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ABSTRACT

In the Kobayashi-Maskawa model electric dipole moments of the *W*-boson and the electron do not arise in the two-loop and three-loop approximations respectively.

The Kobayashi-Maskawa (KM) model looks now as the most natural description of CP-violation. It predicts extremely tiny magnitude of CP-odd effects in the processes without flavour changing. E.g., the neutron electric dipole moment (EDM) d_n in this model constitutes [1] (see also Ref. [2])

$$d_n/e \sim 10^{-32} \div 10^{-31} \text{ cm.} \qquad (1)$$

It is far beyond the present experimental facilities. The last experimental upper limit on the neutron EDM is [3]

$$d_n/e < 1.2 \cdot 10^{-25} \text{ cm.} \qquad (2)$$

The predictions of the KM model for dipole moments of other particles, in particular, the *W*-boson and the electron, should also lie far beyond the facilities of the modern experiment. The best of the published upper limits on the electron EDM d_e [4] is on the same level as (2):

$$d_e/e < 1.3 \cdot 10^{-25} \text{ cm.} \qquad (3)$$

Nevertheless, the calculation of dipole moments of the *W*-boson and the electron in the KM model is of certain methodological interest.

The electron EDM cannot arise via diagram of the type 1 since there the quark interaction vertices are complex conjugate to each other, and the CP-odd phase drops out from the answer. In recent paper [5] the contribution to the electron EDM was considered from diagrams of the type 2. They differ from one another by the arrangement of the quarks in the closed fermion loop. And the external photon can be connected to different quarks, as well as to the W-boson. The calculations in Ref. [5] have led to the prediction for the electron EDM on the level $d_e/e \sim 10^{-38}$ cm.

We shall demonstrate, however, that in fact the electron EDM does not arise at all to this approximation. More precisely, we shall prove that the sum of diagrams 3-9 turns to zero for the W-boson both on-mass-shell and off-mass-shell. In other words, not only the electron EDM vanishes in the three-loop approximation, but as well the W-boson EDM vanishes in the two-loop one. We follow the approach used previously in paper [6] for the proof of vanishing of the quark EDM in the two-loop approximation.

Let us start from the flavour structure of the quark loop. It CP-odd part looks as (see, e. g., Ref. [7]):

$$2i\tilde{\delta}[u(dcs-scd+scb-bcs+bcd-dcb)+c(dts-std+stb-bts+btd-dtb)+$$
$$+t(dus-sud+sub-bus+bud-dub)]. \qquad (4)$$

For the KM matrix we use the standard parametrization [8]

$$\tilde{\delta} = \sin\delta\cdot c_1c_2c_3s_1s_2s_3;$$

the letters u, d, s, c, b, t denote here the Green's functions of the corresponding quarks. It follows from expression (4), in particular, that any diagram of the type 3 should be in fact antisymmetrized in the masses m_1, m_3 of the quarks connected to the upper block of this diagram. This block can be either the mass operator or the vertex part, which depends on where the external field is connected to.

The general structure of the mass operator in the V-A theory looks by itself as

$$\hat{\Sigma} = \hat{p}\,(1+\gamma_5)f(p^2), \qquad (5)$$

the dependence of f on the mass of the internal quark is of no interest to us. This expression should be renormalized in such a way that it turns to zero if one of the quarks, incoming 3 or outgoing 1 is on-mass-shell. In other words, the renormalization should be performed in such a way that a free outgoing or ingoing particle is not mixed with another one. Such a mass operator, which turns to zero at $\hat{p} = m_1$ from the left and $\hat{p} = m_3$ from the right, equals (see [6])

$$\hat{\Sigma}=\hat{p}(1+\gamma_5)\tilde{f}(p^2)-f_{13}[\hat{p}(1-\gamma_5)-m_1(1-\gamma_5)-m_3(1+\gamma_5)]. \qquad (5a)$$

Here

$$\tilde{f}(p^2)=f(p^2)-\frac{m_1^2f_1-m_3^2f_3}{m_1^2-m_3^2},\quad f_{13}=\frac{m_1m_3(f_1-f_3)}{m_1^2-m_3^2};$$

$$f_i = f(p^2 = m_i^2), \quad i = 1,\ 3.$$

It can be checked easily that both for the nonrenormalized mass operator (5) and for the renormalized one (5a) the expression

$$(1 + \gamma_5)\ S_1(p)\ \Sigma\ (p)\ S_3\ (p)\ (1 - \gamma_5), \tag{6}$$

is symmetric under the permutation $m_1 \leftrightarrow m_3$. Here

$$S_i\ (p) = (\hat{p} - m_i)^{-1}, \quad i = 1,\ 3;$$

the projection operators $(1 \pm \gamma_5)$ originate from the external W-boson. Due to the above mentioned antisymmetrization in m_1, m_3, it means that diagrams 3-5 vanish.

The sum of diagrams 6-9 can be evidently written as

$$A_\mu (1+\gamma_5)\{e_1 S_1(p-k)\gamma_\mu S_1(p)\Sigma(p)S_3(p)+S_1(p-k)\Gamma_\mu(p-k,\ p)S_3(p)+$$
$$+e_1 S_1(p-k)\Sigma(p-k)S_3(p-k)\gamma_\mu s_3(p)\}(1-\gamma_5). \tag{7}$$

Here A_μ is the vector-potential of the external field, e_1 is the charge of the quark 1 and, naturally, of the quark 3, $\Gamma_\mu(p-k,\ p)$ is (nondiagonal in quarks) vertex part related to the mass operator $\Sigma(p)$ by the Ward identity

$$\Gamma_\mu\ (p,\ p) = -\ e_1 \frac{\partial \Sigma\ (p)}{dp_\mu}. \tag{8}$$

We are interested in fact in the structure proportional to the field strength $F_{\mu\nu} = i(A_\nu k_\mu - A_\mu k_\nu)$ and can therefore restrict to the expansion of expression (7) in k_μ up to the

first order included. Taking into account the evident identity

$$S\ (p)\ \gamma_\mu S\ (p) = -\frac{\partial S\ (p)}{dp_\mu}, \tag{9}$$

we find that the zeroth order term of the expansion in k of the curly bracket in formula (7) is reduced to

$$\frac{\partial}{\partial p_\mu}\ [(1 + \gamma_5)\ S_1(p)\ \Sigma\ (p)\ S_3(p)\ (1 - \gamma_5)]. \tag{10}$$

This expression, as it has been pointed out above, is symmetric under the permutation $m_1 \leftrightarrow m_3$ and vanishes therefore at the antisymmetrization $1 \leftrightarrow 3$.

The first term of the expansion in k is

$$A_\mu k_\nu\ (1 + \gamma_5)\ \{e_1 S_1\ (p)\ \gamma_\nu S_1(p)\ \gamma_\mu S_1(p)\ \Sigma\ (p)\ S_3\ (p)\ +$$
$$+\ S_1(p)\ \gamma_\nu S_1(p)\ \Gamma_\mu(p,\ p) S_3(p)\ -\ S_1(p) \left.\frac{\partial \Gamma_\mu}{\partial k_\nu}\right|_{k=0} S_3(p)+$$
$$+e_1 S_1(p)\gamma_\nu S_1(p)\Sigma(p)S_3(p)\gamma_\mu S_3(p) -\ e_1 S_1(p)\ \frac{\partial \Sigma(p)}{\partial p_\nu}\ S_3(p)\gamma_\mu S_3(p)+$$
$$+\ e_1 S_1(p)\ \Sigma\ (p)\ S_3(p)\ \gamma_\nu S_3(p)\ \gamma_\mu S_3(p)\}\ (1-\gamma_5). \tag{11}$$

It can be easily shown that the sum

$$(1 + \gamma_5)\ \{S_1\gamma_\nu S_1\gamma_\mu S_1\Sigma S_3\ +\ S_1\Sigma S_3\gamma_\nu S_3\gamma_\mu S_3\}\ (1 - \gamma_5) \tag{12}$$

at any, nonrenormalized or renormalized, expression for the mass operator Σ is symmetric under the permutation $m_1 \leftrightarrow m_3$ and drops out from the answer.

Then, the term with $\partial\Gamma_\mu(p-k,p)/\partial k_\nu|_{k=0}$ where we expand in k the same block to which the photon is connected directly, certainly does not require a renormalization and therefore conserves chirality. Due to it, the expression

$$F_{\mu\nu}(1+\gamma_5)S_1 \left.\frac{\partial\Gamma_\mu}{\partial k_\nu}\right|_{k=0} S_3(1-\gamma_5)$$

is symmetric in $m_1 \leftrightarrow m_3$ and as well cancels at the antisymmetrization.

Somewhat more tedious calculations are necessary to prove the symmetry in $m_1 \leftrightarrow m_3$ of the expression left

$$e_1 F_{\mu\nu}(1+\gamma_5)\left\{-S_1\gamma_\nu S_1 \frac{\partial\Sigma}{\partial p_\mu} S_3 + S_1\gamma_\nu S_1\Sigma\, S_3\gamma_\mu S_3 - \right.$$
$$\left. -S_1 \frac{\partial\Sigma}{\partial p_\nu} S_3\gamma_\mu S_3\right\}(1-\gamma_5). \qquad (13)$$

Thus the sum of diagrams 6-9 vanishes indeed. Together with it, turn to zero both the W-boson EDM in the two-loop approximation and the electron EDM in the three-loop one.

We cannot get rid of the feeling that this simple result, as well as the vanishing of the quark EDM in the two-loop approximation [6], should have a simple transparent explanation. Unfortunately, we have not been able to find it.

One can expect however that switching on the first gluon correction will induce dipole moments both for the W-boson and the electron as it is the case with the quark [7].

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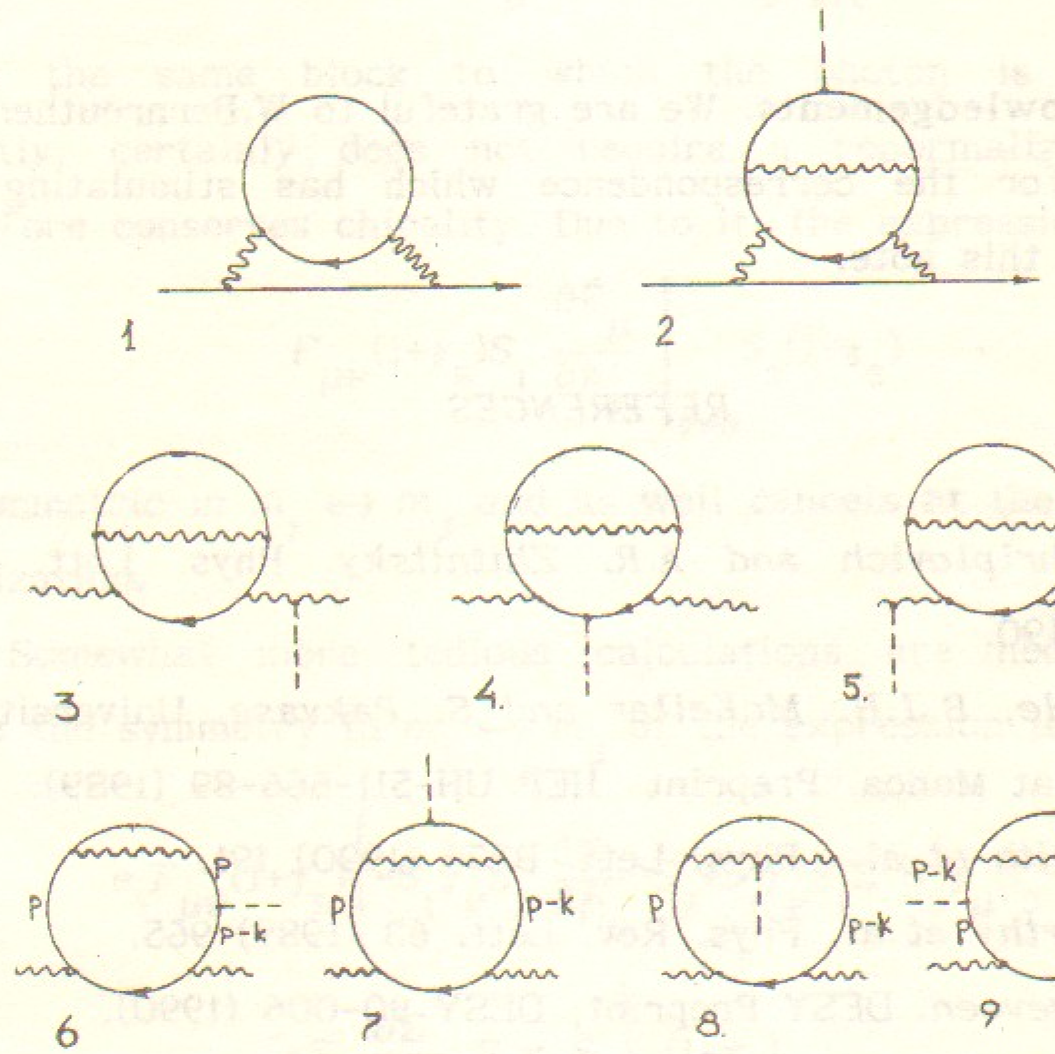

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