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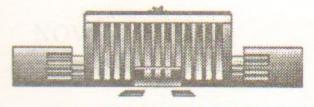
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# ELECTRON-PHOTON INTERACTIONS IN HIGH ENERGY BEAM PRODUCTION AND COOLING

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#### Electron-photon interactions in high energy beam production and cooling<sup>1</sup>

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#### Abstract

In this review we consider three important applications of lasers in high energy physics:  $\gamma\gamma$ ,  $\gamma$ e colliders, laser cooling, positron production. These topics are actual now due to plans of construction linear  $e^+e^-$ ,  $e^-e^-$ ,  $\gamma\gamma$ ,  $\gamma$ e colliders with energies 0.3–1 TeV. High energy photons for  $\gamma\gamma$ ,  $\gamma$ e collisions can be obtained using laser backscattering. These types of collisions considerably increase physics potential of linear colliders. Very low emittance of electron beams required for achieving ultimate  $\gamma\gamma$  luminosity can be obtained using a laser cooling of electron beams. Combining a laser-electron Compton scattering with subsequent conversion of these photons to  $e^+e^-$  pairs on the target (it can be a laser target) one can get a nice source of polarized positron. In this paper, we briefly considered these subjects with emphasis on underlying physics of photon-electron interactions.

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# 1 Introduction

By "Electron-photon interactions ..." in the title of this paper we imply the electron – laser beam interactions. Fantastic progress in laser technique makes it possible now to consider seriously many various applications of lasers in particle physics. In this talk I will consider only three subjects connected with development of high energy linear colliders:

- High energy gamma-gamma, gamma-electron colliders.
- · Laser cooling of electron beams.
- Positron production for e<sup>+</sup>e<sup>-</sup> colliders.

### 2 Photon colliders

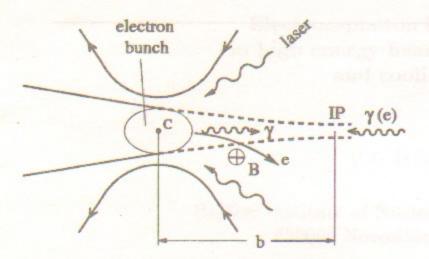
# 2.1 Goals and principles of photon colliders

It is very likely that linear colliders with the c.m.s energies of 0.3–2 TeV will be built in about ten years from now [1]. Besides  $e^+e^-$  collisions, linear colliders give us an unique possibility to study  $\gamma\gamma$  and  $\gamma$ e interactions at energies and luminosities comparable to those in  $e^+e^-$  collisions [2]-[7].

The basic scheme of a photon collider is shown in fig.1. Two electron beams after the final focus system are traveling toward the interaction point (IP) and at a distance of about 0.1–1 cm from the IP collide with the focused laser beams. Photons after the Compton scattering have the energies comparable with the energy of the initial electrons and follow their direction (to the IP) with some small additional angular spread of the order  $1/\gamma$ . With

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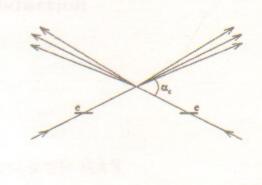


Figure 1: Scheme of  $\gamma \gamma$ ;  $\gamma$ e collider.

Figure 2: Crab-crossing scheme

reasonable laser parameters one can "convert" most of the electrons into high energy photons. The luminosity of  $\gamma\gamma$ ,  $\gamma$ e collisions will be of the same order of magnitude as the "geometric" luminosity of the basic ee beams. Luminosity distributions in  $\gamma\gamma$  collisions have the characteristic peaks near the maximum invariant masses with a typical width about 10 % (and a few times smaller in  $\gamma$ e collisions). High energy photons can have various polarizations, which is very advantageous for experiments.

The physics at high energy  $\gamma\gamma$ ,  $\gamma$ e colliders is very rich and no less interesting than that in e<sup>+</sup>e<sup>-</sup> or pp collisions [7, 8]. Some examples are given below:

- 1. Some phenomena can be better studied at photon colliders than with pp or  $e^+e^-$  collisions, for example, the measurement of the two-photon decay width of the Higgs boson. Some Higgs decay modes and its mass can be measured at  $\gamma\gamma$  colliders more precisely than in  $e^+e^-$  collisions due to larger production cross sections and very sharp edge of the luminosity spectrum.
- 2. Cross sections for production of charged scalar, lepton and top pairs in  $\gamma\gamma$  collisions are larger than those in e<sup>+</sup>e<sup>-</sup> collisions approximately by a factor of 5; for WW production, this factor is even larger, about 10-20.
- 3. In  $\gamma$ e collisions, charged supersymmetric particles with masses higher than in e<sup>+</sup>e<sup>-</sup> collisions can be produced (a heavy charged particle plus a light neutral),  $\gamma\gamma$  collisions also provide higher accessible masses for particles which are produced as a single resonance in  $\gamma\gamma$  collisions (such as the Higgs boson).

These examples together with the fact that the luminosity in  $\gamma\gamma$  collisions is potentially higher than that in e<sup>+</sup>e<sup>-</sup> collisions (due to difference in collisions effects) are very strong arguments in favor of photon colliders. This option has been included now into the Conceptual Design Reports of NLC [9],

TESLA-SBLC [10], and JLC [11] linear colliders. All these projects foresee the second interaction regions for  $\gamma\gamma$ ,  $\gamma$ e collisions.

### 2.2 Effects in the conversion region

In the conversion region, the main process is the Compton scattering of laser photons on high energy electrons. Besides, there are several other important effects.

- 1) The first one is  $e^+e^-$  pair creation in collisions of high energy photons with laser photons. Above the threshold, the process  $\gamma\gamma_L \to e^+e^-$  has the cross section even larger than that of the Compton scattering. The optimum laser wave length for photon colliders corresponds to this threshold.
- 2) Nonlinear QED effects (multiphoton processes) in interactions of electrons with laser wave. Strong laser field leads to change of the energy spectra of the scattered photons, makes possible e<sup>+</sup>e<sup>-</sup> pair creation at electron beam energies lower than that in the case of weak laser field.
- 3) A laser target is a medium with some anisotropic refraction index. As a result, a polarization of high energy photons after the Compton scattering can be noticeably changed during its propagation through a laser target.
- 4) Polarization of electrons is changed obviously when they scatter. But it is less obvious that the polarization is changed for electrons which passed a laser target without Compton scattering.

#### 2.2.1 Compton scattering (linear)

Basic formulae for the Compton scattering in a form convenient for the case of our interest are well known [3, 4].

In the conversion region a photon with an energy  $\omega_0$  is scattered on an electron with an energy  $E_0$  at a small collision angle  $\alpha_0$  (almost head-on). The energy of the scattered photon  $\omega$  depends on its angle  $\vartheta$  with respect to the motion of the incident electron as follows:

$$\omega = \frac{\omega_m}{1 + (\vartheta/\vartheta_0)^2}; \qquad \omega_m = \frac{x}{x+1} E_0; \qquad \vartheta_0 = \frac{mc^2}{E_0} \sqrt{x+1}, \quad (1)$$

$$x = \frac{4E_0\omega_0\cos^2\alpha_0/2}{m^2c^4} \simeq 15.3 \left[\frac{E_0}{TeV}\right] \left[\frac{\omega_0}{eV}\right],$$

 $\omega_m$  is the maximum photon energy,

For example:  $E_0 = 300$  GeV,  $\omega_0 = 1.17$  eV (neodymium glass laser)  $\Rightarrow$  x=5.37 and  $\omega/E_0 = 0.84$ . The value x = 4.8 is the threshold for e<sup>+</sup>e<sup>-</sup>

production (see next section). The angles of scattered particles as functions of the photon energy for x = 4.8 are displayed in fig.3.

The energy spectrum of the scattered photons (without multiple scattering) is given by the Compton cross section

$$\frac{1}{\sigma_c} \frac{d\sigma_c}{dy} = \frac{2\sigma_0}{x\sigma_c} \left[ \frac{1}{1-y} + 1 - y - 4r(1-r) + 2\lambda_e P_c r x (1-2r)(2-y) \right]; \quad (2)$$

$$y = \le y_m = \frac{x}{x+1}; \quad r = \frac{y}{x(1-y)} \le 1; \quad \sigma_0 = \pi \left(\frac{e^2}{mc^2}\right)^2 = 2.5 \times 10^{-25} \text{cm}^2,$$

 $\lambda_e(|\lambda_e| \leq 1/2)$  is the electron longitudinal polarization and  $P_c$  is the mean helicity of laser photons. The total Compton cross section is

$$\sigma_c = \sigma_c^{np} + 2\lambda_e P_c \sigma_1, \tag{3}$$

$$\sigma_e^{np} = \frac{2\sigma_0}{x} \left[ \left( 1 - \frac{4}{x} - \frac{8}{x^2} \right) \ln(x+1) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(x+1)^2} \right],$$

$$\sigma_1 = \frac{2\sigma_0}{x} \left[ \left( 1 + \frac{2}{x} \right) \ln(x+1) - \frac{5}{2} + \frac{1}{x+1} - \frac{1}{2(x+1)^2} \right].$$

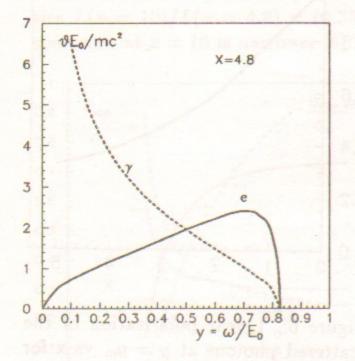
In the region x=1-10 the ratio  $|\sigma_1/\sigma_c| < 0.2$ , i.e. the total cross section only slightly depends on the polarization. However, the energy spectrum does essentially depend on the value of  $2\lambda P_c$ . At  $2\lambda P_c = -1$  and x=4.8 the relative number of hard photons doubles (see fig.4), improving essentially the monochromaticity of the photon beam.

Using the polarized initial electrons and (or) laser photons, one can obtain high energy photons with various polarizations [4]. In particular, the average helicity is

$$\lambda_{\gamma}(y) = \frac{2\lambda_e xr[1 + (1-y)(1-2r)^2] + P_c(1-2r)((1-y)^{-1} + 1-y)}{(1-y)^{-1} + 1 - y - 4r(1-r) + 2P_c\lambda_e rx(1-2r)(2-y)}.$$
 (4)

It is shown in fig.5 for x=4.8. Note, if polarization of laser photons  $P_c=\pm 1$ , then  $\lambda_{\gamma}=-P_c$  at  $y=y_m$ . In the case of  $2P_c\lambda_e=-1$  (the case with good monochromaticity) all the photons in the high energy peak have a high degree a like-sign polarization. Note that for low x the photon helicity  $\lambda_{\gamma}$  depends on  $\lambda_e$  very slightly and high energy photons have very high circular polarization in a wide energy range near the maximum energy even at  $\lambda_e=0$ .

High degree of the circular photon polarization is essential for suppression of the QED backgrounds for the Higgs production, because  $\sigma(\gamma\gamma \to H) \propto 1 + \lambda_{\gamma 1}\lambda_{\gamma 2}$ , while the main QED background  $\sigma(\gamma\gamma \to q\bar{q}) \propto 1 - \lambda_{\gamma 1}\lambda_{\gamma 2}$ .



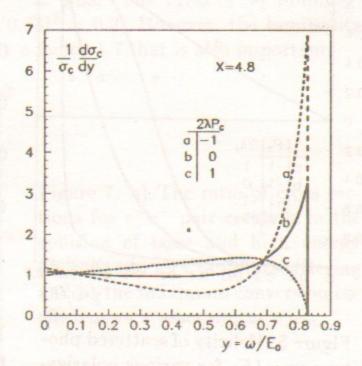


Figure 3: Electron and photon scattering angles vs photon energy for x = 4.8.

Figure 4: Spectrum of the Compton scattered photons for different polarizations of laser and electron beams.

The degree of the linear polarization of the Compton scattered photons is

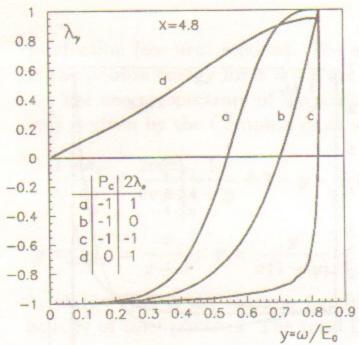
$$l_{\gamma} = \frac{2r^2 P_t}{(1-y)^{-1} + 1 - y - 4r(1-r) + 2P_c \lambda_e r x(1-2r)(2-y)},$$
 (5)

where the linear polarization of laser photons  $P_t$  and their helicity  $P_c$  for the case of complete polarization are connected by relation  $P_t^2 + P_c^2 = 1$ .

The dependence of  $l_{\gamma}$  at  $y=y_m$  on the parameter x is shown in fig.6 for  $P_t=1$  and unpolarized electron beams. At large x is not sufficiently high. It is of interest that one can get larger  $l_{\gamma}$ , up to  $l_{\gamma}=1$ , it is so for  $2\lambda_e P_c=x(x+2)/(x^2+2x+2)$ . However, in this case  $2\lambda_e P_c\approx +1$ , that corresponds to curve c in fig.4, when the number of photons with the energy near  $\omega_m$  is small.

Linear polarization of photon beams will be very useful for determinating the Higgs CP parity. The cross section of the Higgs production by two photons  $\sigma(\gamma\gamma \to H) \propto 1 \pm l_{\gamma_1}l_{\gamma_2}$  for CP=  $\pm$  respectively.

General formulae for the polarization of scattered photons summed over the polarization of the final electrons are given in refs.[12, 4]. However, simulation the multiple Compton scattering requires knowledge of final electron polarization. This problem was considered recently in ref.[13].



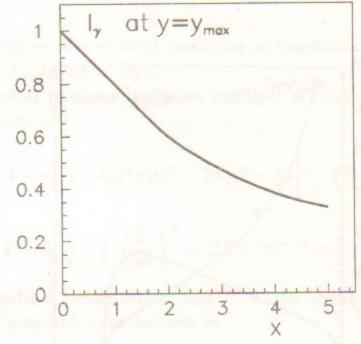


Figure 5: Helicity of scattered photons vs  $\omega/E_0$  for various polarizations of laser and electron beams.

Figure 6: Linear polarization of the scattered photons at  $y = y_m$  vs x for  $P_t = 1$  and  $\lambda_e = 0$ 

Finally, it turns out that the polarization of electrons and high energy photons traveling in a laser target is changed even for particles which have not scattered. This unusual and important phenomena will be discussed later.

#### 2.2.2 e+e- pair production

With increase of the laser photon energy the energies of the scattered photons also increases and monochromaticity of the spectrum improves. However, besides Compton scattering, other processes become possible in the conversion region [3, 5]. The most important one is the process of  $e^+e^-$  pair production in a collision of a laser photon with high energy scattered photon:  $\gamma_0 + \gamma \rightarrow e^+e^-$ . The threshold of this reaction is  $\omega_m\omega_0 > m^2c^4$ , i.e.  $x = 2(1+\sqrt{2}) \approx 4.83$ , the corresponding wavelength and laser photon energy are

$$\lambda = 4.2E_0[\text{TeV}] \ \mu m; \qquad \omega_0 = 0.3/E_0[\text{TeV}] \text{ eV}.$$
 (6)

Above the threshold region the pair production cross section exceeds the Compton one by a factor of 1.5-2 [5, 6], see fig.7. Due to this fact the maximum conversion coefficient at  $x \sim 10-20$  is limited by 35-25% respectively. For these reasons it is preferable to work at  $x \approx 4.8$ . Although, this limit is not absolute. Usually, at x < 4.8 we assume k = 1 that corresponds

to the conversion probability  $1 - e^{-1} = 0.63$ . The ratio of  $\gamma \gamma$  luminosities  $L(x = 10)/L(x = 4.8) = (0.35/0.63)^2 = 0.3$ . However, the luminosity spectrum at x = 10 is narrower [6] by a factor 1.7 that is also important.

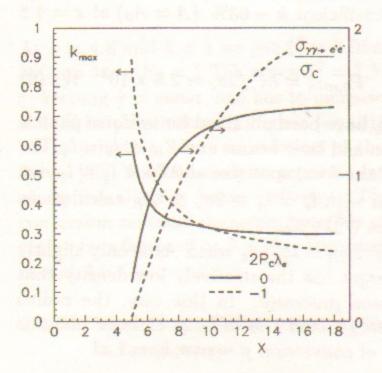


Figure 7: a) The ratio of cross sections for  $e^+e^-$  pair creation in the collision of laser and high energy photons and for Compton scattering and b) the maximum conversion coefficient vs x assuming  $\omega = \omega_m$ .

Besides, e<sup>+</sup>e<sup>-</sup> pairs can be produced in a collision of an electron with a laser photon (Bethe-Heitler process):  $e+\gamma_0 \rightarrow e+e^+e^-$ . However, at x<20 its cross section is less than that of the Compton scattering by two orders of magnitude [3].

#### 2.2.3 Conversion coefficient

The conversion coefficient depends on the energy of the laser flash A as  $k = N_{\gamma}/N_e \sim 1 - \exp(-A/A_0) \ (\sim A/A_0 \ at \ A < A_0).$  (7)

Let us estimate  $A_0$ . At the conversion region the r.m.s radius of the laser beam depends on the distance z to the focus (along the beam) in the following way [3]:  $r_{\gamma} = a_{\gamma} \sqrt{1 + z^2/Z_R^2}$ , where the Rayleigh length  $Z_R = 2\pi a_{\gamma}^2/\lambda$ ,  $a_{\gamma}$  is the r.m.s. focal spot radius. The density of laser photons  $n_{\gamma} = (A/\pi r_{\gamma}^2 \omega_0) \times \exp(-r^2/r_{\gamma}^2) F_{\gamma}(z+ct)$ , where  $\int F_{\gamma}(z) dz = 1$ .

Let us assume the linear density of photons to be uniform along the beam:  $F_{\gamma} = 1/l_{\gamma}$  and  $Z_R \ll l_{\gamma}$ . The probability of the Compton scattering for the electron moving through a laser target along the axis [6]

$$p = 2 \int n_{\gamma} \sigma_c dz \sim \frac{2A\sigma_c}{\pi a_{\gamma}^2 \omega_0 l_{\gamma}} \int_{-\infty}^{\infty} \frac{dz}{1 + \frac{z^2}{Z_R^2}} = \frac{2A\sigma_c}{c\hbar l_{\gamma}}.$$
 (8)

One collision length corresponds to p=1 that gives  $A_0 = \hbar c l_{\gamma}/2\sigma_c$ ,  $P=A_0c/l_{\gamma}=\hbar c^2/2\sigma_c$ .

The minimum laser flash energy (corresponding to  $l_{\gamma} = l_{e}$ ) and power needed for obtaining the conversion coefficient  $k \sim 63\%$  ( $A = A_{0}$ ) at x = 4.8 ( $\sigma_{c} \approx 1.9 \times 10^{-25} \,\mathrm{cm}^{2}$ )

$$A_{0,min} = \hbar c l_e / 2\sigma_c = 8.4 l_e [\text{cm}] J$$
,  $P_{0,min} = \hbar c^2 / 2\sigma_c \approx 2.5 \times 10^{11} W$ . (9)

These minimum values for  $A_0$  and  $P_0$  have been obtained for uniform photon density distribution along the electron and laser beams and  $Z_R \ll l_e = l_\gamma$ . The value of  $A_0$  is almost independent of the focal spot size at  $2Z_R < l_e = l_\gamma$ , i.e.  $a_\gamma < \sqrt{\lambda l_e/4\pi}$ . For Gaussian beams with  $l_e = l_\gamma \equiv 2\sigma_z \gg Z_R$  calculations gives [6]  $A_0 \approx \sqrt{\pi} A_{0,min}$  and  $P_{peak} \approx \sqrt{2} P_{0,min}$ 

Let us consider another example:  $2Z_R \approx l_\gamma \geq l_e$  when  $A_0$  is only slightly larger than  $A_{0,min}$ , but the laser target has the relatively low density that is important for avoiding multiphoton processes. In this case, the radius of laser beam  $r_\gamma \sim a_\gamma$  along the target and the density of laser photons  $n_\gamma \sim A/(\pi\omega_0 a_\gamma^2 l_\gamma)$ . The probability of conversion  $p \sim n_\gamma \sigma_c l_\gamma = 1$  at

$$A_0 \sim \pi \hbar c l_\gamma / 2\sigma_c = \pi A_{0,min} (l_\gamma / l_e). \tag{10}$$

# 2.2.4 Nonlinear effects in Compton scattering in the conversion region.

In the conversion region the density of laser photons can be very high that leads to multiphoton processes [14]-[24] which were studied recently at SLAC: [25]

 $e + n\gamma_0 \rightarrow e + \gamma$ ,  $\gamma + n\gamma_0 \rightarrow e^+e^-$ ,

where  $e, \gamma$  are high energy electron and photon respectively,  $\gamma_0$  denotes a laser photon. Nonlinear effects depend (beside x) on the parameter

$$\xi^2 = (eB\hbar/m\omega_0 c)^2, \tag{11}$$

where B is the field strength,  $\omega_0$  is the photon energy. At  $\xi^2 \ll 1$  an electron interacts with one photon (Compton scattering), while at  $\xi^2 \gg 1$  an electron interacts with a collective field (synchrotron radiation).

What values of  $\xi^2$  are acceptable? In a strong field, an electron has transverse motion, which increases the effective electron mass: [14]  $m^2 \to m^2(1+\xi^2)$ . Due to this fact the maximum energy of scattered photons is decreased by  $\Delta \omega_m/\omega_m = \xi^2/(x+1)$ , that is 5% at  $\xi^2 = 0.3$  and x = 4.8. Let us estimate  $\xi^2$  in our case. Assuming  $2Z_R \approx l_\gamma$  (as for eq.10) and taking into

account that the field in the laser focus is  $B^2 = 4A/a_{\gamma}^2 l_{\gamma}$  and  $A = kA_0$  one get [5, 6]

 $\xi^2 \approx \frac{2}{\pi \alpha} \frac{\sigma_0}{\sigma_c} \frac{\lambda}{l_{\gamma}} k \tag{12}$ 

At x = 4.8 and k = 1 we get  $\xi^2 = 0.05E_0[\text{TeV}]/l_{\gamma}$ . For example, if  $l_{\gamma} = l_e = 200 \,\mu\text{m}$  and  $E_0 = 1$  TeV, then  $\xi^2 = 2.5$ . This is not acceptable. To decrease  $\xi$ , keeping k = const, one has to increase  $l_{\gamma}$ , correspondingly the flash energy should be proportionally higher (eq.10).

In principle, there is solution of this problem, it is stretching the laser focus. [32]. One can focus n separate lasers to n adjacent conversion regions along the electron beam pathway. The total flash energy needed for the conversion remains the same, but  $\xi^2$  is smaller than that with one conversion region by a factor of n. Another way, stretching a "chirped" pulse, is discussed in sect.4.

# 2.2.5 Variation of high energy electrons and photons polarization in laser wave

#### Polarization of unscattered electrons

Let us consider first the following example: an unpolarized electron beam is collided with a circularly polarized laser pulse. Some electrons pass this target without the Compton scattering. What happened with their polarization? It is changed, since the cross section of the Compton scattering depends on the product  $P_c\lambda_e$  and unscattered electron beam contains already unequal amount of electrons with the forward and backward helicities. Considering the multiple Compton scattering we should take this effect into account.

General formulae for this effect has been obtained in ref.[26] (see V.Serbo talk at this Workshop), where the variation in polarization of the unscattered electrons was considered as the result of interference of the incoming electron wave and the wave scattered at the zero angle.

#### Variation of \gamma-quanta polarization in a laser wave

It is well known that a region with an electromagnetic field can be regarded as an anisotropic medium [14]. Strong laser field also has such properties. As a result, the polarization of high energy photon produced in the Compton scattering may be changed during its propagation through the polarized laser target [27] (see V.Serbo talk at this Workshop). This effect is large at  $x \approx 4.8$  (the threshold for e<sup>+</sup>e<sup>-</sup> production). Note, that in the case most important for us  $2P_C\lambda_e = 1$  the polarization of high energy circu-

larly polarized photons is not changed when they propagate in the circularly polarized laser wave.

In principle, using two adjacent conversion regions one can produce first circular polarized photons (using a circularly polarized laser) and then change the circular polarization to the linear one using a linearly polarized laser.

A similar effect, influence of the electromagnetic field on the polarization of high energy photons, exists also at the interaction region of photon colliders [28].

# 3 Effects at the interaction region, ultimate $\gamma\gamma$ luminosity

Due to the absence of beamstrahlung, beams in  $\gamma\gamma$  collisions can have much smaller horizontal beam sizes than in e<sup>+</sup>e<sup>-</sup> collisions. However, even after optimization of the final focusing system the attainable  $\gamma\gamma$  luminosity in current LC projects is determined by the attainable "geometric" ee-luminosity. For the "nominal" beam parameters (the same as in e<sup>+</sup>e<sup>-</sup> collisions) and optimum final focus systems the luminosity  $L_{\gamma\gamma}(z>0.65)\sim 10^{33}~{\rm cm}^{-2}{\rm s}^{-1}$  ( $z=W_{\gamma\gamma}/2E_0$ . Obviously, this is not a fundamental limit.

The only collision effect restricting  $\gamma\gamma$  luminosity at photon colliders is the coherent pair production which leads to the conversion of a high energy photon into e<sup>+</sup>e<sup>-</sup> pair in the field of the opposing electron beam [29, 5, 6]. The probability of this process is high when  $B \geq B_{cr} = \alpha e/\gamma r_e^2 = 2.2 \times 10^7/E[\text{TeV}] \text{ G}.$ 

There are three ways to avoid this effect: a) use flat beams; b) deflect the electron beam after conversion at a sufficiently large distance from the IP; c) under certain conditions (low beam energy, long bunches) the beam field at the IP is below the critical one due to the repulsion of electron beams [30]. The problem of ultimate luminosities for different beam parameters and energies was analyzed recently in ref.[31] analytically and by simulation. The resume is following. At "low" energies (2E < 0.3 - 1 TeV), depending on the beam length), the coherent pair production is not very important, even for a very small  $\sigma_x$ 's, it is because electron beams are repulsing each other so that the field on the beam axis (which affects high energy photons) is below the critical field. It means that the  $\gamma\gamma$  luminosity is simply proportional to the geometric electron-electron luminosity  $L_{\gamma\gamma}(z>0.65)\sim 0.1L_{ee}$ . Having electron beams with sufficiently low emittances one can obtain, in principle,  $L_{\gamma\gamma}(z>0.65)$  above  $10^{35}$  cm<sup>-2</sup>s<sup>-1</sup>.

One of the main problems here is the generation of electron beams with small emittances in both horizontal and vertical directions. One of promising techniques is discussed below.

# 4 Laser cooling of electron beams

The transverse beam sizes at the interaction point are determined by the emittances  $\epsilon_x$ , and  $\epsilon_y$ :  $\sigma_i = \sqrt{\epsilon_i \ \beta_i}$ , where  $\beta_i$  is the beta function at the IP. With an increase of the beam energy the emittance of the bunch decreases:  $\epsilon_i = \epsilon_{ni}/\gamma$ , where  $\gamma = E/mc^2$ ,  $\epsilon_{ni}$  is the normalized emittance. The beams with a small  $\epsilon_{ni}$  are usually prepared in damping rings which naturally produce bunches with  $\epsilon_{ny} \ll \epsilon_{nx}$ . Laser RF photoguns can also produce beams with low emittances. However, for linear colliders it is desirable to have smaller emittances. Recently a new method of electron beam cooling was proposed – laser cooling – which allows further reduction of the transverse emittances after damping rings or guns by 1–3 orders of magnitude [32].

# 4.1 One pass laser cooling

The idea of laser cooling of electron beams is very simple. During a collision with optical laser photons (in the case of strong fields it is more appropriate to consider the interaction of an electron with an electromagnetic wave) the transverse distribution of electrons  $(\sigma_i)$  remains almost the same. Also, the angular spread  $(\sigma'_i)$  is almost constant, because electrons loss momenta almost along their trajectories (photons follow the initial electron trajectory with a small additional spread). So, the emittance  $\epsilon_i = \sigma_i \sigma'_i$  remains almost unchanged. At the same time, the electron energy decreases from  $E_0$  down to E. This means that the transverse normalized emittances have decreased:  $\epsilon_n = \gamma \epsilon = \epsilon_{n0} (E/E_0)$ . One can reaccelerate the electrons up to the initial energy and repeat the procedure. Then after N stages of cooling  $\epsilon_n/\epsilon_{n0} = (E/E_0)^N$  (if  $\epsilon_n$  is far from its limit). There are several questions to this method: 1) requirements on laser parameters 2) an energy spread of the beam after cooling 3) a limit on the final normalized emittances; 4) depolarization.



#### 4.1.1 Flash energy

Physics in the cooling region is the same as in the conversion region of photon colliders sect.2.2. Typical values of parameters in this problem:  $E_0 \sim 5 \text{ GeV}, x \ll 1, \xi^2 \sim 0.5 - 10.$ 

Passing the laser target the electron gradually loss its energy and at the exit [32] (it is assumed that  $Z_R \ll l_\gamma \simeq l_e$ )

$$\frac{\epsilon_{n0}}{\epsilon_n} \simeq \frac{E_0}{E} = 1 + \frac{64\pi^2 r_e^2 \gamma_0}{3mc^2 \lambda l_e} A; \quad A[J] = \frac{25\lambda [\mu \text{m}] l_e [\text{mm}]}{E_0 [\text{GeV}]} \left(\frac{E_0}{E} - 1\right). \quad (13)$$

For example: at  $\lambda = 0.5 \ \mu\text{m}$ ,  $l_e = 0.2 \ \text{mm}$ ,  $E_0 = 5 \ \text{GeV}$ ,  $E_0/E = 10 \ \text{the}$  required laser flash energy  $A = 4.5 \ \text{J}$ . To reduce the laser flash energy in the case of long electron bunches, one can compress the bunch before cooling as much as possible and stretch it after cooling up to the required value.

Assume  $Z_R \sim 0.25l_e$  we can find the parameter  $\xi^2$ :

$$\xi^{2} = \frac{16r_{e}\lambda A}{\pi l_{e}^{2}mc^{2}} = \frac{3\lambda^{2}}{4\pi^{3}r_{e}l_{e}\gamma_{0}} \left(\frac{E_{0}}{E} - 1\right) = 4.3 \frac{\lambda^{2}[\mu \text{m}]}{l_{e}[\text{mm}]E_{0}[\text{GeV}]} \left(\frac{E_{0}}{E} - 1\right).$$
(14)

In the previous example  $\xi^2 = 9.7$ . In principle, both the "undulator" and "wiggler" cases are possible. We will see later that in order to have lower limit on emittance and smaller depolarization it is necessary to have a low  $\xi^2$ . With a conventional optics one can reduce  $\xi^2$  only by increasing  $l_{\gamma}$  (and  $Z_R$ ) with a simultaneous increase of the laser flash energy. From (13) and (14) we get  $A \propto (\lambda^3/\gamma_0^2\xi^2)(E_0/E-1)^2$ .

In sect.2.2 we already discussed the way of reducing  $\xi^2$  keeping the flash energy constant, it is stretching of the conversion region without changing the radius of this area. One of possible solutions [32] uses chirped laser pulses (wave length depends linearly on longitudinal position) and chromaticity of the focusing system. In this scheme, the laser target consists of many laser focal points (continuously) and light comes to each point exactly at the moment when the electron bunch is there. The required flash energy is determined only by diffraction and at the optimum wave length it does not depend on the collider energy.

The electron energy spread arises from the quantum-statistical nature of radiation. After energy loss  $\Delta E$ , the increase of the energy spread  $\Delta(\sigma_E^2) = \int \varepsilon^2 \dot{n}(\omega) d\omega dt = -aE^2 \Delta E$ , where  $\dot{n}(\omega)$  is the spectral density of photons emitted per unit time,  $a = 14\omega_0/5m^2c^4 = 7x_0/10E_0$  for the Compton case and  $a = 55\hbar eB_0/(8\pi\sqrt{3}m^3c^5)$  for the "wiggler" case.

There is the second effect which leads to decreasing the energy spread. It is due to the fact that  $dE/dx \propto E^2$  and an electron with higher (lower)

energy than the average loses more (less) than on average. This results in the damping:  $d(\sigma_E^2)/\sigma_E^2 = 4dE/E$  (here dE has negative sign). The full equation for the energy spread is  $d\sigma_E^2 = -aE^2dE + 4(dE/E)\sigma_E^2$ , with solution  $\sigma_E^2/E^2 = \sigma_{E_0}^2 E^2/E_0^4 + aE_0(E/E_0)(1 - E/E_0)$ . In our case

$$\frac{\sigma_E^2}{E^2} \sim \frac{\sigma_{E_0}^2 E^2}{E_0^4} + \frac{7}{10} x_0 \left(1 + \frac{275\sqrt{3}}{336\pi} \xi\right) \frac{E}{E_0} \left(1 - \frac{E}{E_0}\right),\tag{15}$$

here the results for the Compton scattering and SR are joined together. Example: at  $\lambda = 0.5 \ \mu\text{m}$ ,  $E_0 = 5 \ \text{GeV}$  ( $x_0 = 0.19$ ) and  $E_0/E = 10$ , the Compton term alone gives  $\sigma_E/E \sim 0.11$  and with the "wiggler" term ( $\xi^2 = 9.7$ , see the example above)  $\sigma_E/E \sim 0.17$ . What  $\sigma_E/E$  is acceptable? In the last example  $\sigma_E/E \sim 0.17$  at  $E = 0.5 \ \text{GeV}$  (after cooling). This means that at the collider energy  $E = 250 \ \text{GeV}$  we will have  $\sigma_E/E \sim 0.034\%$ , that is better than necessary (about 0.1 %).

In a two stage cooling system, after reacceleration to the initial energy  $E_0 = 5$  GeV the energy spread is  $\sigma_E/E_0 \sim 1.7\%$ . For this value there may be a problem with focusing of electrons which can be solved using a focusing scheme with correction of chromatic aberrations.

#### 4.1.2 The minimum normalized emittance

It is determined by the quantum nature of the radiation. Let us start with the case of pure Compton scattering at  $\xi^2 \ll 1$  and  $x_0 \ll 1$ . In this case, the scattered photons have the energy distribution (see eq.2:)  $dp = (3/2)[1 - 2\omega/\omega_m + 2(\omega/\omega_m)^2]d\omega/\omega_m$ , where  $\omega_m = 4\omega_0\gamma^2$ . The angle of the electron after scattering is [3]  $\theta_1^2 = (\omega_m\omega - \omega^2)/(\gamma^2E^2)$ . After averaging over the energy spectrum we get the average  $\theta_1^2$  in one collision:  $\langle \theta_1^2 \rangle = 12\omega_0^2/(5m^2c^4)$ . After many Compton collisions  $(N_{coll})$  the r.m.s. angular spread in i=x,y projection  $\Delta \langle \theta_i^2 \rangle = 0.5\Delta \langle \theta^2 \rangle = 0.5N_{coll} \langle \theta_1^2 \rangle = -0.5(\Delta E/\bar{\omega})\langle \theta_1^2 \rangle = -3\omega_0\Delta E/5E^2$ .

The normalized emittance  $\epsilon_{ni}^2 = (E^2/m^2c^4)\langle r_i^2\rangle\langle \theta_i^2\rangle$  does not change when  $\Delta\langle \theta_i^2\rangle/\langle \theta_i^2\rangle = -2\Delta E/E$ . Taking into account that  $\langle \theta_i^2\rangle \equiv \epsilon_{ni}/\gamma\beta_i$  we get the equilibrium emittance due to the Compton scattering

$$\epsilon_{ni,min} \approx 0.5 \gamma E \beta_i \Delta \langle \theta_i^2 \rangle / \Delta E = \frac{3\pi}{5} \frac{\lambda_c}{\lambda} \beta_i = \frac{7.2 \times 10^{-10} \beta_i [mm]}{\lambda [\mu \text{m}]} \text{ m·rad,}$$
(16)

where  $\lambda_c = \hbar/mc$ . For example:  $\lambda = 0.5 \mu m$ ,  $\beta = l_e/2 = 0.1 \text{ mm (NLC)} \Rightarrow \epsilon_{n,min} = 1.4 \times 10^{-10} \text{ m} \cdot \text{rad}$ . For comparison in the NLC project the damping rings have  $\epsilon_{nx} = 3 \times 10^{-6} \text{ m} \cdot \text{rad}$ ,  $\epsilon_{ny} = 3 \times 10^{-8} \text{ m} \cdot \text{rad}$ .

If  $\xi^2 \gg 1$ , the electron moves as in a wiggler. Assuming that the wiggler is planar and deflects the electron in the horizontal plane one can obtain the equilibrium normalized emittance [32]

$$\epsilon_{nx} = \frac{11e^3\hbar c\lambda^2 B_0^3 \beta_x}{24\sqrt{3}\pi^3 (mc^2)^4} = \frac{11}{3\sqrt{3}} \frac{\lambda_C}{\lambda} \beta_x \xi^3 \approx \frac{8 \times 10^{-10} \beta_x [\text{mm}] \xi^3}{\lambda [\mu\text{m}]} \text{ m·rad.}$$
 (17)

For our previous example we have  $\xi^2 = 9.7$  and  $\epsilon_{nx} = 5 \times 10^{-9}$  m·rad (in the NLC  $\epsilon_{nx} = 3 \times 10^{-6}$  m·rad). Stretching the cooling region with  $n_f = 10$ , further decreases the horizontal emittance by a factor 3.2.

For the minimum vertical normalized emittance at  $\xi^2 \gg 1$  estimations give the value [32]

$$\epsilon_{ny_{min}} \sim 3 \left(\frac{\lambda_c}{\lambda}\right) \beta_y \xi \approx \frac{1.2 \times 10^{-9} \beta_y [\text{mm}] \xi}{\lambda [\mu \text{m}]} \text{ m·rad.}$$
(18)

For arbitrary  $\xi^2$  the minimum emittances can be estimated as the sum of (16) and (17) for  $\epsilon_{nx}$  and sum of (16) and (18) for  $\epsilon_{ny}$ 

$$\epsilon_{nx} \approx \frac{3\pi}{5} \frac{\lambda_C}{\lambda} \beta_x (1 + 1.1\xi^3); \quad \epsilon_{ny} \sim \frac{3\pi}{5} \frac{\lambda_C}{\lambda} \beta_y (1 + 1.6\xi).$$
 (19)

Finally, the depolarization of the electron beam during the laser cooling [32]

$$\Delta \zeta/\zeta \approx 0.3x_0(1+1.8\xi). \tag{20}$$

For the previous example with  $\xi^2 = 9.7$  and  $x_0 = 0.19$  we get  $\Delta \zeta/\zeta = 0.057 + 0.32 = 0.38$ , that is not acceptable. This example shows that the depolarization effect imposes the most demanding requirements on the parameters of the cooling system. The main contribution to depolarization gives the second term. Stretching the cooling region by a factor of ten we can get  $\Delta \zeta/\zeta = 0.057 + 0.1 \sim 15\%$ .

Possible sets of parameters for the laser cooling is the following:  $E_0 = 4.5$  GeV,  $l_e = 0.2$  mm,  $\lambda = 0.5$   $\mu$ m, flash energy  $A \sim 5-10$  J, focusing system with stretching factor  $n_f = 10$ . The final electron bunch will have an energy of 0.45 GeV with an energy spread  $\sigma_E/E \sim 13\%$ , the normalized emittances  $\epsilon_{nx}$ ,  $\epsilon_{ny}$  are reduced by a factor 10, the limit on the final emittance is  $\epsilon_{nx} \sim \epsilon_{ny} \sim 2 \times 10^{-9}$  m·rad at  $\beta_i = 1$  mm, depolarization  $\Delta \zeta/\zeta \sim 15\%$ . The two stage system with the same parameters gives 100 times reduction of emittances (with the same limits).

For the cooling of the electron bunch train one laser pulse can be used many times. According to (13)  $\Delta E/E = \Delta A/A$  and even 25% attenuation of laser power leads only to small additional energy spread.

The proposed scheme of laser cooling of electron beams seems very promising for future linear colliders (especially for photon colliders) and allows to reach ultimate luminosities. Perhaps this method can be used for X-ray FEL based on high energy linear colliders.

# 4.2 Laser cooling in storage rings

Application of the laser cooling for damping rings has one problem: an electron loses in each Compton scattering a large amount of its energy (up to  $4\gamma^2\omega_0$ ) that leads to very large beam energy spread or knock out of the electrons from the beam. Nevertheless, Huang and Ruth have shown [33] that the laser cooling can successfully be used for storage rings in the energy range from a few MeV to a few hundred MeV.

The minimum transverse emittance in a laser cooled storage ring is given by eq.16: For example, for  $\lambda = 1 \, \mu \text{m}$  and  $\beta = 1 \, \text{cm}$  the minimum normalize emittance is about  $7.3 \times 10^{-9}$  m rad, much better than that given by other sources

The energy spread is determined by fluctuations in the Compton scattering. For the Compton scattering (see two paragraphs above eq.15)  $\Delta(\sigma_E^2) = -(14\omega_0/5m^2c)E^2dE$ . In equilibrium this growth of emittance is compensated by the radiation damping  $d(\sigma_E^2)/\sigma_E^2 = 2dE/E$  (due to synchrotron oscillations in storage rings the cooling rate is smaller than that in linear cooling by a factor of 2. By comparing heating and cooling rate one gets [33]

$$\sigma_E/E = \sqrt{14\pi\lambda_c\gamma/5\lambda}, \quad \lambda_C = \hbar/mc.$$
 (21)

For example, when E=100 MeV and  $\lambda=1~\mu\mathrm{m}$  the r.m.s energy spread is about 2.6 %. The momentum acceptance for such ring should be about 15%.

Beside effects in the Compton scattering, we should not forget the beam space charge and intrabeam scattering problems inherent to low energy rings. In ref.[33] two configuration were considered: a) E = 100 MeV,  $N = 1.3 \times 10^{10}$ , R = 1 m; b) E = 8 MeV,  $N = 1.1 \times 10^{10}$ , R = 0.5 m. For these parameters they obtained the equilibrium normalized emittances of  $1 \times 10^{-7}$  m and  $2 \times 10^{-5}$  m or by factor 15 and 3000 larger than it is following from eq.16. For more details see ref.[33] and Huang and Ruth talks at this workshop. In any case, laser cooling is a very new field and requires more detailed study. Both schemes of cooling are attractive and have complimentary areas of application.

# 5 Positron production using a laser

Production of positrons for linear e<sup>+</sup>e<sup>-</sup> colliders is a challenging problem. Several possible solutions are based on a laser technique:

- 1. Positron production by high energy electron beam in a laser wave.
- 2. Two step scheme, similar to that of ref.[34], but instead of an undulator a laser wave is used.

# 5.1 Positron production by electrons in a laser wave

In the section 2.2 we have seen that high energy electrons at the conversion region can produced  $e^+e^-$  pairs [3, 5]. This is the a two-step process: 1) the high energy photon is produced in the Compton scattering of a laser photon on high energy electron, 2)  $e^+e^-$  pair is produced in collisions of high energy photon with a laser photon. The threshold of this process is x = 4.8. Baier [35] has considered this scheme for the production of polarized positrons. Practical use of this scheme is a big question. It requires electron beams with high energy ( $E_e \sim 60$  GeV for laser with  $\lambda = 0.25 \ \mu m$ ), the produced positrons have large energies and too large energy spread.

Chen and Palmer [36] proposed to use the same scheme but with a very strong laser field ( $\eta > 1$  and  $\Upsilon > 1$ ). In this case the  $\gamma, e^+, e^-$  shower is developed which stops when the photon energies of became below the coherent pair production threshold. As a result, one can obtain (one of examples)  $N_{e^+} \sim 3N_e$  with the energy about 1.5 GeV and 7% r.m.s energy spread. It is remarkable that the normalized emittance of the final beam can be much lower than that of the initial electron beam. Unfortunately, final positrons and electrons in this scheme are unpolarized. This method also requires high energy initial electron beams (above 50 GeV).

# 5.2 Two step scheme: Compton scattering + W-target

This scheme was described recently by Hirose, Omori, Tsunemi et al. at LC97 in Zvenigorod and it is prepared now for testing at KEK. The 6.7 GeV electron beam is collided with a focused circularly polarized  $CO_2$  laser and produces photons with the maximum energy of 80 MeV. Photons in the high energy part of spectrum have the high degree of the circular polarization. Further these photons are brought to the W target where  $e^+e^-$  pairs are produced. Positrons with  $E_{e^+} > 35$  MeV have the average longitudinal polarization about 80 %. What is essential in this scheme:

1. Each electron produces more than one (about 6) photons.

2. The laser target consists of 40 conversion regions (created by 40 lasers) with 5 cm spacing. This is done for two reason: a) it is a way to create a thick laser target using several low power lasers, b) in this case, the nonlinear parameter  $\xi^2 \ll 1$  (an electron is scattered on one laser photon). This is very important to have collisions with only one laser photon. If the number of absorbed laser photons is fluctuating the maximum energy of scattered photons is fluctuating as well, polarization is no longer an unique function of the photon energy and as a result, the photons at any energy have low degree of polarization.

The total flash energy of 40 CO<sub>2</sub> lasers in the KEK scheme is about 10 J, each electron produces 5.7 photons with the energy 1-80 MeV, the yield of polarized positrons per one initial electron is about 0.2. For the JLC positron injector the total power consumption from plug is 17 MW for linac and 3.2 MW for the laser system. Numbers are reasonable.

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