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IMAGE CURRENT MONITOR FOR BUNCHED BEAM PARAMETERS MEASUREMENTS

Budker INP 95-39

НОВОСИБИРСК

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ABSTRACT

An azimutal distribution of image current induced by bunched beam of charged particles in conducting walls of vacuum chamber contains all the information on the beam parameters accessible for electromagnetic contactless methods of measurements. The main purpose of the paper is to propose a theory of the well known effect of the image current spreading due to introduced resistance which is the adequate base for the development of practical devices. The advantages of the multipole structure of such kind of monitors are also the concern of the present paper. An accuracy of the suggested method for processing an information on the azimuthal distribution of image current is properly estimated. One of the realized designs of the monitor with 8 image current propagation lines to measure the centre-of-mass displacement and quadrupole momenta of the electron and positron subnanosecond beams cross-section is described.

IMAGE CURRENT MONITOR FOR BUNCHED BEAM PARAMETERS MEASUREMENTS

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A bunched beam of charged particles induces a current in the conducting walls of vacuum chamber, so called "wall current" [1-3,5-8]. As for the present paper, the term "image current" (IC) [4] will be used as seems to be more suitable. An azimuthal distribution of IC contains all the information on the beam parameters accessible for the contactless methods of measurements, namely, on the full current [1,2] (on the full charge in the case of an individual bunch), on the centre of mass [3-6], and on the higher multipole momenta of the beam cross-section [4,9].

At present, at INP, in order to measure the displacement and quadrupole momenta of electron and positron beams in the beam transfer channel from the booster storage ring VEPP-3 to the storage ring VEPP-4M 14 IC-monitors of the 3rd modification are used [9]. Fig.1 shows a sketch of the monitor design. The monitor has 8 IC propagation lines located in the break of vacuum chamber (in the area of ceramics on Fig.1; the line lengths are 53mm). The lines are placed homogeneously along the chamber azimuth with the gaps of 2mm. The electron and positron beams in a channel are the single current pulses about 1ns in duration. The lines loads are the primary windings of transformers 1:20 on the ferrite rings (d7xd4x2mm, μ1000). The transformer secondary windings are loaded to the diodes and further to the sections of radio-frequency cables matched for high frequencies (RC-50-1.5-12, 1m). As a result of detection, the charges $\bar{Q}_k$ proportional to respective ICs are memorised on the input capacitances of the multi-channal detecting device IPP-32R (made in INP [10]). Connected to these devices are the buffer capacitances desined for matching the dynamic range of beams and electronics (as one can see on Fig.1).

An information on beam position with respect to X and Y (as well as on the beam charge value) is derived from the azimuthal distribution of charges $\bar{Q}_k$. Primarily, the nonlinearity of detector characteristics is taken into account. In our case, it is well interpolated by the following expression for the detector transmitting characteristics:

$$\bar{Q}_k = Q_k - Q_t[1 + \ln(Q_k/Q_t)],$$

here, Q_k is a charge on the input of k-th detector; Q_t=.54±.05pK is a "threshold" charge defined during the calibration of monitors, which can be interpolated as a charge on the detector shunt capacitance. In order to linearize the detection characteristics one can use the next expression:

$$Q_k^* = \bar{Q}_k + 2.16\, Q_t (\bar{Q}_k/Q_t)^{0.22}.$$

As a result, Q_k^* differs from the initial Q_k less then by 1% at $Q_k > 2Q_t$ (threshold charge Q_t=.54pK corresponds to the beam charge 86.4pK or, to .35mA in terms of average VEPP-3 current, while VEPP-3 nominal current is about 100mA). The beam position with respect to X and Y is calculated in the following way:

$$X = \frac{M\cdot R}{\Sigma}\sum_{k=1}^{8} Q_k^* \cos(\vartheta_k), \quad Y = \frac{M\cdot R}{\Sigma}\sum_{k=1}^{8} Q_k^* \sin(\vartheta_k), \quad \Sigma = \sum_{k=1}^{8} Q_k^*,$$

here $\vartheta_k = \frac{2\pi}{8}k$ -is the azimuth of k-th IC propagation line; $M = \frac{\pi/8}{\sin(\pi/8)}$ -is the scale factor correcting the "filtration" of the azimuthal harmonic because of the finite azimuthal size of the lines; R=27mm is a radius of the lines location. An estimated accuracy of X and Y calculated in this way at 8 IC propagation lines is better than ±.3mm with the beam displacement within the half of an aperture of their location, i.e. within the range ±13.5mm. How to calculate quadrupole parameters of the beam one can find in section 2 of the present paper.

1. IC azimuthal distribution for the round idially conducting chamber.

There isn't necessity for the cross-section of the vacuum chamber of the transfer channel to be round, but practically there is always a possibility to make it so in the monitor area. So the round chamber is cosidered below, since in this case the solution of the problems on IC azimuthal distribution is distinguished for its simplicity and is more illustrative. It is also obvious that, if not entirely quantitavely but at least qualitatively, the main results of this work are applicable to the chambers of an arbitrary cross-section.

Let the vacuum chamber inside of which a bunched beam of charged particles propagates along its z axis be an ideally conducting cilinder of R1=1 in radius. IC induced by the beam in the chamber walls is determined by the vector potential value produced by the beam current:

(1.1) $$\Delta\vec{A} - \frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\vec{A} = -\mu_0\vec{j}.$$

Let the beam current has only z-component and let its dependence on (t,z) has

the form exp(iωt-ikz). Since the boundary conditions ($\vec{A}$=0 at r=R1) are gomogeneous over z, the forced solution of (1.1) contanes only z-th component of vector potential (below, for the sake of simplicity, the factor exp(iωt-ikz) is omitted). The latter we will find in the form of expantion in Fourier row along the azimuth ϑ:

(1.2) $$A_z(r,\vartheta) = A_0(r) + \sum_{m=1}^{\infty}\left\{A_m^c(r)\cos(m\vartheta) + A_m^s(r)\sin(m\vartheta)\right\},$$

here A_0, A_m^c, A_m^s satisfies the equation:

(1.3) $$A_m'' + \frac{1}{r}\cdot A_m' + \left\{\varkappa^2 - \frac{m^2}{r^2}\right\}\cdot A_m = -\mu_0 j_m,$$

here $\varkappa^2 = -k^2/\gamma^2$, γ is a relativistic factor, $j_m(r)$ are the corresponding azimuthal harmonics of $j_z(r,\vartheta)$.

It is known a partial solution of dissimilar equation (1.3) with homogeneous boundary conditions [11,p.77]:

(1.4) $$A_m(r) = -\mu_0\cdot\sum_{k=1}^{\infty}\frac{J_m(\lambda_{mk} r)}{\varkappa^2 - \lambda_{mk}^2}\cdot a_{mk},$$

here λ_{mk} are the radicals of equations $J_m(r)$=0, and a_{mk} are factors of expantion $j_m(r)$ into the Fourier-Bessel row [11,p.136]:

(1.5) $$a_{mk} = \frac{2}{J_{m+1}^2(\lambda_{mk})}\int_0^1 J_m(\lambda_{mk}\xi)j_m(\xi)\xi d\xi.$$

Substituting (1.5) into (1.4) and using the known relations [12,p.119] we get:

(1.6) $$A_m(r) = -\mu_0\cdot\frac{\pi}{2}\left\{J_m(\varkappa)N_m(\varkappa r) - J_m(\varkappa r)N_m(\varkappa)\right\}\int_0^1\frac{J_m(\varkappa\xi)}{J_m(\varkappa)}j_m(\xi)\xi d\xi,$$

here J_m, N_m are the Bessel and Neiman functions respectively.

Image current $j_z^*(\vartheta)$ is determing by the relations:

(1.7) $$\begin{cases} j_z^* = H_\vartheta \quad \text{at } r=1 \\ H_\vartheta = -\frac{1}{\mu_0}\frac{\partial}{\partial r}A_z \end{cases}$$

By differentiation (1.6) with respect to r and assuming r=1 with an account of the known relation for the Bessel functions [11,p.31] we get:

(1.8) $$j_m^* = -\int_0^1\frac{J_m(\varkappa\xi)}{J_m(\varkappa)}j_m(\xi)\xi d\xi.$$

According to (1.8), IC dependence on the azimuth ϑ is described in the following way:

$$(1.9)\qquad j^*(\vartheta) = -\frac{1}{2\pi}\int_0^{2\pi}\int_0^1\left\{\frac{J_0(\varkappa\xi)}{J_0(\varkappa)} + 2\sum_{m=1}^{\infty}\frac{J_m(\varkappa\xi)}{J_m(\varkappa)}\cos(m(\vartheta-\alpha))\right\}j(\xi,\alpha)\xi d\xi d\alpha.$$

Of the main interest is an asymptotic of expression (1.9) at $|\varkappa|\ll 1$. The latter condition takes place either in the case of a beam whose longitudinal size is mach larger than chamber aperture ($k\ll 1$), or, in the case of small longitudinal size of the beam, this condition can be satisfied because of substantial relativism of the beam (i.e. due to $\gamma\gg 1$). Taking into account the corresponding asymptotic of the Bessel functions [13,p.192] we get:

$$(1.10)\qquad j^*(\vartheta) = -\frac{1}{2\pi}\int_0^{2\pi}\int_0^1\left\{1 + 2\sum_{m=1}^{\infty}\xi^m\cos(m(\vartheta-\alpha))\right\}j(\xi,\alpha)\xi d\xi d\alpha.$$

For the narrow beam having the transverse coordinates (ρ,φ) the expression (1.10) gets the following form:

$$(1.11)\qquad j^*(\vartheta) = \frac{1}{2\pi}\left\{1 + 2\sum_{m=1}^{\infty}\rho^m\cos(m(\vartheta-\varphi))\right\}.$$

Here and below, for sake of simplicity, it is assumed that:

$$(1.12)\qquad \frac{1}{2\pi}\int_0^{2\pi}\int_0^1 j(\xi,\alpha)\xi d\xi d\alpha = 1.$$

Summing the row (1.11) we get:

$$(1.13)\qquad j^*(\vartheta) = \frac{1}{2\pi}\cdot\frac{1-\rho^2}{1-2\rho\cos(\vartheta-\varphi)+\rho^2}.$$

The expression like (1.13) one can find in Ref. [3,4,8], and like (1.11) in Ref. [5].

2. Beam parameters defined through image current.

Let us integrate (1.10) with the weight $\cos(m\vartheta)$ using the change in order of integration:

$$(2.1)\qquad I_m^c \equiv \int_0^{2\pi} j^*(\vartheta)\cos(m\vartheta)d\vartheta = \int_0^{2\pi}\int_0^1 \xi^m\cos(m\alpha)j(\xi,\alpha)\xi d\xi d\alpha \equiv \langle\xi^m\cos(m\alpha)\rangle.$$

By integrating (1.10) with the weight $\sin(m\vartheta)$ we get:

$$(2.2)\qquad I_m^s = \langle\xi^m\sin(m\alpha)\rangle.$$

In particular,

$$(2.3)\qquad \begin{cases} I_1^c = \langle\xi\cos\alpha\rangle \equiv \langle x\rangle \\ I_1^s = \langle\xi\sin\alpha\rangle \equiv \langle y\rangle \end{cases} \quad \begin{cases} I_2^c = \langle\xi^2\cos(2\alpha)\rangle \equiv \langle x^2-y^2\rangle \\ I_2^s = \langle\xi^2\sin(2\alpha)\rangle \equiv 2\langle xy\rangle \end{cases}.$$

Integrals I_1^c, I_1^s give the values of coordinates for the beam "centre of mass". For m>1 integrals I_m^c, I_m^s give the value for highest multipole momenta. With the presence of some certain additional information on the beam cross-section geometry, from the latter values one can judge about the beam transverse dimentions.

So, if one knows that the beam is distributed normally in some coordinates (x',y'), i.e.:

$$j(x',y') = \frac{1}{2\pi\sigma_x\sigma_y}\cdot\exp\left\{-\frac{(x')^2}{2\sigma_x}\right\}\cdot\exp\left\{-\frac{(y')^2}{2\sigma_y}\right\},$$

but in coordinates (x,y) it is displaced to $(\delta x,\delta y)$ and turned at an angle ε, i.e.:

$$\begin{cases} x = \delta x + x'\cos(\varepsilon) - y'\sin(\varepsilon) \\ y = \delta y + x'\sin(\varepsilon) + y'\cos(\varepsilon) \end{cases},$$

then, according (2.3) we get:

$$(2.4)\qquad \begin{cases} I_1^c = \delta x,\ I_2^c = \iint_S (x^2-y^2)j(x,y)dxdy = \delta x^2 - \delta y^2 + (\sigma_x^2-\sigma_y^2)\cos(2\varepsilon) \\ I_1^s = \delta y,\ I_2^s = \iint_S 2xyj(x,y)dxdy = 2\delta x\delta y + (\sigma_x^2-\sigma_y^2)\sin(2\varepsilon) \end{cases},$$

here S is an area occupied by the beam. Let us introduce the following notations:

$$\begin{cases} I_2^c - (I_1^c)^2 + (I_1^s)^2 = A \\ I_1^s - 2I_1^cI_1^s = B \end{cases},$$

Then, according to (2.4), $a \equiv \sqrt[4]{A^2+B^2} = \sqrt{|\sigma_x^2-\sigma_y^2|}$ is a half of a focal distance of an ellips of the beam rms dimentions.

Similarly one can demonstrate that for the band-form beam (having cross-section as a line of 2d in length) $a=d/\sqrt{3}$, and for two-component beam (with components spaced by 2d) a=d. In any case, the parameter "a" is an estimate from below for the transverse dimentions of a beam. But for the beam with the strong asymmetry of the transvers dimentions in the mutually orthogonal directions the "a" parameter characterizes its rms size along the asymmetry axis. The bending angle of the asymmetry ε is defined from the system of equations (2.4).

3. Image current spreading.

In order to measure the IC azimuthal distribution, one should introduce by one way or another the loads into the conducting chamber brake whose active component of impedance cannot in practice reach zeroth value. The presence of introduced resistance results in IC "spreding" [3,4,5] - IC homogeneously distributes along the azimuth and an information on beam parameters (exept for the beam current total value) vanishes.

Let us give a simple techniques for evaluation the spreading time constant [4]. An introduced resistance per unit length $r_1=r/l$ of a chamber of radius R one can interpret as a resistance of some imaginary material of a chamber having the volume conductivity σ, taking into account that IC is concentrated in the area of δ thik skin layer:

(3.1)
$$\begin{cases} \delta = \sqrt{\dfrac{2}{\mu_0 \sigma \omega}} \\ r_1 = \dfrac{1}{2\pi R \delta \sigma} \end{cases}$$

Eliminating σ from these expressions we get the relation between r_1 and δ:

(3.2)
$$r_1 = \frac{\mu_0}{4\pi}\cdot\omega\cdot\frac{\delta}{R}.$$

It is obvious that IC azimuthal distribution will remain of the form (1.10) if $\delta \ll R$. Hence, the limitation to the value r_1 takes the form $r_1 \ll \mu_0 \omega / 4\pi$. Or else $\omega \gg 1/\tau$, where

(3.3)
$$\tau = \frac{\mu_0}{4\pi r_1} = 10^{-7}/r_1$$

is the sought constant of spreading time. In the present work an accurate solution of the IC spreding problem within simple but practically sufficient model is proposed.

Let the vacuum chamber be a thin wall cilinder of radius R1=1 with the surface conductivity $\sigma=1/(2\pi r_1)$. Let around and in line with the chamber be an ideally conducting cilindrical shield with radius R2=R. The space between the chamber and shield (area 2) be filled with a material with magnetic permeability μ. The beam propagates inside the chamber (area 1) along its z-axis. The boundary conditions are still gomogeneous over z, therefor, the vector potential produced by the beam current and IC will have only a z-component. It is known [14,p.341] that the solution of equation (1.3) is the

sum of common solution of the corresponding gomogeneous equation with non-gomogeneous boundary conditions (free solution) and a patial solution of equation (1.1) with homogeneous boundary conditions (forced solution $\tilde{A}$):

(3.4)
$$\begin{cases} A^{(1)}_m = C_1 J_m + \tilde{A}_m & \text{- area (1)} \\ A^{(2)}_m = C_2 N_m + C_3 J_m & \text{- area (2)} \end{cases}.$$

The boundary conditions have the form:

(3.5)
$$\begin{cases} \tilde{A}_m = 0 & \text{at } r=R1 \\ A^{(2)}_m - A^{(1)}_m = 0 & \\ \dfrac{1}{\mu_0}\left\{ \dfrac{d}{dr}\left[\dfrac{A^{(2)}_m}{\mu} - A^{(1)}_m \right]\right\} - i\omega\sigma A^{(1)}_m = 0 & \\ A^{(2)}_m = 0 & \text{at } r=R2 \end{cases}$$

The substitution of (3.4) into (3.5) gives the system of equations for the coefficients C_1, C_2 and C_3. Solving the system we find IC according to (1.7):

(3.6)
$$j^{\bullet}_m = -\frac{1}{\mu_0}\cdot \varkappa \tilde{A}'\cdot\left\{ 1+\lambda\left[\frac{J'_m(\varkappa)}{J_m(\varkappa)} - \frac{1}{\mu}\cdot\frac{J'_m(\varkappa)N_m(\varkappa R) - N'_m(\varkappa)J_m(\varkappa R)}{J_m(\varkappa)N_m(\varkappa R) - N_m(\varkappa)J_m(\varkappa R)} \right]\right\}^{-1},$$

here $\lambda=\varkappa/(i\omega\mu_0\sigma)$.

From the viewpoint of IC spreading it is of interest the new in distribution $j^{\bullet}(\vartheta)$ caused by the finite conductivity of the chamber. Let us introduce the "transfer factor" of m-th azimuthal harmonic of IC: $K_m = j^{\bullet}_m(\sigma)/j^{\bullet}_m(\infty)$. According to (3.6) we get:

(3.7)
$$K_m = \frac{1}{1 + 1/(i\omega\tau_m)},$$

here $\tau_m(R,\mu)$ is the spreading time constant taking real values at $|\varkappa| \ll 1$:

(3.8)
$$\begin{cases} \tau_0(R,\mu) = \dfrac{\mu_0}{2\pi r_1}\cdot\mu\cdot\ln(R) \\ \tau_m(R,\mu) = \dfrac{1}{m}\,\dfrac{\mu_0}{2\pi r_1}\cdot\left\{ 1 + \dfrac{1}{\mu}\cdot\dfrac{R^{2m}+1}{R^{2m}-1} \right\}^{-1} \end{cases}.$$

According to (3.7) the spreading is of inductive character. Note that the total inductance of spreading $L_m=\tau_m/r_1$ (per unit length of a chamber) is formed by the parallel connection of the "inner" $L^{(1)}_m$ and "external" $L^{(2)}_m$ spreading inductances:

(3.9)

$$
\begin{cases} L_0^{(1)} = \infty \\ L_m^{(1)} = \frac{1}{m}\cdot\frac{\mu_0}{2\pi} \end{cases}
\qquad
\begin{cases} L_0^{(2)} = \frac{\mu_0}{2\pi}\cdot\mu\cdot\ln(R) \\ L_m^{(2)} = \frac{1}{m}\cdot\frac{\mu_0}{2\pi}\cdot\mu\cdot\frac{R^{2m}-1}{R^{2m}+1} \quad (m\neq 0) \end{cases}
$$

If $\mu=1$ and $R\gg 1$, $L_m^{(1)}=L_m^{(2)}$. In this case, the spreading time value for $m=1$ coincides exactly with the evaluation (3.3) obtaned above with the qualitative way.

Although the real monitor has a finite length of the introduced resistance region, the estimated spreading time constant very closely agrees with experimentally obtained value even though the length of the region is not greater than half of the monitor aperture [4].

4. Image current evolution.

If the beam has a step form, i.e. $j_z(r,\vartheta)\cdot h(t-z/v)$, the form of transfer coefficient $K_m(\omega)$ indicates the following form of the time dependance of the m-th azimuthal harmonic (at some fixed value z, here at z=0):

(4.1)
$$j_m^{*}(t) \sim \exp(-t/\tau_m).$$

According to (4.1) the expression (1.10) for IC takes the form:

(4.2)
$$j^{*}(\vartheta,t) = -\frac{1}{2\pi}\int_0^{2\pi}\int_0^{1}\left\{\exp(-t/\tau_0)+2\sum_{m=1}^{\infty}\xi^m\exp(-t/\tau_m)\cos(m(\vartheta-\alpha))\right\}j(\xi,\alpha)\xi d\xi d\alpha\cdot h(t-z/v)$$

If one takes $\tau_0=\infty$ (because of $R\gg 1$ or $\mu\ll 1$), $\tau_m=\frac{1}{m}\mu_0/(2\pi)\frac{\mu}{\mu+1}/r_1$ (because of $R\gg 1$) and the beam to be narrow, the expression (4.2) can be reduced to the next simple expression:

$$j^{*}(\vartheta,t) = \frac{1}{2\pi}\cdot\frac{1-\rho^2\exp(-2t/\tau)}{1-2\rho\exp(-t/\tau)\cos(\vartheta-\varphi)+\rho^2\exp(-2t/\tau)}\cdot h(t-z/v),$$

here $\tau=\mu_0/(2\pi)\frac{\mu}{\mu+1}/r_1$. Thus in this case, if the distribution $j^{*}(\vartheta)$ is detected at time δt after the beam front of the form $h(t-z/v)$ passing the point of observation z, at the calculation of beam parameters with the expressions (2.1) and (2.2) one should replace ρ by $\rho\cdot\exp(-\delta t/\tau)$.

5. The way to form "resistance per unit length".

As mentioned in section 3, for the measurement of IC azimuthal distrobution it is necessary into the break of conducting vacuum chamber introduce by some or other way some cirtain finite number of "submonitors".

The set of the submonitors will form the monitor itself. One should have quite large number of submonitors, and they should be placed homogeneously along the chamber azimuth. These requirements have two aims. First, due to azimuthal compactness of individual submonitor, whose transverse dimentions could be much less than the chamber aperture, quite low signal rise time are achieved in the submonitor loads. Second, for extracting the information on the beam parameters one could obtain good accuracy using sums insted of integrals (2.1) and (2.2).

An active component of the submonitor load impedance cannot be made to be equel to zero. As it was shown in section 3 of this paper, this leads to IC spreading. However, since in relations of (3.10) kind for the spreading time constants an introduced resistance is included in the form of resistance per unit length of a chamber, the largest (however as any other) spreadung time constant achieved because of sufficient length of submonitor at some cirtain fixed value of the introduced resistance.

In the solid conducting chamber some cirtain number n of longitudinal cuts of l-length are made. The signal from the IC propagation lines obtained in this way is taken with the current transformers (thus, in this case, these lines with corresponding transformers serve as submonitors) The suggested method for the creation of "resistance per unit length" is correct unless the signal propagation time along the submonitor line is much less than the spreading time, i.e. $l/c\ll\tau_m$. Hence, the limit to the total value of the resistance introduced by a set of submonitors is:

$$r \ll \frac{\mu_0}{2\pi}\cdot c/m \cong 60/m \quad [\text{Om}]$$

6. The accuracy of the IC distribution processing.

If the IC propagation lines have the form described in section 5 of this paper, then in case of a narrow beam with the transvers coordinates (ρ,φ) an input current of the k-th submonitor has the form:

(6.1)
$$i_k = \int_{\vartheta_k-\pi/n}^{\vartheta_k+\pi/n} j^{*}(\vartheta)d\vartheta = \frac{1}{n}\left\{1+2\sum_{l=1}^{\infty}\rho^l\cos(l(\vartheta_k-\varphi))\frac{\sin(l\pi/n)}{l\pi/n}\right\}$$

Shifting in (2.1) and (2.2) from integrals to the corresponding sums, we get:

$$S_m^c \equiv \sum_{k=1}^{n} i_k\cos(m\vartheta_k) = \frac{2}{n}\sum_{l=1}^{\infty}\frac{\rho^l}{M_n^l}\sum_{k=1}^{n}\cos(m\vartheta_k)\cdot\cos(l(\vartheta_k-\varphi))=$$

$$= \sum_{l=1}^{\infty} \frac{\rho^{l}}{M_n^{l}} \cos(l\varphi)\cdot\delta_{l,pn\pm m} = \frac{1}{M_n^m} I_m^c + \sum_{p=1}^{\infty} \frac{1}{M_n^{pn\pm m}} \rho^{pn\pm m} \cos((pn\pm m)\varphi),$$

here $M_n^m = \frac{m\pi/n}{\sin(m\pi/n)}$, $I_m^c = \rho^m \cos(m\varphi)$. Thus, The folloving expression is an accurate estimate of the method of replacing integrals by summs:

$$|M_n^m \cdot S_m^c - I_m^c| < |M_n^m| \cdot \frac{\rho^{n-m} + \rho^{n+m}}{1 - \rho^n}.$$

There is similar expression for S_m^S.

Acknowledgments.

The author is thankful to A.S.Medvedko for the initiation and support of the present work, to N.I.Zinevich for the traning in the practical approach to the solution of the systems of differettial equations, to A.S.Kalinin for the hard critical notes, and to M.M.Karliner for attention in reading manuscript and for usful remarks.

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Fig. 1 *A sketch of the image current monitor design.*

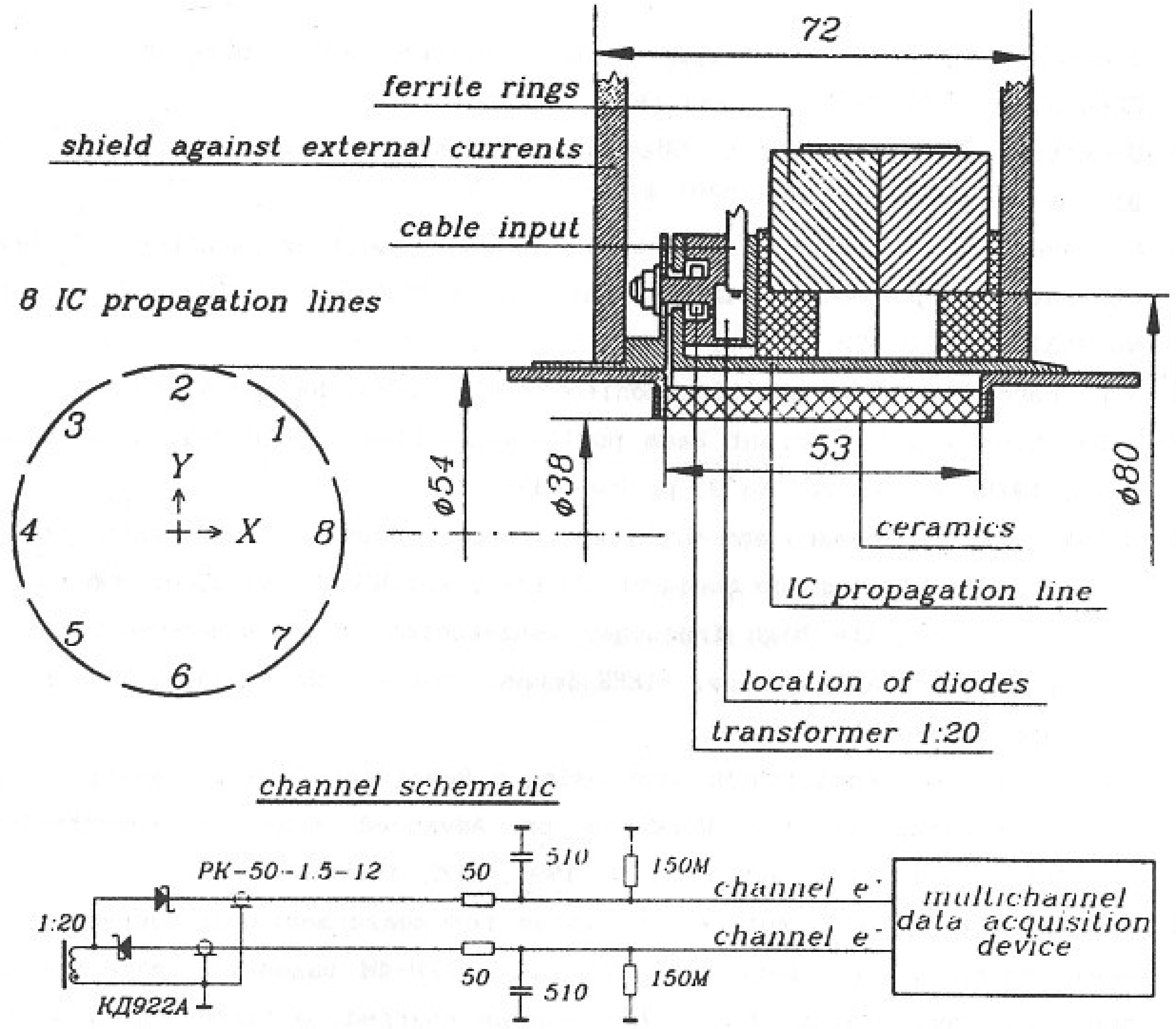

Fig. 2 *On the left: beam displacements in 12 points of the VEPP-3 - VEPP-4M channel. On the right: original set of the monitor signals and calculated 1st and 2nd azimuthal harmonics diagrams. On the picture DG corresponds to parameter "a" and F2 - to ε in the text (part 1).*

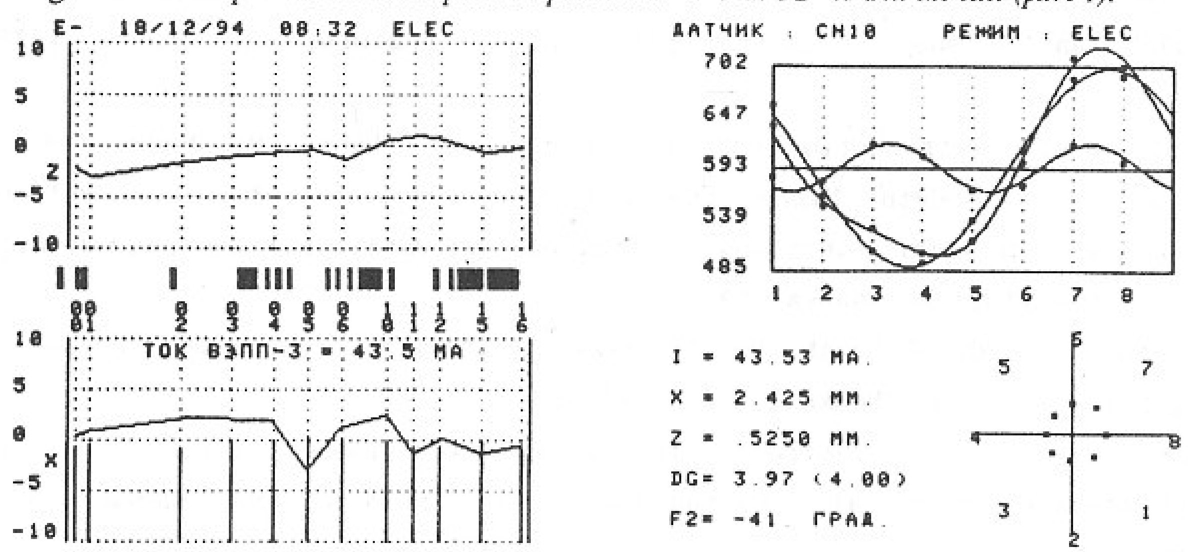

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Image Current Monitor for Bunched Beam Parameters Measurements

Budker INP 95-39

В.П. Черепанов

Датчик тока изображения для измерения параметров сгруппированного пучка

Ответственный за выпуск С.Г. Попов
Работа поступила 11.04.1995 г.

Сдано в набор 13.04.1995 г.
Подписано в печать 19.04.1995 г.
Формат бумаги 60×90 1/16 Объем 0.9 печ.л., 0.9 уч.-изд.л.
Тираж 200 экз. Бесплатно. Заказ N° 39

Обработано на IBM PC и отпечатано на
ротапринте ГНЦ РФ "ИЯФ им. Г.И. Будкера СО РАН",
Новосибирск, 630090, пр. академика Лаврентьева, 11.