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IN LAZER WAKEFIELD ACCELERATOR

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DRIVER CHANNELING IN LASER WAKEFIELD ACCELERATOR

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Abstract

Guiding the laser pulse in a metal waveguide filled with a rare plasma is proposed as a tool for overcoming driver diffraction in the laser wakefield accelerator. The wakefield wavelength in this case is determined by the density of the inner plasma and can be made large enough to provide effective wave generation. For the circular waveguide of the radius about the anomalous plasma skin-depth, most of the driver energy is contained in TE_{11} mode, and the waveguide does not drastically diminish the dephasing length.

Laser-driven plasma-based accelerators have already demonstrated, tremendous accelerating gradients ($\sim$ 200 GV/m, [1]) that are three orders of magnitude greater than those obtained in conventional RF linacs (see reviews [2,3] and references therein). The net energy gain of accelerated particles ($\sim$ 50 MeV, [4]) is not so impressive since acceleration distance is rather short for typical laser and plasma parameters. The main obstacle on the way to plasma-based high energy accelerators is laser diffraction.

To overcome the diffraction, several mechanisms of optical guiding were proposed [2, p. 271]. Each of them has some imperfections. Relativistic optical guiding, plasma wave guiding, and ponderomotive self-channeling are unable to prevent diffraction of short laser pulses. Tailored pulse [5,6] is a halfway solution to the problem since it can only increase the diffraction length several times. Preformed plasma channels can perfectly guide the laser pulse, but, for effective wakefield excitation, the density of channel walls should be much less than the density of a condensed medium. Channel formation in gas or in plasma is a separate, not simple, problem. It is clear that, with present-day plasma channels, it is not possible to achieve a high precision (of precise metallic surfaces level) field structure necessary for high energy physics applications of laser-driven accelerators.

We propose to use a metal tube filled with a uniform plasma for preventing laser diffraction. In this case, different media are responsible for channeling of the driver (precise metal waveguide) and for wakefield excitation (rare inner plasma). This scheme is easy to realize; in particular, the plasma needs not to be preformed since it will appear for a short time as a result of tunneling ionization of a room-temperature neutral gas filling the waveguide.

Let us consider the circular waveguide of radius R. For the laser strength parameter $a_0 \ll 1$, the plasma behaves as a linear medium with the dielectric constant $\varepsilon = 1 - \omega_p^2/\omega^2 \approx 1$, where $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the plasma electron frequency and ω is the laser frequency ($\omega \gg \omega_p$). Fields inside the waveguide are a superposition of TM and TE modes which are well known (see, e.g., [7,8]):

$$\text{TM:} \quad \vec{E}_{mn} = \nabla \text{div}\, \vec{\Pi}_{mn} + \frac{\omega^2 \varepsilon}{c^2} \vec{\Pi}_{mn}, \quad \vec{H}_{mn} = -\frac{i\omega\varepsilon}{c} \text{rot}\, \vec{\Pi}_{mn}; \qquad (1)$$

$$\text{TE:}\quad \vec{H}_{mn} = \sqrt{\varepsilon}\left(\nabla \mathrm{div}\,\vec{\Pi}_{mn} + \frac{\omega^2\varepsilon}{c^2}\vec{\Pi}_{mn}\right),\quad \vec{E}_{mn} = \frac{\imath\omega\sqrt{\varepsilon}}{c}\mathrm{rot}\,\vec{\Pi}_{mn}; \tag{2}$$

$$\vec{\Pi}_{mn} = A_{mn} J_n\left(\frac{\mu_{mn} r}{R}\right)\cdot\begin{pmatrix}\cos n\varphi\\ \sin n\varphi\end{pmatrix} e^{\imath k_z z - \imath\omega t}\,\vec{e}_z. \tag{3}$$

Here J_n is Bessel function of the order n; μ_{mn} is mth zero of J_n for TM modes, or mth zero of the derivative of J_n with respect to its argument for TE modes; A_{mn} is a normalization factor;

$$k_z^2 = \frac{\omega^2\varepsilon}{c^2} - \frac{\mu_{mn}^2}{R^2}; \tag{4}$$

c is the speed of light, and cylindrical coordinates (r, φ, z) with $\vec{e}_z$ being the waveguide axis are used.

When the laser beam enters the waveguide, it excites many modes at once. For the case of a wide laser beam entering a wide waveguide ($R \gg \lambda = 2\pi c/\omega$), the energy falling into dominant modes can be found as follows. Consider, for example, a linearly polarized beam with the Gaussian electric field distribution:

$$E_x = E_0\, e^{-r^2/a^2}\, e^{\imath kz - \imath\omega t},\quad E_y = 0, \tag{5}$$

$$k = \frac{\omega}{c},\quad x = r\cos\varphi,\quad y = r\sin\varphi. \tag{6}$$

For $a \gg \lambda$, we can neglect the longitudinal component of the laser electric field both before the waveguide and inside it, neglect the difference between k_z and k, and put $H_y = E_x$ and $H_x = -E_y$ everywhere. In this approximation, the amplitude of the transverse electric field inside the waveguide is the sum of vortex-free and solenoidal parts arising from TM and TE modes, respectively:

$$\vec{E} = \vec{E}_{TM} + \vec{E}_{TE} = \imath k e^{\imath kz - \imath\omega t}\left(\nabla_\perp \Pi_{TM} + \mathrm{rot}\,(\Pi_{TE}\vec{e}_z)\right). \tag{7}$$

The scalar functions Π_{TM} and Π_{TE} satisfy two-dimensional Laplace equations which follow from (7):

$$\imath k e^{\imath kz - \imath\omega t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Pi_{TM} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}, \tag{8}$$

$$\imath k e^{\imath kz - \imath\omega t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Pi_{TE} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}. \tag{9}$$

The functions Π_{TM} and Π_{TE} obtained from (8)-(9) should then be expanded into Fourier-Bessel series (Π_{TM}) or Dini series (Π_{TE}) [9] to yield the amplitudes of TM or TE modes, respectively.

The boundary condition for (8) is

$$\Pi_{TM}(R, \varphi) = 0; \tag{10}$$

otherwise the Fourier-Bessel series will not converge. The problem (8), (10) thus have the unique solution. The solution to (9) is chosen so that to give correct electric fields (5) after substitution into (7) and to provide $\Pi_{TE}(0, \varphi) = 0$ (for convergence of the series). It is also unique.

For the electric field (5),

$$\Pi_{TM} = \cos\varphi\,\frac{E_0 a^2}{2}\left(\frac{1}{r}(1 - e^{-r^2/a^2}) - r(1 - e^{-1/a^2})\right), \tag{11}$$

$$\Pi_{TE} = \sin\varphi\,\frac{E_0 a^2}{2}\left(\frac{1}{r}(1 - e^{-r^2/a^2}) + r(1 - e^{-1/a^2})\right). \tag{12}$$

Only the modes with the azimuthal number $n = 1$ are excited. The fractions of the laser energy falling into dominant waveguide modes are shown in Fig. 1. The structure of the electric field for these modes is shown in Fig. 2. It is seen from Fig. 1 that for $a \approx 0.75R$ up to 85 % of the incident energy falls into TE_{11} mode.

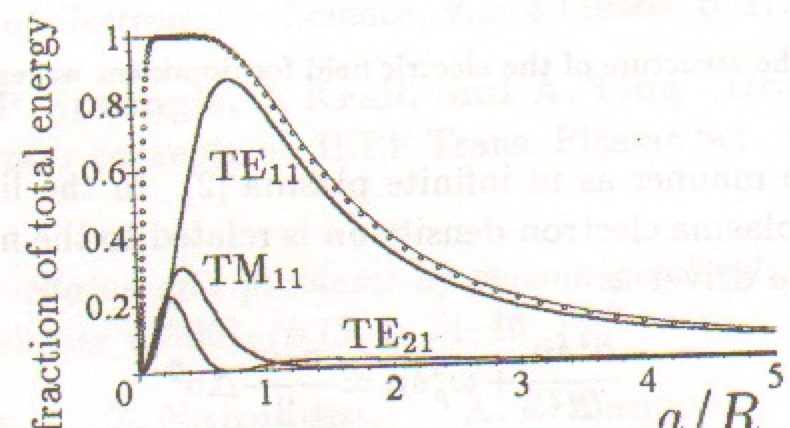

Figure 1: Laser energy falling into separate waveguide modes as functions of the incident beam radius. Thin curve shows the total energy falling into the hole. It nearly coincides with the total energy contained in first five TM and TE modes (shown by dots).

If the incident laser pulse is short, the localized in z-direction electromagnetic field of excited modes drives the plasma wave inside the waveguide in

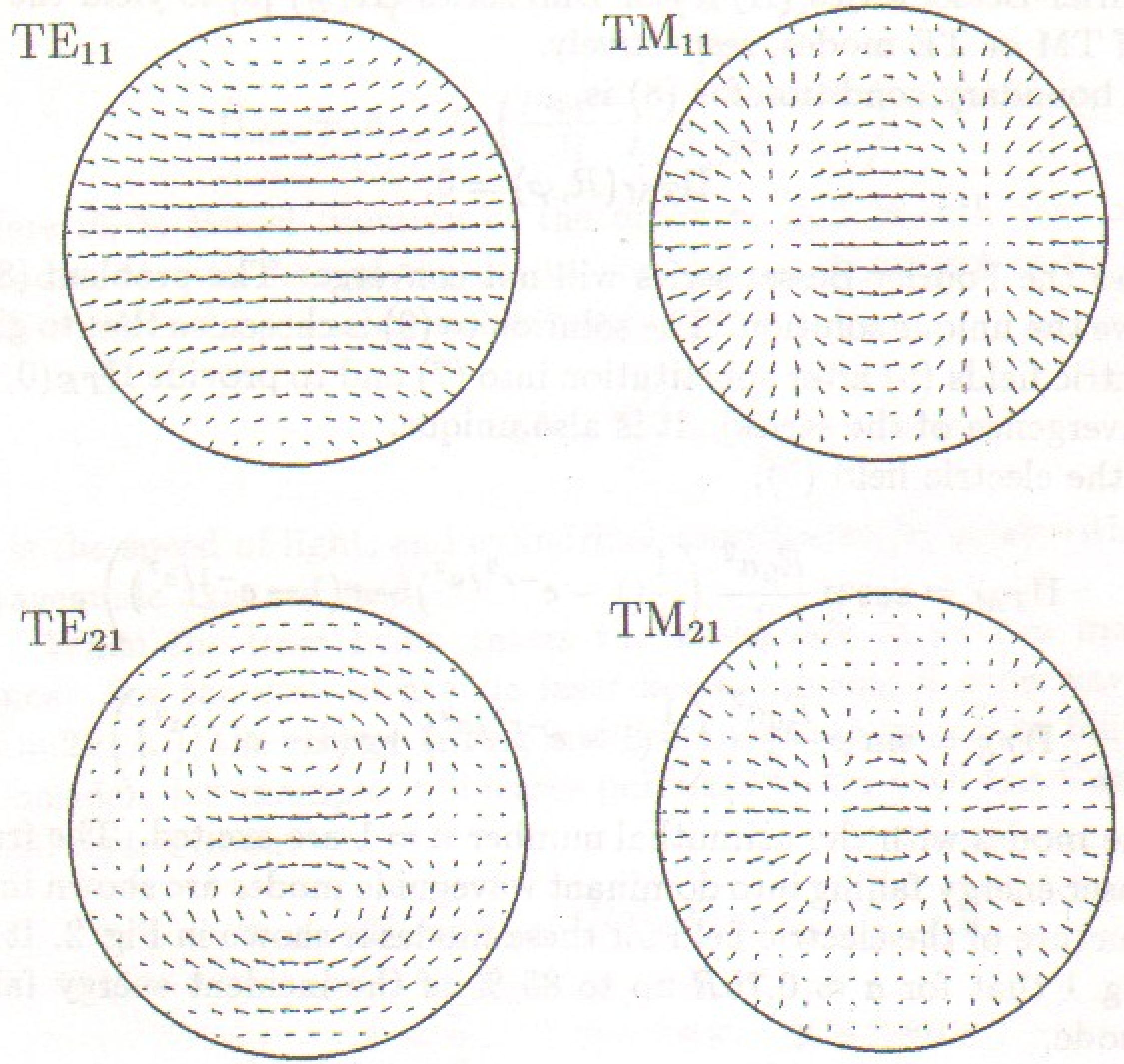

Figure 2: The structure of the electric field for dominant waveguide modes.

almost the same manner as in infinite plasma [2]. In the linear regime, the perturbation of plasma electron density δn is related to the normalized vector potential $\vec{a}$ of the driver as

$$\frac{\partial^2 \delta n}{\partial t^2} + \omega_p^2 \delta n = \frac{n_0 c^2}{2} \Delta a^2 \tag{13}$$

(where Δ is the Laplacian); behind the driver the force $\vec{F}$ acting on accelerated ultra-relativistic electrons is the gradient of some scalar function Φ,

$$\vec{F} = -e(\vec{E} + [\vec{e}_z \times \vec{H}]) = \nabla\Phi, \tag{14}$$

which is determined by δn:

$$\Delta_\perp \Phi - \frac{\omega_p^2}{c^2}\Phi = 4\pi e^2 \delta n. \tag{15}$$

The only difference between the waveguide and the infinite plasma is in the boundary condition for (15): here Φ should turn to zero at $r = R$.

For $R \gtrsim c/\omega_p$, the second term in the left-hand side of (15) is dominant, and the accelerating electric field is little affected by the presence of the metal walls:

$$-eE_z = \frac{\partial \Phi}{\partial z} \approx \frac{\imath\omega_p}{c}\Phi \sim -\imath\frac{4\pi e^2 c\,\delta n}{\omega_p}. \tag{16}$$

Otherwise ($R \ll c/\omega_p$), the electric field in the plasma wave is mainly transversal, and E_z is roughly $(R\omega_p/c)^2$ times less than in infinite plasma with the same δn.

Electromagnetic wavelets in waveguides propagate slower than in the free space. For $R \gtrsim c/\omega_p$, the waveguide contributes to decrease of the group velocity roughly as much as the plasma. Thus, the waveguide does not drastically diminish the length of dephasing between the driver and accelerated particles.

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