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НОВОСИБИРСК

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ABSTRACTS

For high energy Linear Colliders the method of obtaining of polarized particles with help of radiation from helical wiggler was proposed few years ago.

In this paper there is represented new calculations, connected with choice of the optimal parameters of undulator for conversion system. There is described the influence of structure of undulator, influence of the distance from undulator to the target, perturbation of emittance.

There was made estimations of parameters of undulator for testing at SLAC linac.

НЕКОТОРЫЕ ОСОБЕННОСТИ ФОРМИРОВАНИЯ ОНДУЛЯТОРНОГО ИЗЛУЧЕНИЯ ДЛЯ КОНВЕРСИОННОЙ СИСТЕМЫ ЛИНЕЙНОГО КОЛЛАЙДКРА

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Аннотация

Метод получения поляризованных частиц для использования в Линейных коллайдерах с помощью излучения из спирального ондулятора был предложен много лет назад.

В этой работе представлены новые аналитические вычисления, связанные с выбором оптимальных параметров ондулятора для конверсионной системы. Изучено влияние структуры ондулятора, сделаны расчеты величины возмущения эмиттанса, расстояния от ондулятора до мишени и т.д.

Сделаны также оценки параметров ондулятора для возможного тестирования метода на линейном ускорителе SLAC.

1.INTRODUCTION.

Polarization is very important parameter for high energy physics. A lot of papers contains numerous descriptions of this subject. At that time when there are few proposals to obtain highly polarized electrons and experimental data (see for example [5]), obtaining the polarized positrons for the Linear Collider was still open until 1979 [1], when there was invented the method to provide polarized positrons (and electrons) of necessary amount.

On Fig.1 there is represented the components of the conversion system [1].

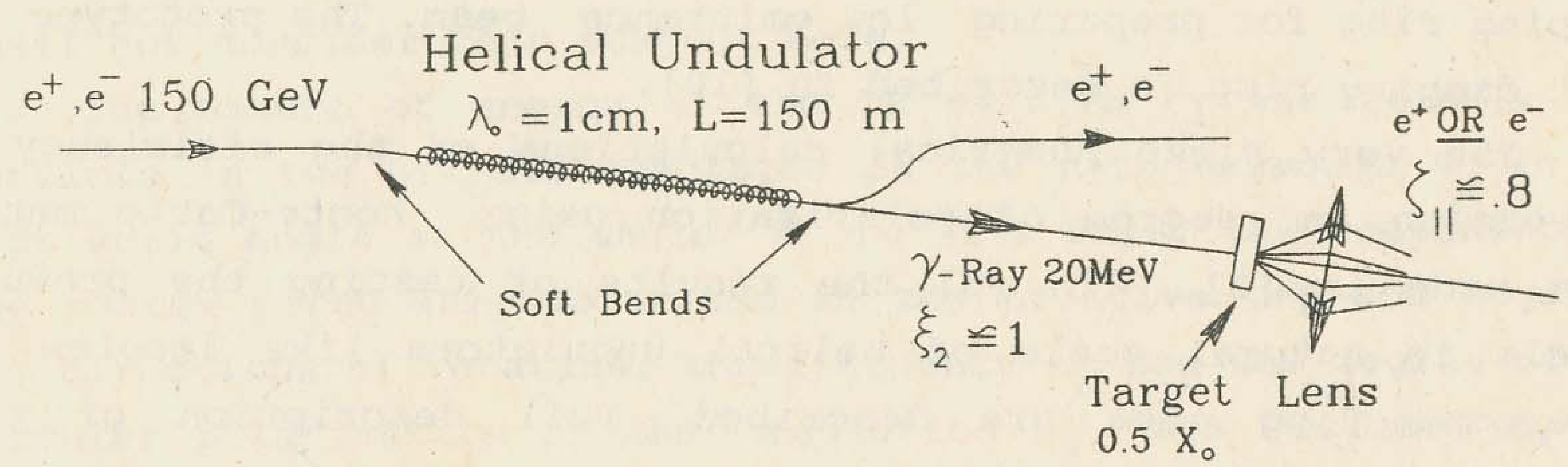

Fig.1 General layout of the Conversion System.

The method is that *circularly polarized photons* are converted into positrons and electrons in thin heavy material target. At the top boundary of energy spectrum that provides a high yield of *longitudinally polarized particles*. Circularly polarized photons are radiated by initial particles (e^+ or e^-) in helical undulator. So, here we use correlation between energy and polarization of the particles, and for separation of the polarized ones it is necessary to select the particles with highest energy. Unpolarized particles can be used as initial ones and after their passage through undulator they are loose only about 1.5% of its energy. Minimal starting energy for normal operation of this conversion system is about 100 GeV. This level mostly defined by the minimal possible period of the undulator, which is enable to fabricate with limitation of minimal aperture of the undulator.

The mean degree of polarization ζ_2 (see page 17) of created positrons or electrons by radiation with polarization ξ_2 is

$$\langle \zeta_{\|} \rangle \cong \frac{\int \zeta_{\|}(E_+,\xi_2)\, \frac{d\sigma(E_\gamma,E_+)}{dE_+}\, dE_+}{\int \frac{d\sigma(E_\gamma,E_+)}{dE_+}\, dE_+},$$

where $d\sigma/dE_+$ is differential cros-section of the pair production and the integrals are taken from maximum of energy of the positrons $E_+^{max} = E_\gamma - 2m_0c^2$ down to the spectrum of energy. The function $\zeta_{\|}(E_+,\xi_2)$ is practically independent of energy and $\zeta_{\|}(E_+,\xi_2) \cong \xi_2\, f(E_+,E_{0+}^{max})$ [8]. So it is desirable to have ξ_2 factor as high as possible. After preliminary acceleration in RF structure and final separation over energy, collected particles goes to damping ring for preparing low emittance beam. The prototype for such damping ring is described in [10].

The very first numerical calculations of the efficiency of conversion and degree of polarization using Monte-Carlo method were made in [2]. In [3] the results of testing the produced models in natural scale of helical undulators like impulse and superconducting ones are described. Full description of this subject were made in [16].

Now the method of conversion with wiggler radiation is also under consideration at DESY [4].

Here we summarize in one place description of the properties of the radiation and some ideas of the undulator design. The formulas are easy to use in codes for calculations with PC's.

2. PROPERTIES OF THE UNDULATOR RADIATION

Here we mostly used the results obtained in previous papers [7,13,18 - 20] described Undulator Radiation, but accommodate its for usage in conversion system of any Linear Collider. Mostly it concerned by approximation of the functions of radiation for the case of extremely high energy and representation of the results in convenient form. So we begin with consideration of the properties of Undulator Radiation (UR).

The mostly important parameters are:

$$\beta_\perp = p_\perp/\gamma, \qquad p_\perp = eH_\perp\lambda_0/2\pi mc^2 = 93.4\, H_\perp[\text{Tesla}]\, \lambda_0[\text{m}],$$

$H_\perp$- is the intensity of the magnetic field of the undulator, λ_0 is the period of the undulator, $\gamma = E/m_0c^2$ is the relativistic factor, $\beta_\perp$ is normalized transverse component of normalized velocity of the particle, $\beta = v/c$, $\beta_{\|}$ is longitudinal component of normalized velocity $\beta_{\|} = \beta\sqrt{1-\beta_\perp^2/\beta^2}$.

Typically, the vacuum chamber is the tube of round cros-section, so it has the threshold types of propagated waves. Each type of this waves has its own distribution of the field in cross-section. In principle here can occur condition for coherent radiation (for example back radiation with wavelength of $\lambda_0/2$), but we shall not consider this subject here.

The amount of energy of the UR emitted by the relativistic particle in the helical undulator on the n-th harmonic $\mathcal{E}_n$ in the unit solid angle at the angle Θ to it's axis, the components of the energy circularly polarized in the positive (+) and negative (-) directions of rotation, the frequency ω_n and the degree of the circular polarization of the radiation ζ_2 are defined by the expressions [7,13] (see also [11])

$$\frac{d\mathcal{E}_n}{do} = \frac{6\mathcal{E}_t\gamma^2}{\pi}\,\frac{n^2F_n(p_\perp,\Theta)}{(1+p_\perp^2+\Theta^2)^3}, \qquad \frac{d\mathcal{E}_{n\pm}}{do} = \frac{6\mathcal{E}_t\gamma^2}{\pi}\,\frac{n^2F_{n\pm}(p_\perp,\Theta)}{(1+p_\perp^2+\Theta^2)^3},$$

$$\omega_n = \frac{2n\Omega\gamma^2}{1+p_\perp^2+\Theta^2} = \frac{\omega_n^{max}}{1+\frac{\Theta^2}{1+p_\perp^2}},$$

$$\xi_{2n} = \frac{F_{n+}-F_{n-}}{F_n(p_\perp,\Theta)} = \frac{(1+p_\perp^2-\Theta^2)}{p_\perp\Theta}\,\frac{J_n(n\varkappa)J_n'(n\varkappa)}{F_n(p_\perp,\Theta)},$$

where

$$F_n(p_\perp,\Theta) = F_{n+}+F_{n-} = J_n'^2(n\varkappa) + \left(\frac{1+p_\perp^2-\Theta^2}{2p_\perp\Theta}\right)^2 J_n^2(n\varkappa),$$

$$F_{n\pm}(p_\perp,\Theta) = \frac{1}{2}\left(J_n'(n\varkappa) \pm \frac{1+p_\perp^2-\Theta^2}{2p_\perp\Theta}J_n(n\varkappa)\right)^2,$$

$\mathcal{E}_t = 4\pi K e^2\Omega\gamma^2 p_\perp^2/3c$ is total energy, emitted by the particle in the undulator, $\omega_{n\,max} = 2n\Omega\gamma^2/(1+p_\perp)^2$, $\Omega = 2\pi c/\lambda_0$ is the frequency of the oscillation of the particle in the undulator, $\Theta = \gamma\theta$, θ - is the angle between longitudinal coordinate and direction of observation, $\varkappa = 2p_\perp\Theta/(1+p_\perp^2+\Theta^2) < 1$, J_n, J_n' is the Bessel's function and it's derivative. K is number of periods of undulator, $K = L/\lambda_0$, L is total length of undulator.

The values

$$\frac{d\mathcal{E}_{n\pm}}{do} = \frac{1 \pm \xi_{2n}}{2}\,\frac{d\,\mathcal{E}_n}{do}.$$

The derivative of the Bessel's function can be found from the relationships

$$J_n'(x) = J_{n-1}(x) - \frac{n}{x}J_{n+1}(x) = \frac{n}{x}J_n(x) - J_{n+1}(x).$$

The degree of circular polarization of the UR emitted at the angle Θ has one sign at all harmonics n simultaneously and changes the sign when crossing the angle $\Theta = \Theta^* = \sqrt{1+p_\perp^2}$. It decreases from $\xi_{2n} = 1$ at $\Theta = 0$ to $\xi_{2n} = 0$ at $\Theta = \Theta^*$ and then to $\xi_{2n} \cong -1$ at $\Theta \gg 1$. At the angle $\Theta = \Theta^*$ the UR has linear polarization.

The angle $\Theta = \Theta^*$ corresponds to the angle $\theta = \pi/2$ in the reference system moving with the average velocity of the particle in the undulator. In this system the particles trajectory is the circle and the UR turn into SR.

The frequency of the UR takes the maximal value ω_n^{max} at the angle $\Theta = 0$. The deviation of the frequency from maximal value is less than 10%, when the angle $\Theta < \Theta^*/3$.

Energy of the photon, emitted on the harmonic number n is

$$E_{\gamma n} = \hbar\omega_n = \frac{2n\hbar\Omega\gamma^2}{1+p_\perp^2+\Theta^2} = \frac{E_{\gamma n}^{max}}{1+\dfrac{\Theta^2}{1+p_\perp^2}},$$

where $E_{\gamma n}^{max} = nE_{\gamma 1}^{max}$ and $E_{\gamma 1}^{max} = 2\hbar\Omega\gamma^2/(1+p_\perp^2)$ is the maximal energy of quantum, emitted on the n-th and first harmonics correspondly.

The number of the quantum, emitted in the undulator on the harmonic n in the unit solid angle is determined by the relation

$$\frac{dN_{\gamma n}}{do} = \frac{1}{E_{\gamma n}}\,\frac{d\mathcal{E}_n}{do} = 4\alpha nK\,\frac{p_\perp^2\gamma^2F_n(p_\perp,\Theta)}{(1+p_\perp^2+\Theta^2)^2},$$

where $\alpha = e^2/\hbar c \cong 1/137$.

According to the formula for energy of the photon, the angle Θ, the frequency end energy of the quanta are connected by the relation

$$\Theta = \Theta(p_\perp,s) = \sqrt{(1+p_\perp^2)\left(\frac{\omega_n^{max}}{\omega}-1\right)} = \sqrt{(1+p_\perp^2)(1-s)/s}$$

where $s = \omega_n/\omega_n^{max} = E_{\gamma n}/E_{\gamma n}^{max}$ is the relative energy of the photon.

It means that the last relation determine the dependence of the values $d\mathcal{E}_n/do$, $d\mathcal{E}_{n\pm}/do$, dN_γ/do, ξ_{2n} on the frequency or on energy of the quanta.

In accordance with the previous expressions and the relationship $do = \pi d\Theta^2/\gamma^2 = -\pi(1+p_\perp^2)ds/s^2/\gamma^2$, the spectral distribution and the degree of circular polarization of the emitted photons are

$$\frac{dN_{\gamma n}}{ds} = \frac{\pi(1+p^2)}{s^2\gamma^2}\,\frac{dN_{\gamma n}}{do} = 4\pi\alpha nK\,\frac{p_\perp^2}{1+p_\perp^2}\,F_n\big(p_\perp,\Theta(s)\big)$$

$$\xi_{2n} = \frac{\sqrt{1+p_\perp^2}}{p_\perp}\,\frac{2s-1}{\sqrt{s(1-s)}}\,\frac{J_n(n\varkappa)J_n'(n\varkappa)}{F_n(p_\perp,s)}, \qquad (1)$$

where $\varkappa = 2p_\perp\sqrt{s(1-s)/(1+p_\perp^2)}$,

$$F_n(p_\perp,s) = J_n'^2(n\varkappa) + \frac{1+p_\perp^2}{4p_\perp^2}\,\frac{(2s-1)^2}{s(1-s)}\,J_n^2(n\varkappa),$$

The previous formulas are valid in all cases, and now we shall consider some modification, connected with the assumption of low $p_\perp$ factor.

In the approximation $\varkappa \ll 1$ ($p_\perp \ll 1$ or/and $\Theta \ll 1$) we shall use the expansion of the Bessel's functions

$$J_n(x) = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{\nu!\Gamma(n+\nu+1)} \left(\frac{x}{2}\right)^{n+2\nu} \simeq \frac{1}{n!}\left(\frac{x}{2}\right)^n - \frac{1}{(n+1)!}\left(\frac{x}{2}\right)^{n+2} + \dots ,$$

First term here describes the Bessel's function $J_1(x)$ and $J_2(x)$ with the accuracy, better than 10% when $\varkappa < 0.6$. The accounting the second term increases the accuracy to 2%.

In that case the expansion of the Bessel's functions to an approximation of $\nu = 1$ leads to the functions $F_n(p_\perp,\Theta)$, $\xi_{2n}(p_\perp,\Theta)$ and $F_n(p_\perp,s)$, $\xi_{2n}(p_\perp,s)$ of the form

$$F_n(p_\perp,\Theta) \simeq \frac{n^{2(n-1)}}{2(n-1)!(n-1)!} \left(\frac{p_\perp\Theta}{1+p_\perp^2+\Theta^2}\right)^{2(n-1)} \left\{ \frac{(1+p_\perp^2)^2+\Theta^4}{(1+p_\perp^2+\Theta^2)^2} - \right.$$

$$\left. - \frac{2n}{n+1}\left(\frac{p_\perp\Theta}{1+p_\perp^2+\Theta^2}\right)^2 \left[1 + n\frac{(1+p_\perp^2)^2+\Theta^4}{(1+p_\perp^2+\Theta^2)^2}\right] \right\} ,$$

$$\xi_{2n}(p_\perp,\Theta) = \frac{\left[1-2n\left(\frac{p_\perp\Theta}{1+p_\perp^2+\Theta^2}\right)^2\right]\left[(1+p_\perp^2)^2-\Theta^4\right]}{(1+p_\perp^2)^2+\Theta^4 - \frac{2n}{n+1}(p_\perp\Theta)^2\left[1+n\frac{(1+p_\perp^2)^2+\Theta^4}{(1+p_\perp^2+\Theta^2)^2}\right]} ,$$

and

$$F_n(p_\perp,s) = \frac{n^{2(n-1)}p_\perp^{2(n-1)}}{2(n-1)!(n-1)!}\left[\frac{s(1-s)}{1+p_\perp^2}\right]^{n-1} \left\{1 - 2s + 2s^2 - \right.$$

$$\left. - \frac{2n}{n+1}\frac{p_\perp^2}{(1+p_\perp^2)} s(1-s)\left[1+n(1-2s+2s^2)\right]\right\} ,$$

$$\xi_{2n}(p_\perp,s) = \frac{(2s-1)\left[1 - \frac{2n^2+1}{n+1}\frac{p_\perp^2}{(1+p_\perp^2)}s(1-s)\right]}{1-2s+2s^2 - \frac{2np_\perp^2}{(n+1)(1+p_\perp^2)}s(1-s)\left[1+n(1-2s+2s^2)\right]} .$$

In dipole approximation $p_\perp \ll 1$ and for the harmonics n = 1,2:

$$F_1(\Theta) = \frac{1}{2}\frac{1+\Theta^4}{(1+\Theta^2)^2} , \quad F_2(\Theta) = 2(p_\perp\Theta)^2\frac{1+\Theta^4}{(1+\Theta^2)^4} ,$$

$$\xi_{21} = \xi_{22} = \frac{1-\Theta^4}{1+\Theta^4} ,$$

$$F_1(s) = \frac{1}{2}(1-2s+2s^2), \quad F_2(s) = 2s(1-s)(1-2s+2s^2)p_\perp^2,$$

$$\xi_{21}(s) = \xi_{22}(s) = \frac{2s-1}{1-2s+2s^2}.$$

This function is represented in Table 1.

Table 1.

s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ξ_2	-1.	-.97	-.88	-.68	-.38	0.0	.38	.68	0.88	0.97	1.

It is clear, that it has point antisymmetry around s = 0.5 .

In case $p_\perp \ll 1$ expressions for spectral distribution becomes

$$\frac{dN_{\gamma 1}}{ds} = \frac{2\pi\alpha Kp_\perp^2}{(1+p_\perp^2)^2}(1-2s+2s^2), \quad \frac{dN_{\gamma 2}}{ds} = \frac{8\pi\alpha Kp_\perp^2}{(1+p_\perp^2)^2}s(1-s)(1-2s+2s^2)p_\perp^2,$$

where $\Theta = \Theta(s) = ((1+p_\perp^2)(1-s)/s)^{1/2}$. For angular distribution we obtain ($do = \pi d\Theta^2/\gamma^2$)

$$\frac{dN_{\gamma 1}}{\pi d\Theta^2} = 2\alpha K\frac{p_\perp^2(1+\Theta^4)}{(1+p_\perp^2+\Theta^2)(1+\Theta^2)^2} , \quad \frac{dN_{\gamma 2}}{\pi d\Theta^2} = 16\alpha K\frac{p_\perp^2(p_\perp\Theta)^2(1+\Theta^4)}{(1+p_\perp^2+\Theta^2)(1+\Theta^2)^4}$$

The number of the photons emitted by the particle in the undulator on the harmonic n in the range of angles $(0,\Theta)$ and the corresponding range of the relative frequency (1,s) is

$$\Delta N_{\gamma n}(p_\perp,s) = \int_s^1 \frac{dN_{\gamma n}}{ds}ds = 4\pi\alpha nK\frac{p^2}{1+p_\perp^2}\Phi_n(p_\perp,s) \qquad (2)$$

where $\Phi_n(p_\perp,s) = \int_s^1 F_n(p_\perp,s)ds$.

In the approximation $\varkappa \ll 1$ for the harmonics n=1,2 :

$$\Phi_1(p_\perp,s) = \frac{1}{6}(1-s)(2-s+2s^2) - \frac{p_\perp^2}{2(1+p_\perp^2)}(1-s)^2\left(\frac{4}{15}+\frac{8s}{15}-\frac{s^2}{5}+\frac{2s^3}{5}\right)$$

$$\Phi_2(p_\perp,s) = \frac{p_\perp^2}{10(1+p_\perp^2)}(1-s)^2\Big[(1+2s^2-2s^3+4s) - \frac{20\,p_\perp^2}{21(1+p_\perp^2)}(1-s)\left(\frac{2}{15}+\frac{2}{5}s+\frac{4}{5}s^2-s^3+2s^4\right)\Big] .$$

The total number of the photons, emitted on the first harmonic according to previous relation is

$$N_{\gamma 1} = \Delta N_{\gamma 1}(s{=}0) = \frac{4\pi\alpha K}{3}\,\frac{p_\perp^2}{1+p_\perp^2}\left(1-\frac{2}{5}\,\frac{p_\perp^2}{1+p_\perp^2}\right)$$

On Table 2 there is represented the number of the photons on the first and second harmonics for different $p_\perp$ and s factors.

Table 2.

	$p_\perp$ =	0.7	0.5	0.35	0.2	0.1
s=0.	ΔN_1	87.	56.2	31.9	11.6	3.0
	ΔN_2	38.	14.6	4.3	0.54	0.036
s=0.9	ΔN_1	13.	8.2	4.4	1.59	0.41
	ΔN_2	1.6	0.59	0.18	0.02	.0014

In the case $p_\perp \gg 1$ (the case when particle has the relativistic velocity in the moving reference system and emits the syncrotron radiation) the value $N_{\gamma n}$ can be represented in the form

$$N_{\gamma n} = 2\sqrt[6]{3}\;\Gamma(\tfrac{2}{3})\;\alpha\;K\;n^{-2/3} \cong 3.24\;\alpha\;K\;n^{-2/3} \qquad (\,n \ll p_\perp^3\,)$$

Notice here, that the value $N_{\gamma 1}$ estimated by the expression for $N_{\gamma 1}$ in the case $p_\perp \gg 1$, is only 30% less, than the value $N_{\gamma 1}$ estimated by the exact expression in the last formula.

In the conversion system the monochromatic source must be used. The energy of the photons emitted on the first harmonic must not differ more tan 10% from it's maximal value E_1^{max} and degree of circular polarization must not fall below the value ξ_2 = 0.9 . For this purpose it is necessary to collimate the emission angle by the value $\Theta = \Theta^*/3$.

Let us take for example, $p_\perp$ = 0.5 , K = 10^4. In this case, according to (2), for full spectra $\Delta N_{\gamma 1}(s{=}0) \simeq 56$, $\Delta N_{\gamma 2}(s{=}0) \simeq 14$, $\Delta N_{\gamma 1}(s = 0.9) \simeq 8$, $\Delta N_{\gamma 2}(s = 0.9) \simeq 0.6$. This yields that the collimation of the angle by the value $\Theta = \Theta^*/3$ (s = 0.9) decreases

the relation $\Delta N_{\gamma 2}/\Delta N_{\gamma 1}$ at the converter from 27 % to 13 % . Decreasing of the $p_\perp$ = 0.35 improves this ratio to more acceptable value 4.2% , but also decreases the number of emitted photons $\Delta N_{\gamma 1}(s = 0.9)$ from 8 to 4.5 . The number of the photons emitted in these conditions can gain only by increasing the number of periods of the undulator K.

Spectral-angular distribution of the UR energy emitted by the particle on the n-th harmonic when the *finite number of periods* K is taken into account is determined by the expression

$$\frac{\partial^2 \mathcal{E}_n}{\partial\omega\,\partial o} = \frac{K}{\omega_1}\,\frac{\partial \mathcal{E}_n}{\partial o}\;\mathrm{Sinc}^2\sigma_n\;,$$

where $\mathrm{Sinc}\,\sigma_n = \mathrm{Sin}(\sigma_n)/\,\sigma_n$, $\sigma_n = \pi n K(\omega - \omega_n)/\omega_n$. The value $\mathrm{Sinc}^2\sigma_n \simeq (\omega_1/K)\;\delta(\omega-\omega_n)$ when $K \gg 1$.

Further we shall consider the case, when the distance from the end of the undulator to the observation point, located on the converter, is compared with the length of the undulator $K\lambda_0$. In this case spectral density of energy of the UR, illuminating the converter ($dS = R^2 do$)

$$\frac{\partial^2 \mathcal{E}_n}{\partial\omega\,\partial S} = \frac{1}{K\lambda_0}\int_{y_1}^{y_f} R^{-2}(\Theta)\;\;\frac{\partial \mathcal{E}_n}{\partial o}\;\delta[\omega-\omega_n(\Theta)]\;dy\;,$$

where y is the longitudinal coordinate of the emission point of the particle in the undulator, y_1, y_f ($y_1 > y_f$) is the coordinates of the beginning and the end of undulator, the value y = 0 corresponds to the converter's one. The angle Θ is connected with the transverse coordinate *r* of the observation point on the converter by the relation

$$\Theta = -\gamma\;\mathrm{Arctg}(r/y) \simeq -\gamma r/y$$

According to this last two relations, in the approximation $R(\Theta) \simeq y$, $dy/y^2 \simeq -d\Theta/\gamma r$, we can obtain

$$\frac{\partial^2 \mathcal{E}_n}{\partial\omega\,\partial S} = \frac{1}{K\lambda_0\gamma r}\int_{\Theta_1}^{\Theta_f} \frac{\partial \mathcal{E}_n}{\partial o}\;\delta[\omega-\omega_n(\Theta)]\,d\Theta,$$

where $\Theta_1 \simeq \gamma r/y_1$, $\Theta_f \simeq \gamma r/y_f$.

Using the relations $\delta[\omega-\omega_n(\Theta)] = \delta[\Theta-\Theta(\omega_n)]/(\partial\omega_n/\partial\Theta)$ we can represent last relation in the form

$$\frac{\partial^2 \mathcal{E}_n}{\partial\omega\,\partial S} = \frac{6\mathcal{E}_t\gamma^2}{\pi} P_n(p_\perp,r,s) \qquad (3)$$

where

$$P_n(p_\perp,r,s)=\frac{n^2}{K\lambda_0\gamma r}\int_{\Theta_i}^{\Theta_f}\frac{F_n(p_\perp,\Theta)}{(1+p_\perp^2+\Theta^2)^3}\,\delta(\omega-\omega_n(\Theta))\,d\Theta =$$

$$= \frac{n^2 s^{1.5}\,F_n(p_\perp,s)}{2K\lambda_0\gamma r\,\omega_n^{max}\,(1+p_\perp^2)^{2.5}\sqrt{1-s}},$$

when $s_f < s < s_i$ and $P_n(p_\perp,r,s) = 0$, when $s < s_f$, $s > s_i$, $s = \omega/\omega_n^{max}$,

$$s_i = 1/[1+\gamma^2r^2/(1+p_\perp^2)y_i^2], \qquad s_f = 1/[1+\gamma^2r^2/(1+p_\perp^2)y_f^2].$$

The function $P_n(p_\perp,n)$ in the approximation $\varkappa \ll 1$, $\Theta \ll \Theta^*$ or $r \ll r^* = (1+p_\perp^2)y_f/\gamma$ is

$$P_n(p_\perp,r) = \frac{1}{(1+p_\perp^2)^3 y_i y_f}\left[1-\frac{2p_\perp^2}{3(1+p_\perp^2)^2}(\Theta_f^2+\Theta_f\Theta_i+\Theta_i^2)\right].$$

According to (3) (page 12) the photon flux of the undulator radiation at the point of observation, determined by the radii r, is distributed in the range of relative frequency/energy s_i-s_f. It depends on radii r, the distance from the undulator to the converter y_f, and the length of the undulator $K\lambda_0 = y_i - y_f$. The degree of circular polarization $\xi_{2n}(s)$ defined by (1) (page 7) corresponds to every frequency of the range.

According to (3) the photon spectral flux density is

$$\frac{\partial^2 N_{\gamma n}}{\partial\omega\,\partial S} = \frac{1}{E_\gamma}\frac{\partial^2\mathcal{E}_n}{\partial\omega\,\partial S} = \begin{cases} \dfrac{\alpha p_\perp^2 F_n}{2\pi c\gamma r}\sqrt{\dfrac{s}{(1+p_\perp^2)(1-s)}}, & s_f \le s \le s_i \\ 0, & s < s_f,\ s > s_i \end{cases}$$

Correspondly the photon flux density

$$\frac{\partial N_{\gamma n}}{\partial S} = \frac{1}{\hbar\omega}\frac{\partial\mathcal{E}_n}{\partial S} = \int_0^{\omega_n^{max}}\frac{\partial^2 N_{\gamma n}}{\partial\omega\,\partial S}\,d\omega = \frac{2\alpha\gamma p_\perp^2}{(1+p_\perp^2)^{1.5}\lambda_0}\,q_n(p_\perp,r)$$

where $$q_n(p_\perp,r) = \frac{n}{r}\int_{s_i}^{s_f}\sqrt{\frac{s}{1-s}}\,F_n(p_\perp,s)\,ds$$

The number of the photons emitted by the particle in the range of radii $(0,r)$ in approximation $\Theta \ll \Theta^*$ or $r \ll r^* = (1+p_\perp)^{1/2}y_f/\gamma$ is

$$\Delta N_\gamma = 2\pi\int_0^{r_m}\frac{\partial N_{\gamma n}}{\partial S}\,r\,dr = \frac{4\pi\alpha\gamma p_\perp^2}{(1+p_\perp^2)^{1.5}\lambda_0}\,Q_n(p_\perp,r_m), \qquad (4)$$

where

$$Q_n(p_\perp,r_m) = \int_0^{r_m} r\,q_n(p_\perp,r)\,dr,$$

r_m is the radius of the converter. In this approximation we have

$$q_1 = \frac{\gamma}{\sqrt{1+p_\perp^2}}\left(\frac{1}{y_f}-\frac{1}{y_i}\right) - \frac{5\gamma^3r_m^2}{6(1+p_\perp^2)^{1.5}}\left(1+\frac{4}{5}\frac{p_\perp^2}{1+p_\perp^2}\right)\left(\frac{1}{y_f^3}-\frac{1}{y_i^3}\right),$$

$$Q_1 = \frac{0.5\gamma r_m^2}{\sqrt{1+p_\perp^2}}\left(\frac{1}{y_f}-\frac{1}{y_i}\right) - \frac{5\gamma^3r_m^4}{24(1+p_\perp^2)^{1.5}}\left(1+\frac{4}{5}\frac{p_\perp^2}{1+p_\perp^2}\right)\left(\frac{1}{y_f^3}-\frac{1}{y_i^3}\right).$$

Accordingly in the case $r = r^*/3$ (or $\Theta \le \Theta^*$ $s > 0.9$):

$$\Delta N_{\gamma 1} = \frac{2\pi\,\alpha\,p_\perp^2\,y_i}{9(1+p_\perp^2)\,\lambda_0}\left(1-\left|\frac{y_f}{y_i}\right|\right)\left\{1-\frac{5}{108}\left(1+\frac{4}{5}\frac{p_\perp^2}{1+p_\perp^2}\right)\left[1+\frac{y_f}{y_i}+\left(\frac{y_f}{y_i}\right)^2\right]\right\}$$

It follows from last expression, that the number of the photons, emitted by the particle with the degree of circular polarization $\xi_2 > 0.9$ on the area $\pi(r^*)^2$ increases with the increasing of the length of undulator and has a finite limit. The effective length of the undulator is $K\lambda_0 \cong |y_f|$. It means that the minimum distance from the undulator to the converter must be of the order of the length of the undulator or larger : $y_f \ge y_i - y_f = K\lambda_0$. The installation of the undulator on the distance $y_f = K\lambda_0$ decreases the value $\Delta N_{\gamma n}$ two times in comparison with the case $y_f \gg K\lambda_0$.

The perturbation of emittance of the beam during the motion through the undulator is negligible as it can be pointed out from the analogous calculation made in [8]. The rate of emittance growth due to radiation is

$$\Delta\varepsilon \cong \langle H\rangle \int (E_\gamma/E)^2\ (dN_\gamma/dE_\gamma)dE_\gamma$$

where $H = \frac{1}{\beta x}(\eta_x + (\beta_x\eta'_x - \frac{1}{2}\ \beta'_x\eta_x)^2)$, brackets <...> means average over period, β_x - is envelope function, η_x - is dispersion function, and derivatives are taken over longitudinal distance. Last expression can be rewritten as

$$\Delta\varepsilon \cong \langle H\rangle \sum_n \left(\frac{\hbar\omega_n^{max}}{m_0c^2\gamma}\right)^2 \int_0^1 s^2\ \frac{dN_{\gamma n}}{ds}\ ds.$$

For <H> we can estimate [8] $\langle H\rangle \cong \frac{1}{2}\ \bar\beta\ p_\perp^2/\gamma^2$, where $\bar\beta$ is β_x averaged over period. Here we can take into account only first harmonic

$$\frac{dN_{\gamma 1}}{ds} = \frac{\pi\alpha\ Kp_\perp^2}{(1+p_\perp^2)^2}\ \frac{1}{2}\ \frac{1+\Theta^4}{(1+\Theta^2)^2}\ ,$$

where $\Theta = \Theta(s) = ((1+p_\perp^2)(1 - s)/s)^{1/2}$ and estimate

$$\Delta\varepsilon \cong \langle H\rangle\ \frac{4\hbar^2\Omega^2\gamma^4}{(m_0c^2\gamma)^2}\ \frac{\pi\alpha\ Kp_\perp^2}{(1+p_\perp^2)^4}\ \frac{1}{6} \cong 0.4\pi\alpha^{-1}\ K\ \bar\beta\ \left(\frac{2\pi r_0}{\lambda_0}\right)^2 \frac{p_\perp^4}{(1+p_\perp^2)^4}\ .$$

It is evident, that this perturbation is small.

When particle goes trough undulator it lose its energy by the low (*dl* - along the trajectory)

$$\frac{d\gamma}{dl} = \frac{1}{m_0c^3}\ \frac{d\mathcal{E}_t}{dt} = -\ \frac{2}{3}\ r_0\ \gamma^2\ (p_\perp/\lambda\!\!\!^{-}_0)^2,$$

where $\lambda\!\!\!^{-}_0 = \lambda_0/2\pi$. This yields, that energy of the quantum also changes from the beginning of the undulator to its end. This difference may be up to few percent. To compensate this effect it is possible to *change the period* of the undulator accordingly the losses of energy.

The betatron motion is not considered here, but in [12] there is represented consideration of this question. Some calculations and experimental data with helical undulator are represented in [24]. The general output of this paper is that the *pair* of undulators can eliminate perturbation of transverse motion of each other. So, some procedure of installation of the parts of the

undulator will be enough for normal operation of whole ≅ 100 meter long array of undulators. This question will be illuminate in other place.

3. CONVERSION OF THE UR TO THE LONGITUDINALLY POLARIZED e^+, e^-.

Here we obtain analytical estimation of conversion efficiency, using the formulas of previous chapter.

The differential cros-section of the pair production [6]

$$\frac{d\sigma(E_\gamma,E_+)}{dE_+} \cong \frac{4\ \alpha\ Z(Z+1)r_0^2}{E_\gamma - 2m_0c^2}\ G(E_+,E_+^{max})$$

where Z is the Atomic number of the nuclei of the target, $r_0 = e^2/m_0c^2$, $G(E_+,E_+^{max})$ is the universal function of the ratio E_+/E_+^{max}. Here we shall use an approximate expressions for this function, see later.

The number of the photons in the converter falls with a depth τ by the law

$$\frac{N_\gamma(\tau)}{N_\gamma(0)} = exp(-\ \frac{7}{9}\ \tau),$$

where $\tau = y\ /X_0$, y [g/cm^2], is longitudinal coordinate, measured as fraction of radiation lengths X_0,

$$X_0^{-1} \cong 4\alpha Z(Z+1)NA^{-1}r_0^2\ ln(183Z^{1/3})\ [cm^2/g],$$

here $N \cong 6\ 10^{23}$ is Avogadro number, A is the mass number [12,13].

The number of the positrons produced by one photon in the media layer of the thickness $d\tau$ at the depth of τ at the initial energy E_+ per energy interval dE_+ is

$$\frac{\partial^2 N_+}{\partial E_+\partial\tau} = \frac{N}{A}\ exp(-\ \frac{7}{9}\ \tau)\ \frac{d\sigma(E_\gamma,E_+)}{dE_+}\ .$$

Let us take into account the process of fluctuation of energy losses in the media.

The probability WdE_+, that the positron with initial energy E_{0+} was produced by the photon at the depth of τ in the converter of the thickness δ, and at the exit of the converter has energy in the interval from E_+ to $E_+ + dE_+$, is described by the formula [17]

$$W(E_{0+},E_+,\delta-\tau) = \frac{1}{E_{0+}} \left[\ln \frac{E_{0+}}{E_+} \right]^{\frac{\delta-\tau}{\ln 2} - 1} \left[\Gamma \left(\frac{\delta-\tau}{\ln 2} \right) \right]^{-1},$$

where $\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx$, and δ is also measured as partial of the radiation length.

The number of the positrons produced by one photon of the energy E_γ in the converter at the initial energy $E_{0+} < E_\gamma$ per energy interval dE_+ and coming out of the converter at the energy E_+ per the energy interval dE_+ is determined by the convolution of the previous expressions:

$$\frac{\partial^2 N_+}{\partial E_+ \partial E_{0+}} = \int \frac{\partial^2 N_+}{\partial E_+ \partial \tau} W(E_{0+},E_+,\delta-\tau)\, d\tau =$$

$$= \frac{4\alpha Z(Z+1) N A^{-1} r_0^2}{E_\gamma - 2m_0 c^2} G(E_{0+},E_+)\, I(E_{0+},E_+),$$

where $I(E_{0+},E_+) = \int_0^\delta exp(-\frac{7}{9}\tau)\, W(E_{0+},E_+,\delta-\tau)\, d\tau$ is the probability integrated over the target.

So, the number of the positrons that comes out of the converter in the interval dE_+ around E_+, when the UR photons emitted by single high energy particle in the undulator on the harmonic n is determined by

$$\frac{dN_{+n}}{dE_+} = \int \frac{\partial^2 N_+}{\partial E_+ \partial E_{0+}} \frac{\partial^2 N_{\gamma n}}{\partial \omega\, \partial S} dE_{0+} d\omega\, ds\ .$$

This expression can be rewritten in the form

$$\frac{dN_{+n}}{dE_+} = \frac{\alpha\, p_\perp^2\, R_n(E_+,r_m,y_i,y_f,\delta)}{\hbar c \gamma \ln(183 Z^{-1/3})(1+p_\perp^2)^{1/2}}, \qquad (5)$$

where

$$R_n = \int_0^{r_m} dr \int_{s_f}^{s_i} \frac{F_n(s)}{(s(1-s))^{1/2}} ds \int_0^{E_+^{max}} G(E_0,E_+^{max})\, I(E_0,E_+^{max})\, dE_{0+}.$$

Remind here that $E_+^{max} = sE_{\gamma n}^{max} - 2\, m_0 c^2$, $s_{i,f} = s_{i,f}\ (r,y_i,y_f)$.

Now we consider the expression for polarization.

As it was mentioned, the degree of longitudinal polarization

of the positrons or electrons produced is determined by the expression [8]

$$\zeta_\parallel = \xi_2\, f(E_+,E_{0+}^{max}),$$

where ξ_2 is the degree of circular polarization of the photon, $f(E_+,E_{0+}^{max})$ is the universal function of the ratio of initial energy of the produced positrons E_{0+} to the maximal energy of the positrons $E_{0+}^{max} = E_\gamma - 2m_0c^2$. To an approximation of ~ 10%, the function

$$f(E_+,E_{0+}^{max}) \cong 1 - 2 \left(\frac{E_{0+}^{max} - E_{0+}}{E_{0+}^{max}} \right)^2, \text{ when } (E_{0+}/E_{0+}^{max}) > 0.5.$$

The value $\zeta_\parallel$ tends to 1 when $\xi_2 = 1$, and $(E_+/E_+^{max}) \sim 1$. The value $\zeta_\parallel \sim 0.9$ when $(E_{0+}^{max} - E_{0+})/E_{0+}^{max} \cong 0.22$, or $(E_{0+}/E_{0+}^{max}) \sim 0.78$. The change of the value $\zeta_\parallel$ in the process of changing of energy of positron from E_{0+} to E_+ in the material of the converter is negligible, cause the length of depolarization is about 3 X_0. This yields that the degree of the longitudinal polarization of the positrons at the end of the converter at the energy E_+ is determined by averaging of the $\zeta_\parallel(E_{0+},E_\gamma)$

$$\zeta_\parallel(E_+) = \frac{\sum_{n=1}^\infty \int_0^{r_m} dr \int_{s_f}^{s_i} \frac{\xi_{2n}(s)}{\sqrt{s(1-s)}} F_n(s) ds \int_{E_+}^{E_\gamma - 2m_0c^2} f\, G\, I \frac{dE_{0+}}{E_{0+}}}{\sum_{n=1}^\infty \int_0^{r_m} dr \int_{s_f}^{s_i} \frac{F_n(s) ds}{\sqrt{s(1-s)}} \int_{E_+}^{E_\gamma - 2m_0c^2} f\, G\, I \frac{dE_{0+}}{E_{0+}}}\ .$$

The function $F_n(s)$ determined in previous chapter ((1), page 7).

Let us approximate the functions f, G,I from the formulas of polarization and R_n from expression (5).

As it was mentioned, the function f at $E_{0+} > 0.5\ E_+^{max}$, can be represented in approximate form

$$f \cong 1 - 2 \left(\frac{sE_{\gamma n}^{max} - 2m_0c^2 - E_{0+}}{sE_{\gamma n}^{max} - 2m_0c^2} \right)^2.$$

The function G depends on the matter of the converter. For example, for the case of ^{82}Pb and energy $E_\gamma \cong 50 m_0 c^2$ it can be represented in the form

$$G \cong \begin{cases} 4.75\sqrt{1-\eta}\,, & 0.89 < \eta \le 1 \\ 1.55\,, & 0.11 \le \eta \le 0.89 \\ 4.75\sqrt{\eta}\,, & 0 \le \eta < 0.11 \end{cases}$$

where $\eta = (sE^{max}_{\gamma n} - 2m_0c^2 - E_{0+})/(sE^{max}_{\gamma n} - 2m_0c^2)$.

Now let us estimate the function $I(E_+,E_{0+})$. It can be rewritten in the form

$$I(E_{0+},E_+) = -\delta \frac{\partial}{\partial E_+} Y(E_{0+},E_+),$$

where $Y(E_{0+},E_+,\delta) = \frac{1}{\delta} \int_{E+}^{E_{0+}} I \, dE_+$.

The function Y determines the share of the positrons, produced with energy E_{0+}, that have energy in the interval (E_+,E_{0+}) at the exit of the converter. Specifically the values $Y(E_{0+}=E_+) = 0$, $Y(E_+ = 2m_0c^2) = 1$ independently on δ. For $\Delta = 0.1$: $Y(\delta \to 0) = 1$, $Y(\delta=0.1) = 0.9$, $Y(\delta = 0.2) = 0.75$, $Y(\delta=0.4) = 0.56$. It means that at thickness $\delta=0.1$ only 75% of the particles produced at the energy interval $\Delta = 0.1$ remain at this interval.

Cause the losses of energy on bremstrahlung [17], the target must be thin ($\delta \ll 1$). In this case we can get

$$exp(-\tfrac{7}{9}\tau) \cong 1, \qquad \left[\Gamma\left(\frac{\delta-\tau}{ln2}\right)\right]^{-1} \cong \left(\frac{\delta-\tau}{ln2}\right).$$

Correspondly, the value I can be presented in the form

$$I \cong \frac{\Delta^{\frac{\delta}{ln2}} ln\frac{\Delta^{\delta}}{2} + ln2}{(E_{0+}-E_+)\, ln^2\Delta}$$

where $\Delta = ln \dfrac{E_{0+}-E_+}{E_{0+}}$.

In case when $(E_{0+}-E_+)/E_{0+} \ll 1$ the value $\Delta \cong (E_{0+}-E_+)/E_{0+}$ and, correspondly, the value

$$Y\left(\frac{E_{0+}}{E_+}\right) \cong \frac{ln2}{\delta ln\Delta}\left(1-\Delta^{\frac{\delta}{ln2}}\right).$$

Notice, that for very thin converter $\delta < 0.1$ we can get the value $I\left(\frac{E_{0+}}{E_+}\right) = \delta(E_{0+} - E_+)$, and hence $Y = 1$.

The number of positrons that will come out of the converter with energy in the interval (E_+,E^{max}_+) can be represented in the

form

$$\Delta N_{+n}(E_+,E^{max}_+) = \frac{\alpha p_\perp^2 \delta}{c\gamma ln(183z^{-1/3})}\Gamma_n,$$

where $\Gamma_n = \int_{E}^{E^{max}_+} R_n\, dE_+ = \int_0^{r_m} dr \int_{s_f}^{s_i} \frac{F_n ds}{\sqrt{s(1-s)}} \int_{E_+}^{E^{max}_+} G(E_{0+},E^{max}_+)\, Y(E_{0+},E_+)\, dE_{0+}$.

The last expression defines the conversion efficiency of the initial positron (entering undulator) to the positrons produced in the media and accepted by the next accelerator in the energy interval $\Delta E_+ = E^{max}_{\gamma nm} - 2m_0c^2 - E_+$.

For very thin converter the value Γ_n becomes

$$\Gamma_n = E^{max}_{\gamma n} \int_0^{r_m} dr \int_{s_f}^{s_i} \sqrt{\frac{s}{(1-s)}}\, F_n(s)\, \bar{G}(s)\, ds,$$

where $\bar{G}(s) = \int_{\xi_1}^{1} G(\xi)d\xi$, $\xi_1 = (E_{+1}-2m_0c^2)/(sE^{max}_\gamma - 2m_0c^2)$, E_{+1} is the lower energy of the produced positron beam that is accepted by the conversion system.

In the simple case of the thin ^{82}Pb converter, monochromatic radiation ($s \simeq 1$, $1 - s \ll 1$) and wide energy interval $E^{max}_+ - E_{+1} > 0.1 E^{max}_+$, $(1 - \xi_1 > 0.1)$ the value $\Gamma_n = Q_n\bar{G}(1)/n$ and the expression can be represented in the form

$$\Delta N_{+n}(E_+ - E^{max}_+) = \frac{\alpha p_\perp^2 E^{max}_+ \; Q_n \bar{G}(1)\delta}{c\gamma n\; ln(183Z^{-1/3})}$$

(Q_n is determined by(4), page 13), $\bar{G}(1) \simeq 1.55(1 - \xi_1)$. For n = 1, $r_m = kr^* = k(1+p_\perp)^{1/2} y_f/\gamma$ according to (4), last relation can be represented in the form

$$\Delta N_{+1} \simeq \frac{2\pi\alpha k^2 K\bar{G}(1)\delta}{ln(183Z^{-1/3})} \frac{p_\perp^2}{1+p_\perp^2} \frac{y_f}{y_i}(1 - {}_1) \cong$$

$$\cong 3\; 10^{-2}\, k^2\, K\delta \frac{p_\perp^2}{1+p_\perp^2} \frac{y}{y_i}\; 1 - \xi_1).$$

For $k = 1/2$, $K = 10^4$, $\delta = 0.2$, $p_\perp = 1$, $y_i = 2y_f = 2K\lambda_0$, $\xi_1 = 0.7$ the conversion efficiency is $\Delta N_{+1} \simeq 1.1$.

For increasing the conversion efficiency it is possible to use *few* thin targets and accept secondary particles from each of them in longitudinal phase-space [16].

4. THE HELICAL UNDULATOR DESIGN.

For obtain helical magnetic field the double helix with opposite supplying currents is more convenient one [11,12,2,16].

The other types of undulators for helical field forming are: The undulators with circular polarized electromagnetic waves [18] and the Undulators with strong elliptically polarized magnetic field and undulators that form the superposition of two strong helical magnetic fields with multiple periods permit to produce more hard circular polarized undulator radiation on higher harmonics [21-23].

The main output of considerations of the field distribution in helical undulators is that the minimal wavelength λ_0 can be of the order of the aperture [11,12,16,24]. Generally it yields from the dependence of the transverse magnetic field $H_\perp$ (supposed constant in previous calculations) of the transverse coordinates, proportional to $K_{0,1}(r/\bar{\lambda}_0)$ and value of the field strength on the axis as

$$H_\perp \cong \frac{I}{\bar{\lambda}_0}\left[\frac{a}{\bar{\lambda}_0} K_0\left(\frac{a}{\bar{\lambda}_0}\right) + K_1\left(\frac{a}{\bar{\lambda}_0}\right)\right],$$

where a is the radius of the helix, I is the current, K_0 and K_1 are modified Bessel functions, which decreases exponentially with increasing the ratio $a/\bar{\lambda}_0$. So it it is necessary increase the current in the helix also *exponentially*, for saving $H_\perp$ constant.

There are few general items which define the minimal diameter of the aperture of the undulator. First - the emittance of the beam which provides the beam size in the undulator end, hence, as minimum 10 σ aperture, where σ is RMS of the beam. Second - limitations due to resistive wall instability of the beam in the chamber of the undulator [14]. Also important item for superconducting undulator is simple heating by the image currents and by radiation of the particles at the angle $\Theta > \Theta^*$, especially for the downstream parts of the undulator [16].

In [3,16] the manufacturing technology and testing results are presented for two versions of undulator unit sections, i.e. the pulsed air-core one with the period λ_0= 0.6 cm and the axis field amplitude $H_\perp$= 6.0 kG and the superconducting one with the period λ_0= 1 cm and field amplitude up to 5 kG.

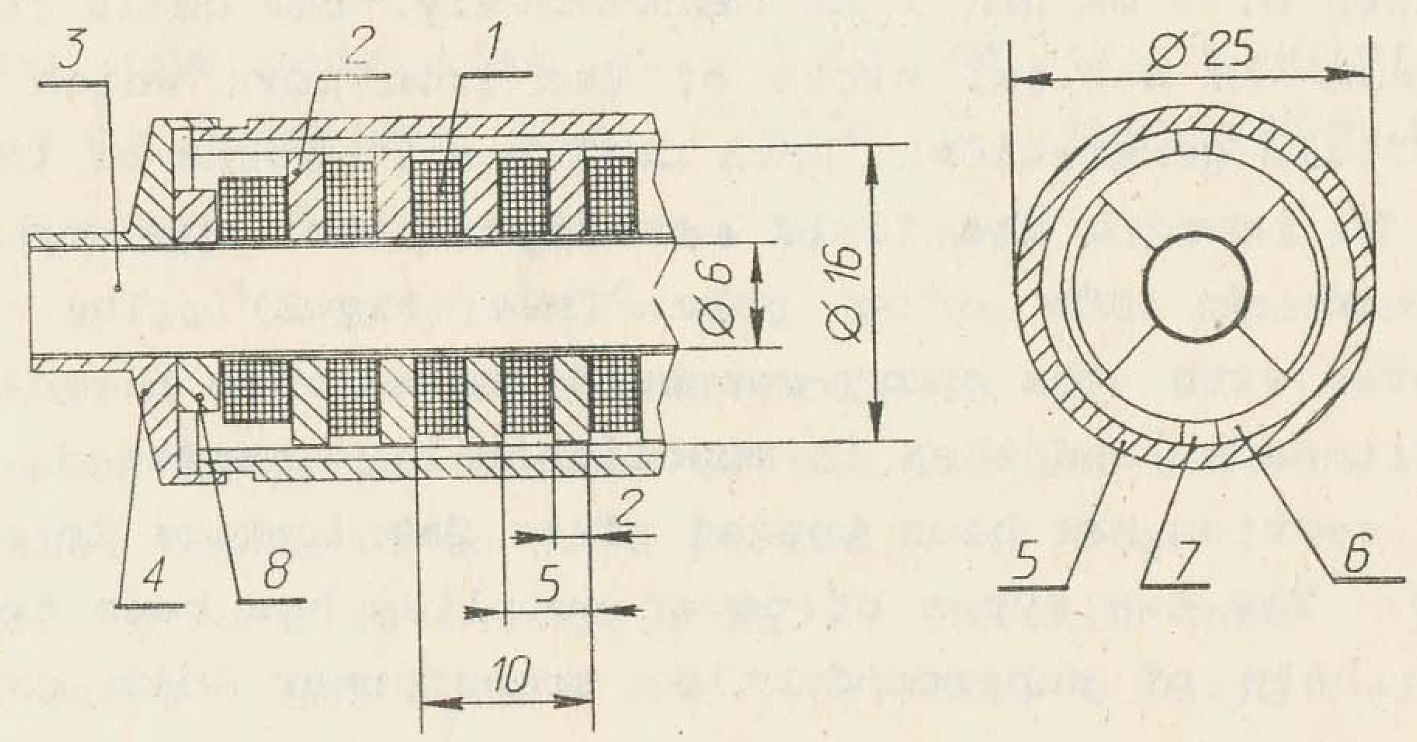

Fig.2. Super conducting undulator core design. 1-turns of the winding,2-core,3-vacuum pipe, 4-end part, 5-core assembly, 6-yoke, 7-helium pipe, 8-end screen.

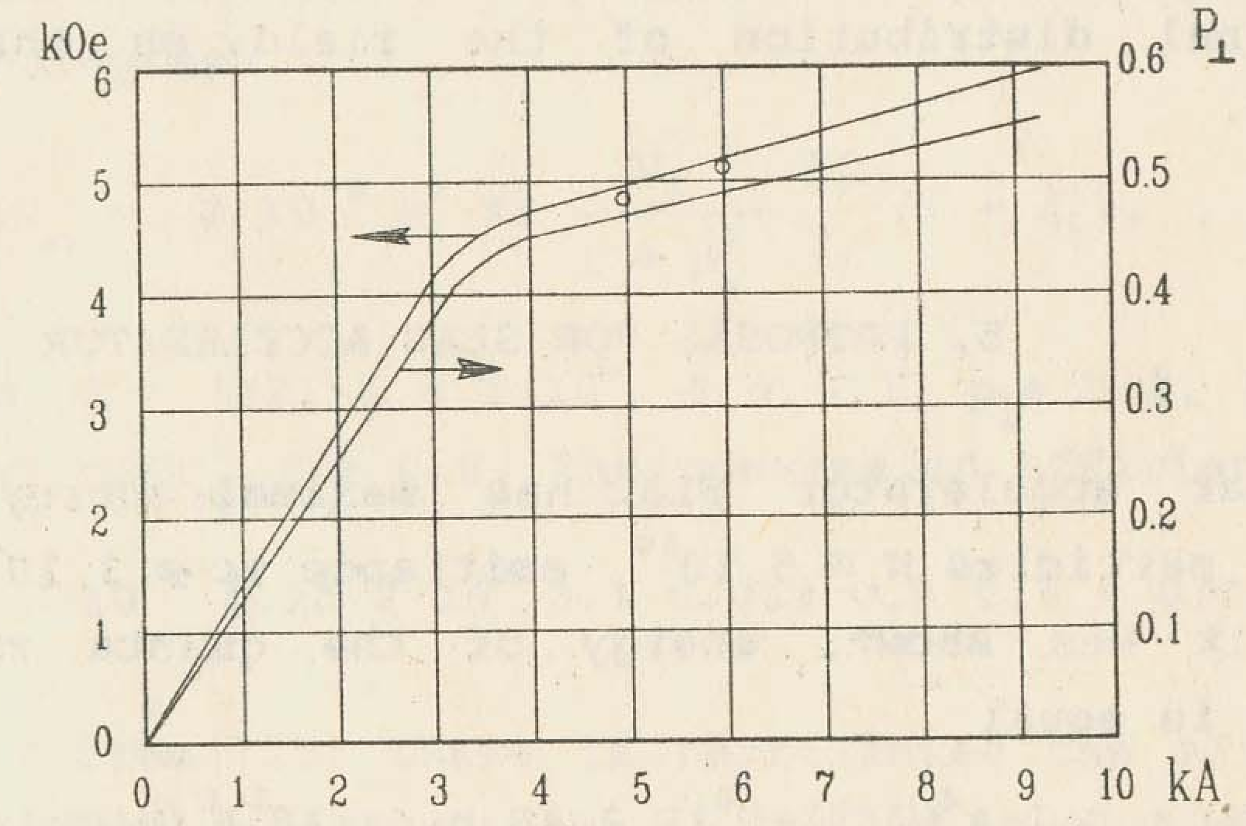

Fig.3. The calculated curve and experimental results with the super conducting undulator. On x axis - total current, on left y axis-field on the axis of the undulator, on right y axis -the $P_\perp$factor.

More attractive is the undulator with the superconducting coils. One of the most promising features of this one is that its normal operation is available irrespectively of the repetition rate of the whole complex.

The superconducting undulator testing section has been fabricated with the bore diameter 6 mm, winding radial and axial sizes of 5 mm and 3 mm respectively. The coils form a double helix wound in helical slots of the iron core which contributes to the field generation. Each coil has 15 turns of the wire 0.7 mm dia. To improve the field configuration the coils and core are closed in the iron yoke (see Fig.2). The winding is impregnated with the epoxy compound loaded with aluminum oxide or barium titanate and then is monolithically hardened.

The section has been tested after fabrication in an immersion cryostat. The two types of power supplies has been tested, usual and with help of superconducting transformer with captured flux [16].

On Fig.3 there is shown the calculated field on the axis of undulator and with points there are shown experimental results. The field amplitude of 5 kOe on the axis is attained at supply current of 400 A i.e. at 6 kA turns total. The field was measured with Hole probe in liquid helium. Here there was also measured longitudinal distribution of the field on the axis of the undulator.

5. PROPOSAL FOR SLAC ACCELERATOR

Linear accelerator SLAC has maximal energy about 50 GeV, number of particles $N \cong 5\ 10^{10}$, emittance $\gamma\varepsilon \cong 3\ 10^{-3}$ rad cm.

As it was shown, energy of the quanta radiated in the Undulator is equal

$$E_{\gamma n} = \hbar\omega = n\ 2\ \gamma^2\hbar\Omega/(1+P_\perp^2) \cong 2.48\ n\ (\gamma/10^5)^2/\lambda[cm]/(1+P_\perp^2)\ [MeV]$$

where $\Omega = 2\pi c/\lambda_0$ -is the angular frequency of transverse oscillations in the wiggler, n =1,2,...

Number of the quantum, radiated by N particles in the Undulator, described by the formula

$$N_{\gamma 1} = \Delta N_{\gamma 1}(s=0) = N\ \frac{4\pi\alpha K}{3}\ \frac{p_\perp^2}{1+p_\perp^2}\left(1 - \frac{2}{5}\ \frac{p_\perp^2}{1+p_\perp^2}\right)$$

If we suppose, that center of the wiggler installed in crossover of beta-function with significance β^*, so $\beta(s) = \beta^* + \frac{l^2}{\beta^*}$, and the beam size in the wiggler will be $a = (\gamma\varepsilon\ \beta/\gamma)^{1/2}$. If we assume, that the beam size at output of the wiggler is ≅10% bigger, than in crossover, i.e. $l^2/\beta^{*2} \cong 0.2$, that gives for $l = 2$ meters $\beta^* \cong l/(0.2)^{1/2} \cong 450$ cm. The beam size will be at energy 50 GeV, $a \cong (3\ 10^{-3}\ 450\ /10^5)^{1/2} = 3.7\ 10^{-3}$cm. The aperture of the vacuum chamber A let be ten times more i.e. $A = 3.7\ 10^{-2}$cm or 0.37 mm. So diameter of inner chamber of the wiggler will be at least 1 mm. This makes possible to have *period* λ of 2 mm , $P_\perp \cong 0.2$ and correspondly, $E_\gamma \cong 2.48\ 5 = 12.4$ MeV. In this case $P_\perp^2/(1+p_\perp^2) = 0.038$, K= 400/0.2 and $N_\gamma/N \cong 4\pi\alpha\ (400./0.2)\ 0.038/3. = 2.3$ or 2.3 quants per one initial particle for the undulator of 4 meters total length. The spectrum of radiation is good forming, cause $P_\perp$ is so small, and represented by the fist harmonic mainly. Polarization of the photon flux in angle $\theta < 1/\gamma$ is about 100%, i.e. parameter $\zeta_2 \cong 1$.

If we estimate the efficiency of conversion using the formula from previous paragraph

$$\Delta N_{+1} \simeq 3\ 10^{-2}\ k^2\ K\delta\ \frac{p_\perp^2}{1+p_\perp^2}\ \frac{y_f}{y_1}\ (1 - \xi_1),$$

with parameters $k \cong 1/2$, $K = 2\ 10^3$, $\delta = 0.1$, $p_\perp = 0.2$, $P_\perp^2/(1+p_\perp^2) = 0.038$, $y_f = 2y_1 = 2K\lambda_0$, $\xi_1 = 0.5$, the conversion efficiency will be

$$\Delta N_{+1} \simeq 3\ 10^{-2}\ 0.25\ 2\ 10^3\ 0.1\ 0.038\ 0.5\ 0.5 = 0.014$$

In Table 3 from [15] there is represented the efficiency of conversion of gammas into positrons K, calculated using numerical codes described there for gammas with $E_\gamma = 10$ MeV, angle of capture ± 0.5 rad, interval of separated energy 0.5 ($\xi_1 = 0.5$). Here also calculated mean polarization, mean energy losses ΔW, MeV per particle and accuracy of calculation ε.

Table 3.

E_γ = 10 MeV, E_+ = 7.5 ±2.5 MeV, ϕ = ± 0.5 rad

t,cm	$\delta=t/l_{X_0}$	$K/10^{-2}$	$\varepsilon/10^{-3}$	$<\zeta>$	ΔW MeV/e^+
0.01	0.028	0.34	0.54	0.78	0.007
0.025	0.07	0.74	0.78	0.79	0.05
0.05	0.14	0.95	0.87	0.79	0.26
0.10	0.28	1.14	0.98	0.79	0.51
0.15	0.43	1.04	0.98	0.786	0.48
0.20	0.57	0.93	0.87	0.79	0.56
0.25	0.71	0.77	0.80	0.792	0.61

So it is possible to obtain here for δ = 0.1

$$N_+/N = (N_+/N_\gamma)(N_\gamma/N) \cong 0.008 \; 2.3 = 0.018,$$

i.e. efficiency, close to estimated analytically, which gives $\cong 1\ 10^9$ positrons per one initial bunch. In principle here it is possible to use more thick target, up to δ = 0.3 .

6. CONCLUSION.

Of cause it was not possible to cover all problems concerning Undulators and its utilization for Conversion System in single paper. This will be done in other publications.

Undulators, mentioned here, were fabricate as a short models with necessary value of wavelength and obtained parameters shows that *technical realization* of such full scale Undulator is possible in any case.

This method of obtaining polarized particles (e^+, e^-) can be easily used in *any* Linear Collider such as CLIC, DESY/THD, JLC, TLC, TESLA, VLEPP.

Proposal for SLAC FFTB shows that it is possible to test this method experimentally.

7. REFERENCES.

1. *V.E.Balakin, A.A.Mikhailichenko*: VLEPP: The Conversion System, Proc. of the 12 Int.Conference on High Energy Accelerators, Batavia, 1983, p.127.
Conversion system for obtaining highly polarized electrons and positrons, Preprint INP 79-85, Novosibirsk 1979.
2. *A.D.Cherniakin et al.*:Development of the conversion system for VLEPP project,ibid.,p.131.
3. *T.A.Vsevolojskaya, A.A.Mikhailichenko, E.A.Perevedentsev, G.I.Silvestrov, A.D.Cherniakin*: Helical Undulator For Conversion system of the VLEPP project, Proc.of the 13 Int. Conference on High Energy Accelerators, Novosibirsk, 1989, vol.1, p.164.
4. *K.Flotmann, J.Roßach*: A high Intensity Positron Source for Linear Collider, DESY M-91-11,1991.
5. *Third International Workshop on Linear Collider (LC91)*, September 17-27, Protvino, 1991.
6. *V.B.Berestetzki et al.*: Quantum electrodynamics, Moscow,Nauka, 1980, p.131.
7. *D.E.Alferov, Yu.A.Bashmakov, K.A. Belovintsev, E.G.Bessonov, P.A.Cherenkov*: The undulator as a source of electromagnetic radiation,Particle accelerators, 1979, vol.9, p.223.
8. *V.N.Baier,V.N.Katkov,V.S.Fadin*: Radiation of the relativistic electrons, Moscow, Atomizdat,1973.
9. *N.S.Dikansky, A.A.Mikhailichenko*: Straightline Cooling System for Obtaining High Energy Beams e^+,e^- With Minimal Emittance, Preprint INP 88-9,1988.
10. *V.V.Anashin,et al.*:Prototypes for the Damping Rings and the Beam Buncher Incorporated in the VLEPP Project, Proc. of XIII Int. Conf.on High Energy Acc., Novosibirsk, 1989, vol.1, p.159.
11. *Brian M.Kincaid* : A short-period helical wiggler as an improved source of syncrotron radiation, J.Appl.Phys, vol.48, N7(2684), 1977.
12. *J.P.Blewett, R.Chasman*: Orbits and fields in the helical wiggler,ibid, p.2692.
13. *D.E.Alferov, Yu.A.Bashmakov, E.G.Bessonov*: Undulator radiation, Trudy FIAN, vol.80,p.100, M.,1975.
14. *Mikhailichenko A.A., Parkhomchuk V.V.*: Transverse resistive instability of a single bunch in Linear Collider, prep. INP 91-55, Novosibirsk, 1991.
15. *Bukin A.D., Mikhailichenko A.A.*: Optimized Target Strategy for Production of Polarized Electrons and Positrons for Linear Collider, Preprint INP 92-aa, Novosibirsk,1992.
16. *A.A.Mikhailichenko*: Conversion system for obtaining polarized electrons and positrons at high energy, Dissertation, Novosibirsk, INP, 1986.
17. *Rossi B.*: High energy particles, N/Y,1952.
18. *E.G.Bessonov*: Undulators, undulator radiation, free-electron lasers. Proc. Lebedev Phys. Inst., Ser.214, (Undulators, free-electron lasers), Ed. M.A.Markov, p.5-151, 1992.
19 *D.F.Alferov, Yu.A.Bashmakov, E.G.Bessonov*: To the theory of the the undulator radiation. Sov. Phys.-Tech. Phys., v.18, p. 1336, 1974.
20. *E.G.Bessonov*: To the theory of the undulator radiation. Preprint FIAN N-18, Moscow, 1982.

21. *E.G.Bessonov, E.B.Gaskevich*: Spontaneous and induced emission of particles at higher harmonics in an undulator with an elliptically polarized magnetic field. Soviet Physics-Lebedev Institute reports, 1995, N-7, p.16-20.
22. *V.I.Alexeev, E.G.Bessonov*: On some methods of generating circularly polarized hard undulator radiation. NIM, 1991, v.A308, p.140-141.
23. *H.Onuki, N.Saito and T.Saito*: Apply. Phys. Lett.52 (1988), p.173.
24. *A.A.Anashin et al*: Superconducting helical undulator for measurements of polarization of interacting beams in VEPP-2M storage ring, Preprint INP 84-11, Novosibirsk,1984.

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