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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
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LOW ENERGY THEOREMS FROM THE EFFECTIVE LAGRANGIAN WITH VECTOR MESONS

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НОВОСИБИРСК

Low Energy Theorems
from the Effective Lagrangian
with Vector Mesons

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ABSTRACT

It is shown that the previously proposed effective Lagrangian with vector mesons correctly reproduces all current algebra low energy theorems in the domain of its validity.

1. INTRODUCTION

It is well known that the approximate chiral symmetry of QCD and its dynamical breaking combined with gauge invariance and the existence of the chiral anomaly to great extent defines the self-interactions of the pseudoscalar mesons and their electromagnetic properties in the low energy limit [1]. All these current algebra results can be reproduced by means of the effective Lagrangian approach [2]. In particular, photon - pseudoscalar meson interactions at low energies can be described by the effective action [3] :

$$\Gamma_{e.m.}(U,A) = \frac{F_\pi^2}{8} \int_{M^4} dx\ Sp\{(D_\mu U)(D^\mu U)^+\} - \frac{i}{80\pi^2} \int_{M^5} Sp(\alpha^5) + \frac{e}{16\pi^2} \int_{M^4} A\ Sp\{Q(\alpha^3 + \beta^3)\} + \qquad (1)$$

$$+ \frac{ie^2}{8\pi^2} \int_{M^4} A(dA)\ Sp\left\{Q^2(\alpha+\beta) + \frac{1}{2} QU^{-1}QdU - \frac{1}{2} QUQ(dU^{-1})\right\},$$

where $U = \exp\{\frac{2i}{F_\pi}\Phi\}$, $F_\pi \approx 135$ MeV is the pion decay constant and $\alpha = (dU)U^{-1}$, $\beta = U^{-1}(dU)$.

It was shown that (1) correctly reproduces all current algebra low energy theorems [4].

But for the energies ~ 1 GeV low - lying vector mesons can also play a dynamical role and must be included in the effective Lagrangian. Different approaches how to treat vector meson degrees of freedom can be found in literature [3, 5, 6, 7, 8]. Any reasonable effective theory, however, should be capable to reproduce current algebra results in the low energy limit. This becomes rather subtle matter than vector mesons are present [9]. An elegant general formalism how to deal with this problem was developed in [10]. Using this method a minimal extension of (1) including vector mesons was constructed [11], valid for the vertexes which contain no more than five particles. The corresponding effective Lagrangian looks like

$$L_{eff.} = L^{N} + L^{WZW} + L^{VDM}, \qquad (2)$$

where

$$L^{VDM} = + \frac{m_v^2}{2}\, Sp\left\{\left(V_\mu - \frac{e}{g} A_\mu Q\right)\left(V^\mu - \frac{e}{g} A^\mu Q\right)\right\},$$

$$L^{N} = \frac{1}{2} Sp\{(D_\mu\Phi)(D^\mu\Phi)\} +$$

$$+ \frac{1}{F_\pi^2}\left(\frac{1}{3} - \alpha_K\right) Sp\{\Phi(D_\mu\Phi)\,\Phi(D^\mu\Phi) - \Phi^2(D_\mu\Phi)(D^\mu\Phi)\}$$

and

$$L^{WZW} = \frac{\varepsilon^{\mu\nu\lambda\sigma}}{\pi^2}\left\{\frac{2}{F_\pi^5}\left(\frac{1}{5} - \alpha_K + \frac{3}{2}\alpha_K^2\right) \times\right.$$

$$\times\, Sp\{\Phi(\partial_\mu\Phi)(\partial_\nu\Phi)(\partial_\lambda\Phi)(\partial_\sigma\Phi)\} - \frac{3}{4}\frac{g^2}{F_\pi} Sp\{(\partial_\mu V_\nu)(\partial_\lambda V_\sigma)\Phi\} -$$

$$- \frac{i\,g}{F_\pi^3}(1-3\,\alpha_K)\, Sp\{V_\mu(\partial_\nu\Phi)(\partial_\lambda\Phi)(\partial_\sigma\Phi)\} +$$

$$+ \frac{g^2}{4\,F_\pi^3} Sp\Big\{V_\mu(\partial_\nu V_\lambda)[\Phi^2(\partial_\sigma\Phi) - \Phi(\partial_\sigma\Phi)\Phi + (1-3\,\alpha_K)(\partial_\sigma\Phi)\Phi^2)] +$$

$$+ (\partial_\mu V_\nu)V_\lambda[(1 - 3\,\alpha_K)\,\Phi^2(\partial_\sigma\Phi) - \Phi(\partial_\sigma\Phi)\Phi + (\partial_\sigma\Phi)\Phi^2)] -$$

$$- (1-6\,\alpha_K)[V_\mu\Phi(\partial_\nu V_\lambda)(\partial_\sigma\Phi)\Phi + (\partial_\mu V_\nu)\,\Phi V_\lambda\Phi(\partial_\sigma\Phi)] +$$

$$+ (1-3\,\alpha_K)[V_\mu\Phi^2(\partial_\nu V_\lambda)(\partial_\sigma\Phi) + (\partial_\mu V_\nu)\,\Phi^2 V_\lambda(\partial_\sigma\Phi)] -$$

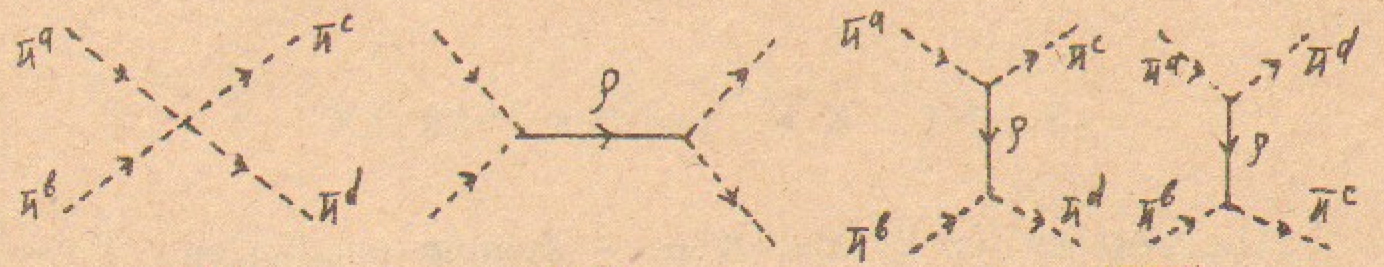
$$- [V_\mu\Phi(\partial_\nu V_\lambda)\Phi(\partial_\sigma\Phi) + (\partial_\mu V_\nu)\Phi V_\lambda(\partial_\sigma\Phi)\Phi]\Big\}\Big\}.$$

In these formulas $g = \frac{g_{\rho\pi\pi}}{\sqrt{2}}$ and $\alpha_K = \frac{g^2 F_\pi^2}{m_v^2}$. Note that $\alpha_K = \frac{1}{2}$ corresponds to well known KSRF relation [12].

Let us show that this effective Lagrangian really reproduces all low energy theorems relevant to the domain of its validity.

2. PION-PION SCATTERING

There are the following diagrams contributing in the pion-pion scattering amplitude:



(I) (II) (III)

Using Feynman rules, given in the appendix, four - momentum conservation and assuming the low energy approximation for the vector meson propagator $iD^{ab}_{\mu\nu} \approx ig_{\mu\nu}\, \delta^{ab} / M^2_V$, we get immediately:

$$A_1 = \frac{i}{f_\pi^2}\left(\frac{1}{3} - \alpha_K\right) \times$$

$$\times \left\{\delta_{ab}\delta_{cd}(2s-t-u) + \delta_{ac}\delta_{bd}(2t-s-u\ ) + \delta_{ad}\delta_{bc}(2u-t-s\ )\right\} ,$$

$$A_2 = \frac{i}{f_\pi^2}\ \alpha_K(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})\ (t-u)\ ,$$

$$A_3 = \frac{i}{f_\pi^2}\ \alpha_K \left\{(\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{cb})(s-u) + (\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd})(s-t)\right\},$$

where $f_\pi = \frac{F_\pi}{\sqrt{2}}$, $s = (p_a+p_b)^2$, $t = (p_a-p_c)^2$ and $u = (p_a - p_d)^2$.

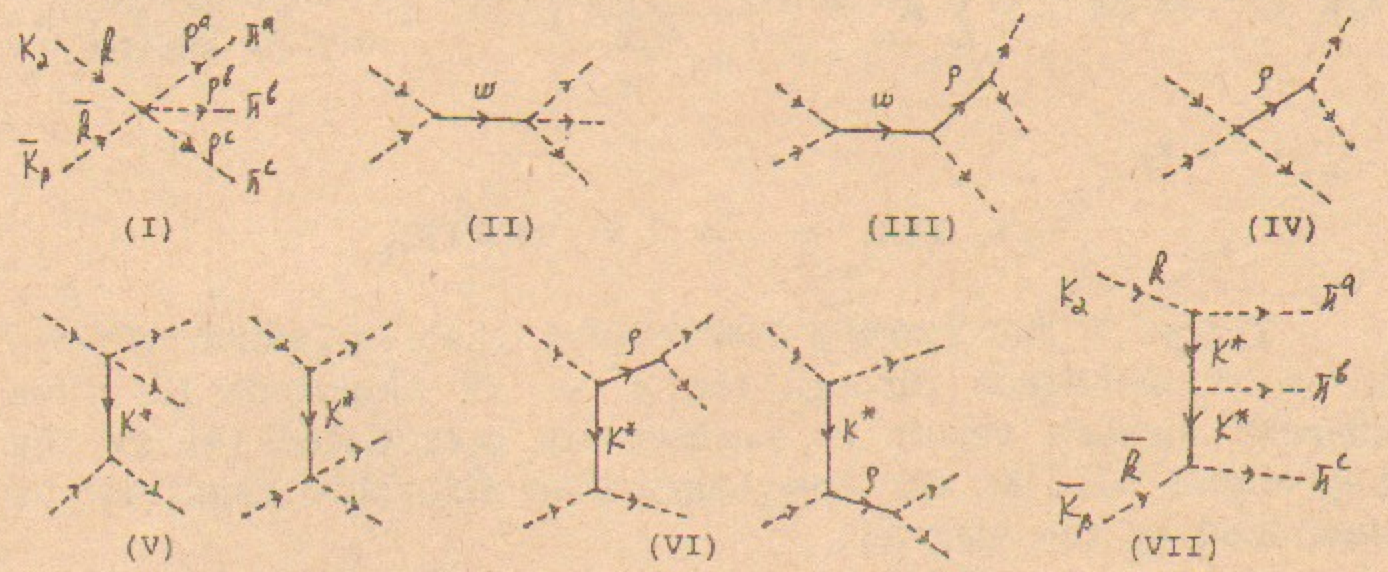
So the effective four-pion vertex in the low energy limit is

$$A(\ \pi^a + \pi^b \longrightarrow \pi^c + \pi^d\ ) = A_1 + A_2 + A_3\ .$$

All reminiscence about vector mesons disappears (α_K terms are canceled) and the result is exactly the same as for the photon-pseudoscalar meson Lagrangian (1) .

3. THE $K\overline{K} \longrightarrow 3\,\pi$ VERTEX

The restoration of the Wess - Zumino prediction [1] for the anomalous $K\overline{K} \longrightarrow 3\,\pi$ vertex is more impressive. There are seven classes of diagrams contributing in this case:



Note that for the classes III, IV, V and VI the sum of diagrams with cyclic permutations of the (a, b, c) isospin indexes is assumed and for the class (VII) the sum extends to all permutations.

As can be easily verified the contribution from each class can be expressed as

$$A_n = -\frac{15\,\delta_{\alpha\beta}}{4\pi^2 f_\pi^5}\ a_n \varepsilon^{\mu\nu\sigma\tau}\ \varepsilon_{abc}\ p^a_\mu\, p^b_\nu\, p^c_\sigma\ \overline{k}_\tau\ ,$$

where

$$a_1 = \frac{1}{5} - \alpha_K + \frac{3}{2}\,\alpha_K^2\ , \qquad a_2 = \frac{1}{5}\,\alpha_K\ (\ 1 - 3\alpha_K)\ ,$$

$$a_3 = \frac{3}{5}\,\alpha_K^2\ , \qquad a_4 = \frac{2}{5}\,\alpha_K\ (\ 1 - 3\ \alpha_K)\ ,$$

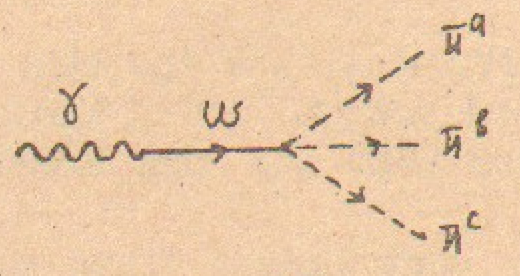
$$a_5 = \frac{2}{5}\,\alpha_K\ (1 - 3\alpha_K)\ , \quad a_6 = \frac{3}{5}\,\alpha_K^2\ , \quad a_7 = \frac{3}{10}\,\alpha_K^2\ .$$

α_K terms are canceled again in the sum and we get the Wess - Zumino result

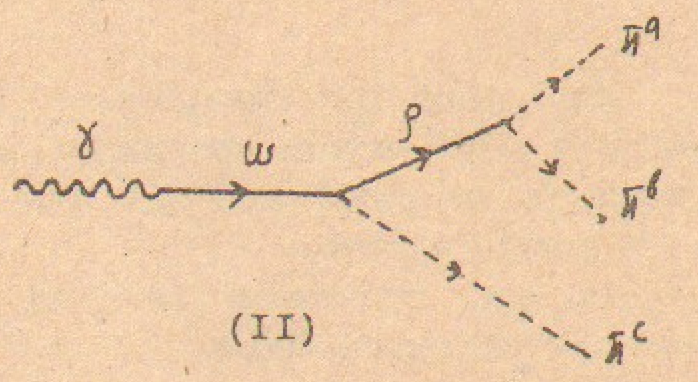
$$A(K\overline{K} \longrightarrow 3\pi) = \sum_{n=1}^{7} A_n = - \frac{3\,\delta_{\alpha\beta}}{4\pi^2 f_\pi^5} \, \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} \, p_\mu^a \, p_\nu^b \, p_\sigma^c \, \overline{k}_\tau \, .$$

4. THE $\gamma \longrightarrow 3\pi$ VERTEX

There is the famous low energy theorem which defines $\gamma \longrightarrow 3\pi$ amplitude through the $\pi^0 \longrightarrow 2\gamma$ amplitude [1]. This current algebra result is successfully reproduced [4] by the Lagrangian (1). As for the Lagrangian (2), this amplitude is described by the diagrams



(I)



(II)

For the second class, the sum over cyclic permutations of (a, b, c) is assumed.

Using Feynman rules, we get

$$A_1^\mu = \frac{e}{4\pi^2 f_\pi^3} (1 - 3\,\alpha_K)\, \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} \, p_\nu^a \, p_\sigma^b \, p_\tau^c \, ,$$

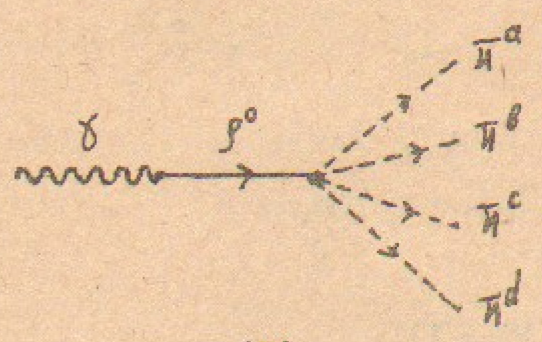
$$A_2^\mu = \frac{e}{4\pi^2 f_\pi^3} (3\,\alpha_K)\, \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} \, p_\nu^a \, p_\sigma^b \, p_\tau^c \, .$$

So the sum coincides with the correct result

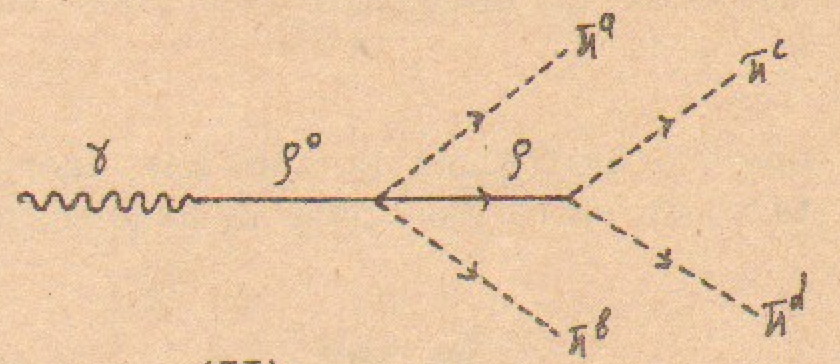
$$A^\mu(\gamma \longrightarrow 3\pi) = \frac{e}{4\pi^2 f_\pi^3} \, \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} \, p_\nu^a \, p_\sigma^b \, p_\tau^c \, .$$

5. THE $\gamma \longrightarrow 4\pi$ VERTEX

There are two classes of diagrams contributing in the low energy limit



(I)



(II)

In the second case it is assumed, that the diagrams with permutations (a, b, c, d)$\longrightarrow$(a, c, b, d), (c, d, a, b), (a, d, c, b), (c, b, a, d) and (b, d, a, c) should be added.

For them we get

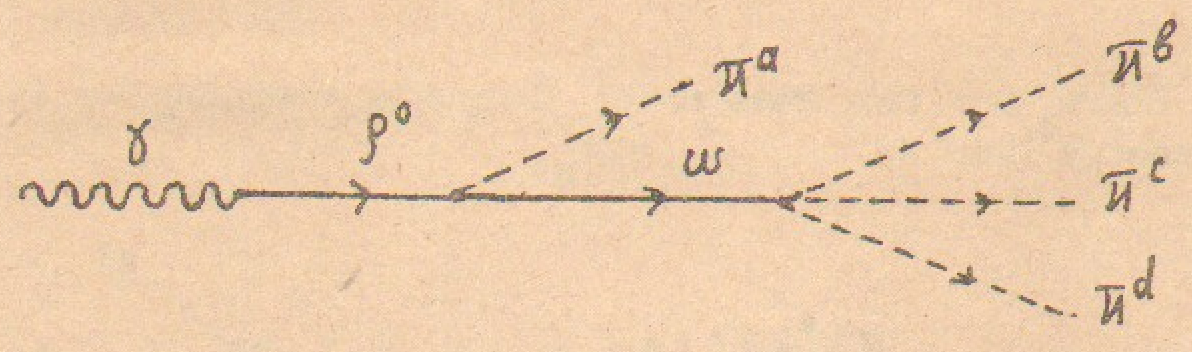
$$A_1 = - \frac{e}{f_\pi^2} \left(\frac{1}{3} - \alpha_K \right) \left\{ \varepsilon_{3ab} \delta_{cd} p_\mu^a + \text{permutations } (a,b,c,d) \right\} ,$$

$$A_2 = - \frac{e}{f_\pi^2} \left(\alpha_K \right) \left\{ \varepsilon_{3ab} \delta_{cd} p_\mu^a + \text{permutations } (a,b,c,d) \right\} ,$$

and the effective $\gamma \longrightarrow 4\pi$ vertex in the low energy limit turns out to be the same as in the purely photon-pseudo-scalar meson case:

$$A^\mu(\gamma \longrightarrow 4\pi) = - \frac{e}{3\, f_\pi^2} \left\{ \varepsilon_{3ab} \delta_{cd} p_a^\mu + \text{permutations } (a,b,c,d) \right\} .$$

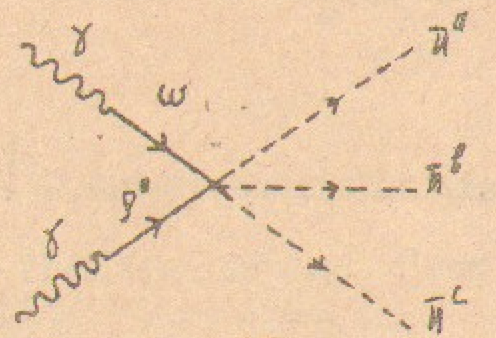
Note that the diagram



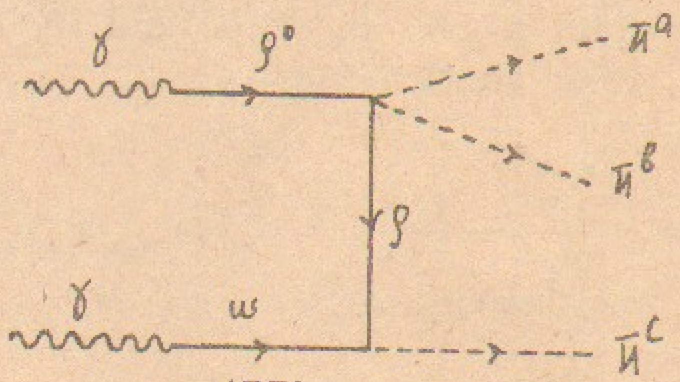
doesn't contribute in the low energy limit, because it contains an extra smallness in momentum ~ p^4.

6. THE $2\gamma \longrightarrow 3\pi$ VERTEX

Finally we discuss $2\gamma \longrightarrow 3\pi$ vertex. In this case there are two classes of diagrams relevant in the low energy limit:



(I)



(II)

For the second class the sum over cyclic permutations of (a, b, c) is assumed and in both cases the diagrams with initial photon permutation should be added.

It is easy again to get their contributions:

$$A_n^{\mu\nu} = \frac{i\, e^2}{24\pi^2 f_\pi^3}\, \varepsilon^{\mu\nu\sigma\tau} a_n (k_1 - k_2)_\sigma \times$$

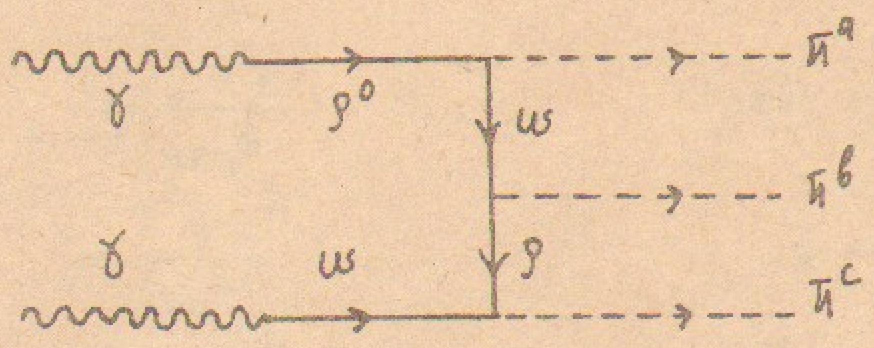
$$\times \left\{ (\delta_{ab}\delta_{c3} - \delta_{ac}\delta_{b3})\, p^c_\tau + \text{permutations of } (a,b,c) \right\},$$

where $a_1 = 2 - 3\alpha_K$, $a_2 = 3\,\alpha_K$, and the sum

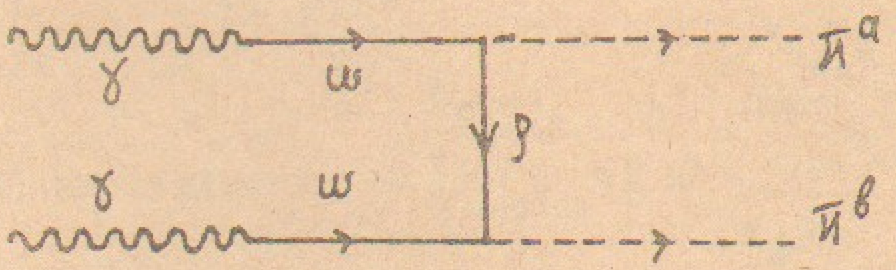
$$A^{\mu\nu}(2\gamma \longrightarrow 3\pi) = A_1^{\mu\nu} + A_2^{\mu\nu}$$

is again free from α_K and coincides with what is expected from Lagrangian (1).

The diagram



contains extra smallness ~ p^4 and doesn't contribute in the low energy limit. The same is true for the only dangerous diagram for the $2\gamma \longrightarrow 2\pi$ amplitude:



7. CONCLUDING REMARKS

As we have seen in all cases the effect of vector mesons disappears completely in the low energy limit and the resulting effective vertexes turns out to be the same as for the Lagrangian (1). Therefore we conclude that the Lagrangian (2) also reproduces all low energy theorems in the domain of its validity and can be viewed as a good starting point for the description of the low energy meson phenomenology.

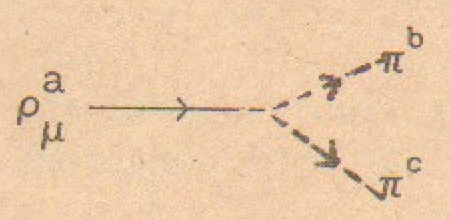
Appendix

Here we collect some Feynman rules for the Lagrangian (2) in terms of the isospin fields $\pi^a = (\pi^1, \pi^2, \pi^3)$, $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$,

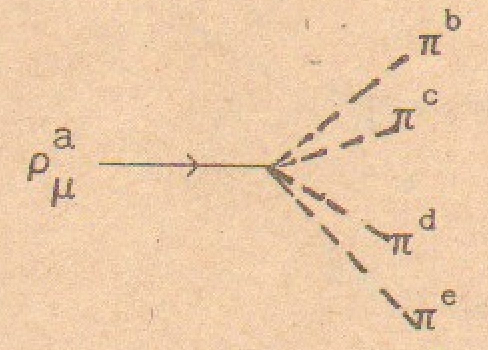
$K^*_\mu = \begin{pmatrix} K^{*+}_\mu \\ K^{*0}_\mu \end{pmatrix}$, $\rho^a_\mu = (\rho^1_\mu, \rho^2_\mu, \rho^3_\mu)$ and isosinglet ω_μ.

$$- i \frac{e}{g_{\rho\pi\pi}} m_V^2$$

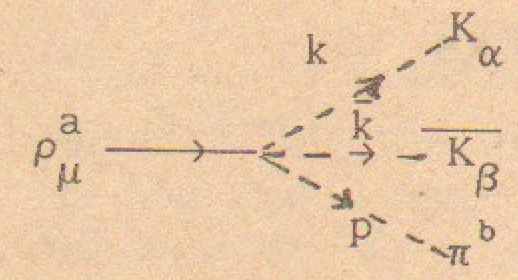
$$- i \frac{e}{3g_{\rho\pi\pi}} m_V^2$$



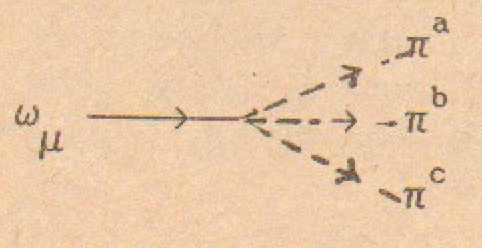
$$g_{\rho\pi\pi} \varepsilon_{abc} (p^b - p^c)_\mu$$



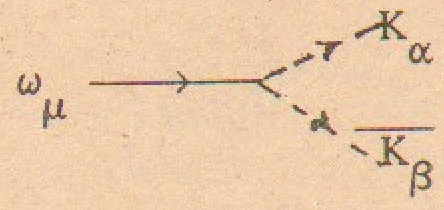
$$\frac{g_{\rho\pi\pi}}{f_\pi^2} \left(\frac{1}{3} - \alpha_K \right) \times$$
$$\times \left\{ \delta_{bc} \varepsilon_{dea} p^e_\mu + \text{permutations } (b,c,d,e) \right\}$$



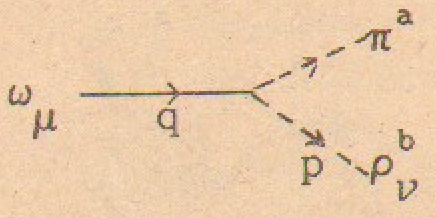
$$i \frac{g_{\rho\pi\pi}}{4\pi^2 f_\pi^3} (1-3\alpha_K) \delta_{ab} \delta_{\alpha\beta} \varepsilon^{\mu\nu\sigma\tau} p_\nu k_\sigma \bar{k}_\tau$$



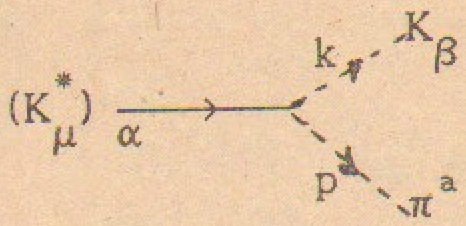
$$\frac{3g_{\rho\pi\pi}}{4\pi^2 f_\pi^3} (1-3\alpha_K) \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} p^a_\nu p^b_\sigma p^c_\tau$$



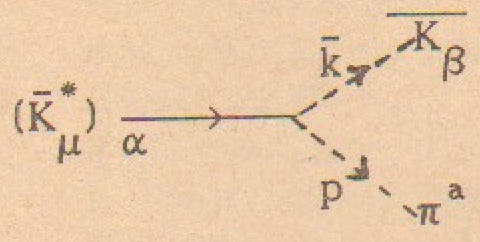
$$i \frac{g_{\rho\pi\pi}}{2} (k - \bar{k})_\mu \delta_{\alpha\beta}$$



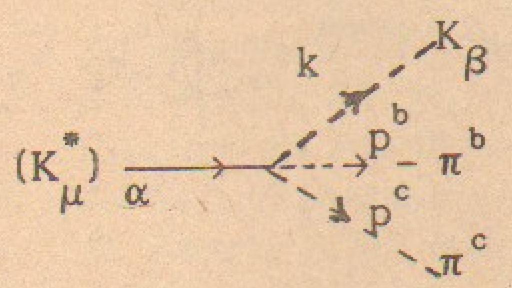
$$i \frac{3g^2_{\rho\pi\pi}}{8\pi^2 f_\pi} \delta_{ab} \varepsilon^{\mu\nu\sigma\tau} q_\sigma p_\tau$$



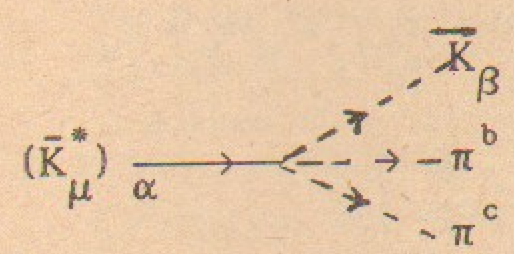
$$i \frac{g_{\rho\pi\pi}}{2} (\tau_a)_{\beta\alpha} (p - k)_\mu$$



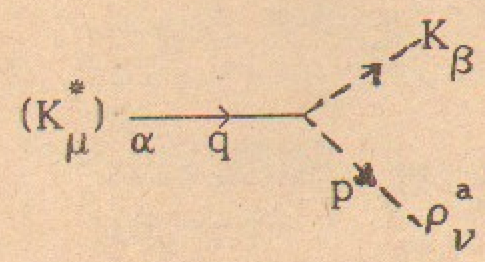
$$- i \frac{g_{\rho\pi\pi}}{2} (\tau_a)_{\alpha\beta} (p - \bar{k})_\mu$$



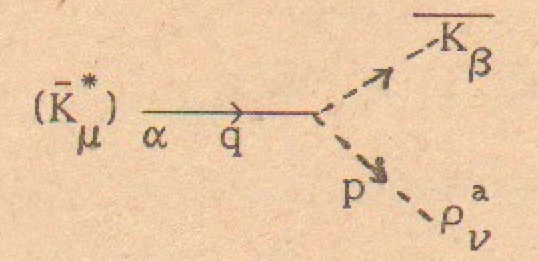
$$\frac{g_{\rho\pi\pi}}{4\pi^2 f_\pi^3} (1-3\alpha_K) \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} (\tau_a)_{\beta\alpha} p^b_\nu p^c_\sigma k_\tau$$



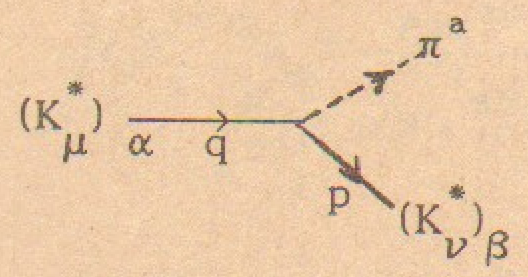
$$\frac{g_{\rho\pi\pi}}{4\pi^2 f_\pi^3} (1-3\alpha_K) \varepsilon^{\mu\nu\sigma\tau} \varepsilon_{abc} (\tau_a)_{\alpha\beta} p^b_\nu p^c_\sigma \bar{k}_\tau$$



$$i \frac{3g^2_{\rho\pi\pi}}{16\pi^2 f_\pi} \varepsilon^{\mu\nu\sigma\tau} (\tau_a)_{\beta\alpha} q_\sigma p_\tau$$

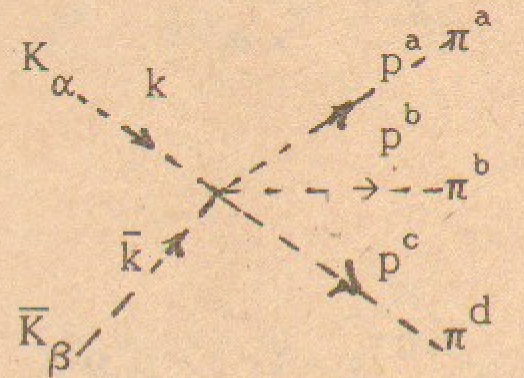


$$i\frac{3g^2_{\rho\pi\pi}}{16\pi^2 f_\pi}\varepsilon^{\mu\nu\sigma\tau}(\tau_a)_{\alpha\beta}\,q_\sigma p_\tau$$

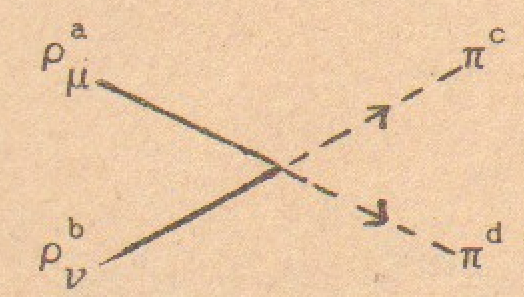


$$i\frac{3g^2_{\rho\pi\pi}}{16\pi^2 f_\pi}\varepsilon^{\mu\nu\sigma\tau}(\tau_a)_{\beta\alpha}\,q_\sigma p_\tau$$

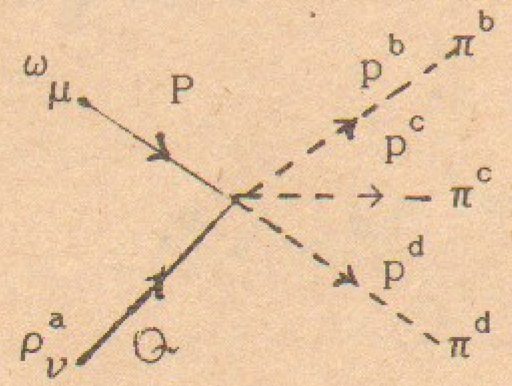
$$i\left(\frac{1}{3}-\alpha_K\right)\Big\{\delta_{ab}\delta_{cd}(2s-t-u)+\delta_{ac}\delta_{bd}(2t-s-u)+\delta_{ad}\delta_{bc}(2u-t-s)\Big\}$$



$$-\frac{15\,\delta_{\alpha\beta}}{4\pi^2 f_\pi^5}\times\left(\frac{1}{5}-\alpha_K+\frac{3}{2}\alpha_K^2\right)\varepsilon^{\mu\nu\sigma\tau}\varepsilon_{abc}\,p^a_\mu p^b_\nu p^c_\sigma\,\overline{k}_\tau$$



$$ig^2_{\rho\pi\pi}(2\delta_{ab}\delta_{cd}-\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})\,g_{\mu\nu}$$



$$i\frac{g^2_{\rho\pi\pi}}{8\pi^2 f_\pi^3}\varepsilon^{\mu\nu\sigma\tau}[(1-3\alpha_K)P-Q]_\sigma\times\left\{(\delta_{bc}\delta_{da}-\delta_{bd}\delta_{ca})\,p^d_\tau+\underset{(b,c,d)}{\text{permutations}}\right\}$$

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