution for a single state interacting with a continuum is also known (see e.g. [16]). A precise treatment of this Breit-Wigner-type

situation, along the lines of ref. [17], yields for  $E_F \gg \varepsilon_N$ ,  $\Gamma$ :

$$E_{tot}^{(0)} = \frac{-\Gamma}{4\pi} \left[ \ln \left( \frac{16E_F^2}{\varepsilon_N^2 + \Gamma^2/4} \right) + 2 \right] + \frac{\varepsilon_N}{\pi} \cot^{-1} \frac{2\varepsilon_N}{\Gamma}, \tag{10}$$

which coincides with Eqs. (8,9) at  $|\varepsilon_N| \gg \Gamma$ .

Let us now consider the branch where the level  $\varepsilon_1$  is occupied. The energy of this electron is  $\varepsilon_1(V_g)$ . However, adding one more electron via the hopping  $t_N$  now costs  $\varepsilon_N(V_g) + U_{CB}$ . The ensuing reduction of the downward shift of the level  $E_{tot}^{(1)}$  is of crucial importance. The analog of Eq. (8) for  $\varepsilon_N + U_{CB} \gg \Gamma$  now reads

$$E_{tot}^{(1)} = \varepsilon_1 - \frac{\Gamma}{2\pi} \ln \left( \frac{4E_F}{\varepsilon_N + U_{CB}} \right). \tag{11}$$

The initial energy of the electron gas in the leads as well as the contribution from  $\varepsilon_i$  with i < 0 may be included as an additive constant in eqs. (9-11).

Within  $-U_{CB} < \varepsilon_{1,N} < 0$ , both eqs. (9) and (11) are valid. Since  $\varepsilon_N > \varepsilon_1$  one expects  $E_{tot}^{(1)}$  to be the true ground state in this region. However, because in our case  $\Gamma \gg \Delta$  the second term in eqs. (9,11) may invalidate this expectation. For small  $V_g$  one has  $E_{tot}^{(0)} < E_{tot}^{(1)}$  and the Eqs. (8-10) describe the true ground state of the system. However, the two functions  $E_{tot}^{(0)}(V_g)$  and  $E_{tot}^{(1)}(V_g)$  cross at

$$\varepsilon_N(V_q) = -U_{CB}/[\exp\{2\pi(\varepsilon_N - \varepsilon_1)/\Gamma\} + 1] \tag{12}$$

and the ground state jumps onto the branch  $E_{tot}^{(1)}$ . The current-transmitting virtual state N now again has a positive energy. Thus, the phase of this transmission has returned to what it was before the process of filling of state N and the subsequent sharp jump into the state where level 1 is filled. It is the latter jump which provides the sharp drop by  $\pi$  of the phase of the transmission amplitude, following the increase by  $\pi$  through the broad resonance. Thus, many  $(\sim (\Gamma/\Delta)ln(U_{CB}/\Gamma))$  consecutive resonances are due to the transition via one and the same level N.

For electrons with spin the Breit-Wigner-related formula (10) does not work. One should consider the Anderson impurity model. However, far from the resonance the perturbation theory may still be used (we assume that the temperature is large enough to be away from the Kondo effect [18-20]). Eq. (8) acquires only the overall factor 2 due to spin  $E_{tot}^{(0)} \rightarrow 2E_{tot}^{(0)}$ . The

logarithm in Eq. (11) is also multiplied by 2. Instead of Eq. (9) one has

$$E_{tot}^{(0)} = \varepsilon_N - \frac{\Gamma}{2\pi} \left\{ \ln \left( \frac{4E_F}{|\varepsilon_N|} \right) + \ln \left( \frac{4E_F}{\varepsilon_N + U_{CB}} \right) \right\}. \tag{13}$$

The first logarithm here is the second-order correction due to the jumps from the occupied orbital in the dot back to the wire. Another logarithm accounts for the jumps from the wire onto the second, unoccupied orbital. The ground state of the system is now doubly degenerate due to two orientations of spin in the dot. The result (12) is basically unchanged.

The crossing of energy levels  $E_{tot}^{(0)}$  (9) and  $E_{tot}^{(1)}$  (11) becomes avoided if one introduces the small individual width  $\Gamma_1$  of the narrow level  $|1\rangle$ , but the phase drop remains sharp  $\delta V_g \sim (\varepsilon_1 - \varepsilon_N) \sqrt{\Gamma_1/\Gamma_N}$  [22].

In the simplified model Eq. (5) all levels in the dot are changed in the same way by the plunger voltage. Taking into account the different sensitivities of longitudinal and transverse modes to the plunger [10, 11] may allow to keep our broad level  $\varepsilon_N$  even longer within the relevant strip of energy. This may provide an explanation of even longer sequences of resonances accompanied by the  $-\pi$  jumps.

Generalization of our approach for N < 0 (still  $|\varepsilon_N| < \Gamma$ , the broad level immersed into the sea of occupied narrow levels) is straightforward.

Also in a more refined approach, adding new electrons into the QD should cause a slow change of the selfconsistent potential V(x, y). The total energy of the dot and the wire will be lowered in the presence of strongly coupled levels. This may cause the potential of the QD to automatically adjust to allow such levels, which will support our explanation of the experiment of Ref. [5].

Even though we have shown in the section 3 that the broad levels are natural for the strongly coupled ballistic QD-s, one may wonder, why it happens that in the experiment such a level was found just close enough to the Fermi energy. We see two possible explanations. First, the self-consistent shape of the QD may indeed be automatically adjusted in order to have such a level. Second, the parameters of the QD might have been tuned in the course of preparation of the experiment while opening up the dot.

#### 4 Kondo Effect

Recent experiments [23-26] observed the increase of the conductance G of an appropriate QD at low temperature in the CB valleys corresponding to the

odd number of electrons, which has stimulated the renewed interest in the Kondo effect in QD-s [18-20]

The correction to G is due to the spin-flip interaction of the unpaired electron in the dot with the leads. For example, at the upper wing of the first charging resonance

$$G_K = G_0 \left( 1 + \frac{const \times \Gamma}{E_F - \varepsilon} \ln \left( \frac{U_{CB}}{T} \right) \right) , \ \Gamma \ll E_F - \varepsilon \ll U_{CB} .$$
 (14)

Here  $\varepsilon$  is the energy of single particle state in the dot and the unperturbed conductance at the wings of resonance is  $G_0 \sim \Gamma_L \Gamma_R / (E_F - \varepsilon)^2$ . However, already in the first experiments, effects which do not find an explanation within the straightforward Kondo mechanism were observed [25, 26].

The correction associated with the spin flip blows up at the Kondo temperature  $T_K = \sqrt{U_{CB}\Gamma} \exp \{\pi(\varepsilon - E_F)(\varepsilon + U_{CB} - E_F)/2\Gamma U_{CB}\}$  [27]. The natural way to observe the Kondo effect is to fabricate the devices with large  $T_K$ .  $T_K$  is large in very small (up to  $\sim 50$  electrons in real experiments) and sufficiently open QD-s.

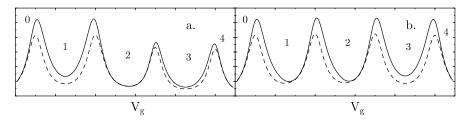


Figure 4: The gate voltage dependence of the conductance (schematic): a. for two narrow resonances ( $\Gamma \ll \Delta$ ), b. for one broad and one narrow resonance. Dashed line  $T \gg T_K$  (but  $\Gamma \gg T$ ), solid line  $T \sim T_K$ . The valley number coincides with the number of electrons added to the dot.

In this section we will consider how the Kondo effect behaves in the case of overlapping resonances. In fact we do not need now to have  $\Gamma \gg \Delta$ . The interesting new effects appear already if one has two levels  $\varepsilon_1$  and  $\varepsilon_2$  one narrow and another broad with  $\Gamma \sim \varepsilon_2 - \varepsilon_1$ . The prediction for the Kondotype correction to the conductance in this case follows immediately from the consideration of the previous section without any additional calculation. The result is shown in fig. 4.

In fig. 4a. we have shown schematically the conductance as a function of gate voltage following the occupation of two narrow levels (the usual Kondo

effect). The first two similar peaks correspond to occupation of level  $\varepsilon_1$ , the other two peaks correspond to occupation of the second level  $\varepsilon_2$ , having slightly different width. At low temperature  $T \sim T_K$  the conductance grows up at the odd valleys 1 and 3.

Quite different is the Kondo effect in the case of the broad peak shown in fig. 4b. At the left peak the broad level crosses the Fermi energy for the first time. Since  $\Gamma \sim \varepsilon_2 - \varepsilon_1$  no matter which energy is larger, the broad level is occupied at this resonance. At low  $T \sim T_K$  the conductance increases at the right wing of the resonance (1 electron in the QD) and does not change at the left wing (no electrons in the QD). Something of interest happens at the middle of the valley 1. Here, in accordance with the theory developed in the previous section, the electron in the dot jumps from the broad to the narrow level and becomes "invisible" for the exchange with the leads. Thus though formally the spin of the QD equals s = 1/2, there is no Kondo effect at the left wing of the second resonance<sup>2</sup>. The second peak is associated again with the population of the broad level by one electron. As for the valley 1, one finds the Kondo effect only in the left half of the second valley. In the middle of this valley the electron configuration is rearranged again and the second electron jumps to the narrow level. After that the occupied narrow level does not play any role in the transmission. The usual Kondo effect takes place in the valley 3 in fig. 4b.

Thus, our main result of this section is the prediction of possibility to have Kondo effect in even valleys in the QD. Remarkably, something very close to this prediction was observed in the recent experiment in Stutgart [25]. The authors of ref. [25] have observed in a few samples the increase of the zero-bias differential conductance for odd number of electrons in the dot (such as the valley 3 on our fig. 4b). In addition, in two samples they have found the Kondo-type peak in the preceding valley, which starts at zero bias and is then (with increase of  $V_g$ ) shifted smoothly to the (small) non-zero bias voltages. In this paper we consider only the zero bias conductance, but already the zero bias part of this observation of ref. [25] agrees well with what we have shown on the valley 2 of fig. 4b.<sup>3</sup>. The explanation of the finite bias part of the experiment should include nonequilibrium effects. The work in this direction is in progress.

<sup>&</sup>lt;sup>2</sup>Some increase of the conductance due to spin exchange still may take place here, but the correction to G is proportional to the small width of narrow level.

<sup>&</sup>lt;sup>3</sup>The "elementary event" shown on the fig. 4b. consists of the sequence of 4 peaks (valleys), but ref. [25] reports measurements for two valleys only.

#### 5 Conclusions

To conclude, motivated by the measurements of the transmission phase in the QD [4, 5] we investigated the charging effects in the QD with a single anomalously broad level. Contrary to the common expectation the pronounced CB takes place here even for  $\Gamma\gg\Delta$ . In this case upon increasing  $V_g$ , it is energetically favorable to first populate in the dot the level strongly coupled to the leads. At a somewhat larger  $V_g$  a sharp jump occurs to a state where the "next in line" narrow level becomes populated. This jump accounts for the sharp decrease by  $\sim \pi$  of the transmission phase observed in the experiment. The absence of significant fluctuations of the strength of resonances seen in the experiment [5] and their large widths are clear within our mechanism. The current transmission through such QD resembles the behavior of rare earth elements, whose chemical properties are determined not by the electrons with highest energy, but by the "strongly coupled" valence electrons. An explicit numerical example shows how such broad levels may appear in ballistic (non-integrable) QD-s.

The overlapping of single-particle resonances may take place also in the Kondo experiments in QD-s, where in order to increase the Kondo temperature the dot is usually sufficiently opened. New low-temperature effects take place in this case, including the Kondo effect in even valleys. This may provide an explanation of the unexpected results of recent experiments [25, 26].

#### 6 Acknowledgements

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