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RELATIVISTIC CORRECTIONS TO THE POSITRONIUM DECAY RATE REVISITED

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НОВОСИБИРСК

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Abstract

We rederive here in a simple and transparent way the master formula for the dominant part of large relativistic corrections to the positronium decay rate.

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1. The strong disagreement between the experimental value of the orthopositronium decay rate [1]

$$\Gamma_{exp}^{o-Ps} = 7.048\,2(16)\ \mu s^{-1} \tag{1}$$

and its theoretical value which includes the order α and $\alpha^2\log(1/\alpha)$ corrections [2–6]

$$\Gamma_{th}^{o-Ps} = m\alpha^6\,\frac{2(\pi^2-9)}{9\pi}\left[1 - 10.2866(6)\frac{\alpha}{\pi} - \frac{1}{3}\alpha^2\log\frac{1}{\alpha}\right]$$
$$= 7.038\,236(10)\ \mu s^{-1} \tag{2}$$

is a real challenge to the modern QED. For the disagreement to be resolved within the QED framework, the correction $\sim (\alpha/\pi)^2$, which has not been calculated completely up to now, should enter the theoretical result (2) with a numerical factor 250(40), which may look unreasonably large.

Though the result of more recent experiment [7]

$$\Gamma_{exp}^{o-Ps} = 7.0398 \pm 0.0025(\text{stat}) \pm 0.0015(\text{syst})\ \mu s^{-1} \tag{3}$$

does not demand by itself such a large second-order correction, the problem of evaluating $\sim (\alpha/\pi)^2$ terms certainly exists.

One class of large second-order corrections arises as follows [8]. The large, ~ -10, factor at the α/π correction to the decay rate (see (2)) means that the factor at the α/π correction to the decay amplitude is roughly -5. Correspondingly, this correction squared contributes about $25(\alpha/\pi)^2$

So, expression (10) can be used as a regulator, and in the now rapidly converging integral

$$\int \frac{d\vec{p}}{(2\pi)^3} 8\pi a^3 \left[\frac{1}{(p^2a^2+1)^2} - \frac{1}{p^4a^4}\right] [M(\vec{p}) - M(0)] \tag{12}$$

we can safely expand $M(\vec{p})$ up to $(p/m)^2$ included. In this way we obtain

$$\int \frac{d\vec{p}}{(2\pi)^3} 8\pi a^3 \left[\frac{1}{(p^2a^2+1)^2} - \frac{1}{p^4a^4}\right] \left(\frac{p}{m}\right)^2 = -\frac{3}{4}\alpha^2. \tag{13}$$

This is an alternative derivation of the master formula (5) used in our article [12]. In our opinion, it leaves no doubts in the correctness of this prescription.

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