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V.N.Baier and Yu.F.Pinelis

QCD POTENTIAL DUE TO SMALL SIZE VACUUM FLUCTUATIONS

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DUE TO SMALL SIZE VACUUM FLUCTUATIONS

V.N.Baier and Yu.F.Pinelis
Institute of Nuclear Physics
630090, Novosibirsk-90, USSR

ABSTRACT

It is shown that the non-perturbative local, static, flavor-independent potential in QCD for heavy-quark systems of the ψ- and Υ-families is reasonable only if the typical size of vacuum fluctuations is rather small. The QCD-parameters are estimated. It is shown that the small size instantons lead to a dynamical freezing of α_s in QCD.

1. As well known now, the QCD-vacuum has a complex non-perturbative structure connected, in particular, with instantons (for the detailed consideration of instanton effects see Ref./1/). Heavy-quark systems can serve very likely as one of the best non-perturbative vacuum probes.

The interaction between heavy quark Q and heavy antiquark $\bar{Q}$ at short distances r is asymptotically free. With the growth of r the asymptotic freedom breaks not only because of the perturbative increase of the coupling constant α_s but because of the non-perturbative effects giving rise to power corrections, as was first shown in Ref./2/. The mechanism of asymptotic freedom breaking, which is connected with long-wave non-perturbative fluctuations /2/ has been considered there with use of the under-threshold sum rules (see in addition Ref./3/, where also the heavy $Q\bar{Q}$-systems have been examined by means of the method of Ref./2/ for states with different quantum numbers).

Owing to the non-relativistic nature of the $Q\bar{Q}$-states a consideration of the non-perturbative effects in these systems is substantially simplified. The effective potential of interaction $V(r)$, if its introduction is possible, is a powerful tool for the description of the non-relativistic systems. However, if we make an attempt to interpret the arising power corrections by means of $V(r)$ for large enough q^2 (and small r), the real difficulties are immediately encountered. So, the power correction to the total cross section $e^+e^- \to$ (hadrons)$_f$ (f is the flavor of the produced $Q,\bar{Q}$) turns out to be negative /4,5/: $\Delta R_f(E) < 0$, although the vacuum fluctuations(VF) are suppresed when $Q,\bar{Q}$ move away one from another.

Then one can expect that this suppression has to result in the $Q, \bar{Q}$ attraction and, therefore, in $\Delta R_{\mathcal{E}}(E) > 0$. The explanation consists in that at fairly high energies when only the lower-dimension operator $\vec{\mathcal{E}}^2$ contributes to $\Delta R_{\mathcal{E}}(E)$ ($\vec{\mathcal{E}}$ is a chromoelectric field), the interaction generated by the VF is, undoubtedly, non-potential.

This fact does not mean, however, the inconsistency of very successful phenomenological potential models (see, e.g., Ref./6/ and the literature cited there) and QCD. The success of the potential models not only in the description of the properties of resonances but in the prediction of a large number of the facts,which are experimentally confirmed, should be explained in QCD. But in the papers /7/ it has been claimed that if $\langle \frac{\alpha_s}{\pi} G^2 \rangle \neq 0$, then a non-perturbative $Q\bar{Q}$ -interaction in QCD cannot be described in terms of the potential in its usual meaning.

In the present paper we analyse those conditions under which the potential in QCD can be introduced in a consistent manner. Unlike the other existing attempts of consideration, it is shown that the potential in QCD has a sense only because of the smallness of the characteristic size*) ρ_c of vacuum fluctuations: $1/\rho_c \sim 1 GeV \gg \Lambda$.

*) The idea of a <u>second scale</u> (unlike the scale of $R_c \sim \frac{1}{\Lambda}$ - confinement radius) was introduced in QCD in the papers by Callan and Dashen and Gross and by Carlitz and Creamer /1/. The breaking of the chiral symmetry caused by instantons has been considered there to occur at the distances $r \ll R_c$. In the important papers of Carlitz and Creamer /1/ it has been shown for the first time that the generation of dynamical masses of light quarks and the breaking of chiral symmetry are due to <u>the small size instantons</u> with $\rho_c \ll \frac{1}{\Lambda}$. Later, it was also pointed in a number of papers /8,9,10,4,5/ to the existence of the second scale in QCD.

2. The principal point which makes it possible to see the emergence of the potential in QCD is taking into account the operators $\vec{\mathcal{E}} \mathcal{D}_0^{2n} \vec{\mathcal{E}}$ $(n \geq 1)$ together with the operator $\vec{\mathcal{E}}^2$. These operators form the leading infinite subsequence of non-perturbative corrections for the $Q\bar{Q}$ -systems /5,4/. Owing to these operators the difference of the Green's function $G(\vec{r}', \vec{r}; E)$ for $Q\bar{Q}$ from the perturbative Green's function

$$\Delta G(\vec{r}', \vec{r}; E) = N_c \cdot \langle \vec{r}' | \frac{1}{E - H_0^s} \hat{V} \frac{1}{E - H_0^s} | \vec{r} \rangle \qquad (1)$$

can be considered as the effect of the potential operator

$$\hat{V} = \frac{1}{N_c} \cdot \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \cdot K_{\mathcal{E}}(p_0) \vec{r} \frac{1}{p_0 - E + H_0^a} \vec{r} \qquad (2)$$

where $K_{\mathcal{E}}(p_0)$ is the Fourier transform of the gluon vacuum correlator $K(\tau)$:

$$K_{\mathcal{E}}(\tau) = \frac{4}{3} \pi \alpha_s \cdot \langle \vec{\mathcal{E}}(\tau) P e^{ig \int_0^{\tau} d\xi B_0(\xi)} \vec{\mathcal{E}}(0) P e^{ig \int_{\tau}^{0} d\xi B_0(\xi)} \rangle \qquad (3)$$

and $H_0^{s,a}$ is the projections of the Hamiltonian $H_0^{Q\bar{Q}} = \frac{\vec{p}^2}{m} + \frac{\alpha_s}{r} t_1 \cdot t_2$ (t_1, t_2 are color generators for $Q, \bar{Q}$) onto the singlet and octet states, respectively. In fact, $K_{\mathcal{E}}(p_0)$ is the energy distribution function of a non-perturbative gluon in the intermediate state $Q\bar{Q}G$,in which the interaction potential is formed in the dipole approximation. The variation scale of $K_{\mathcal{E}}(p_0)$ is $|p_0| \sim 1/\rho_c$, where ρ_c is the characteristic size of the VF. Therefore, one can neglect the operators $\vec{\mathcal{E}} \mathcal{D}_0^{2n} \vec{\mathcal{E}}$ $(n \geq 1)$ only for $Q\bar{Q}$ -systems and sizes ρ_c such that $\rho_c \gg (\omega_c^{Q\bar{Q}})^{-1}$ ($\omega_c^{Q\bar{Q}} = \frac{1}{4} \left(\frac{N_c^2 - 1}{2N_c} \right)^2 \cdot m \alpha_s^2$). This is fulfilled only for very longwave VF (if the ψ - and Υ -families are studied).

As is seen from Eq.(2), for the matrix elements of the potential operator the non-locality due to the operator of kinetic energy takes place only at very short distances:

$$|\vec{r}\,'-\vec{r}| \sim |m\cdot(p_0-E)|^{-1/2} \ll a_B \qquad (4)$$

for not too high levels ($a_B^{-1}=\frac{N_c^2-1}{4N_c}m\alpha_s, \rho_c^{-1}\sim 1\,GeV$). Outside the narrow region of non-locality, if the combination $\frac{p^2}{m}+\frac{\alpha_s}{2N_c r}-E$ in the octet propagator is neglected, we have the local, static, flavor and energy independent potential

$$V(r)=\frac{r^2}{N_c}\cdot\int_0^\infty d\tau\, K_E(\tau) \equiv \lambda\cdot r^2 \qquad (5)$$

As was argued in /4/, by virtue of the smallness of ρ_c , the corresponding VF may be represented by instantons and

$$\lambda=\frac{\pi^2}{3}\cdot\frac{8}{N_c}\cdot\int_0^\infty d\varphi\,\psi(\varphi)\cdot\int_0^\infty\frac{d\rho}{\rho^4}D(\rho) \qquad (6)$$

where $\int_0^\infty d\varphi\cdot\psi(\varphi)\approx 1.28$ /5/, $D(\rho)$ is the density of instantons.

3. The validity of formula (5) is limited from above, formally, by the inequality $r \ll \rho_c$. As will be seen below, the dipole approximation works due to geometrical reasons with an accuracy better than 15% even at $r=\rho_c$. Here, in order to find the quantities $\langle\frac{\alpha_s}{\pi}G^2\rangle$ and ρ_c from the comparison with the phenomenological potentials, we use the known result of Callan, Dashen and Gross /1,11/ for the static (both in the color octet and color singlet states) $Q\bar{Q}$ -Green's function in an approximation of the dilute gas of instantons (DGA). The consideration beyond the static approximation will be publi-

shed elsewhere.

It should be emphasized an important point: the static approximation is not applicable for the longwave mechanism /2/ of the asymptotic freedom breaking.

In the DGA the instantons generate the attractive potential

$$V_I(r)=2\int_0^\infty\frac{d\rho}{\rho}D(\rho)\cdot\Phi\left(\frac{r}{\rho}\right), \qquad (7)$$

$$\Phi(y)=\frac{2}{3}\int d^3\vec{z}\left[1-\cos\frac{\pi|\vec{z}|}{\sqrt{1+\vec{z}^{\,2}}}\cos\frac{\pi|\vec{y}-\vec{z}|}{\sqrt{1+(\vec{y}-\vec{z})^2}}+\right.$$
$$\left.+\frac{\vec{z}\cdot(\vec{y}-\vec{z})}{|\vec{z}|\cdot|\vec{y}-\vec{z}|}\sin\frac{\pi|\vec{z}|}{\sqrt{1+\vec{z}^{\,2}}}\sin\frac{\pi|\vec{y}-\vec{z}|}{\sqrt{1+(\vec{y}-\vec{z})^2}}\right] \qquad (8)$$

Just as in Ref./5/, of importance is taking into account the leading logarithmic corrections $\alpha_s^m\cdot\ln^m r$ to the Coulomb interaction, which lead to perturbative variation of α_s according to the renormalization group

$$V_{Coul}(r)=-\frac{N_c^2-1}{2N_c}\cdot\frac{2\pi}{\beta_0\ln(1/\Lambda_{Q\bar{Q}}\cdot r)}\cdot\frac{1}{r} \qquad (9)$$

$\beta_0=\frac{11}{3}N_c-\frac{2}{3}N_f$. The theoretical curve $V_{th}(r)=V_{Coul}(r)+V_I(r)$ is presented on Fig.1 and corresponds to the parameters Λ, m_b, m_c, $\langle\frac{\alpha_s}{\pi}G^2\rangle$, ρ_c , which are close to those obtained in Ref./5/ ($D(\rho)$ is taken from Ref./4/):

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle=0.12\,GeV^4,\quad \rho_c=1.6\,GeV^{-1}$$
$$\Lambda_{Q\bar{Q}}=0.24\,GeV \qquad (10)$$
$$m_b=4.86\,GeV,\quad m_c=1.47\,GeV$$

Fig.1 also presents the curves for the very successful single-parametric Richardson's potential $V_{Rich}(r)$ /12/ and $V_{Coul}(r)$ and $V_{Dip}(r)=V_{Coul}(r)+\lambda r^2$. The masses of quarks have been fitted by means of the comparison between $V_{th}(r)$ and $V_{Rich}(r)$ by the same way as the other parameters ; $m_c^{Rich}=1.49\,GeV, m_b^{Rich}=4.88\,GeV$. Note that these are close to the masses obtained in Ref./5/. It is worth noting first of all, that as has been expected, the deviations from the dipole approximation ($V_{th}(r)$ from $V_{Dip}(r)$) do not exceed 15% even at $r=\rho_c$.

The turn-down of $V_{th}(r)$ at $r\sim(0.4\,GeV)^{-1}$ is connected with the closeness of the infrared pole. It is evident that in the absense of the nonperturbative effects the perturbative growth of α_s according to the renormalization group cannot to bind $Q\bar{Q}$ and even cannot lead to the formation of Coulomb bound states (for $m=m_c$ or $m=m_b$). The role of small size instantons in our consideration reduces to smoothing the influence of the infrared pole.

A rather small exceeding of $V_{th}(r)$ over $V_{Rich}(r)$ in the region of small r is connected, in our opinion, with a number of reasons. First, the static approximation leads to the overestimation of the nonperturbative effects for lower levels:

$$\left\langle \frac{1}{P_0}\right\rangle > \left\langle \frac{1}{P_0+\frac{\vec{P}^2}{m}+\frac{\alpha_s}{2N_c r}-E}\right\rangle \qquad (11)$$

Second, as is seen from Fig.1, inclusion of the nonperturbative effects results, already at the distances $r \gtrsim (1\ GeV)^{-1}$, in that the potential becomes substantially steeper compared with $V_{Coul}(r)$ at $\Lambda_{Q\bar{Q}}=0.24\,GeV$. This can be imitated by substantially larger Λ such as, for example, in Refs./6,12/. In addition, the behaviour of $V(r)$ at short distances, $r<(2\ GeV)^{-1}$, has little influence on the spectrum of the ψ - and Υ-resonances.

The agreement of the potential $V_{th}(r)$, which takes into account the small size instantons, with the phenomenological potential (Ref./12/) in the region of $(2\,GeV)^{-1}<r<(0.4\,GeV)^{-1}$

distances is very promising and confirms the analysis of the sum rules made in Ref./5/.

4. Of much interest is the fact that such a consideration allows one not only to determine the various QCD parameters (see Eq.(10)) but to demonstrate clearly in the frameworks of QCD (in accordance with the empirical observation of Ref./9/) the dynamical freezing of the coupling constant α_s. That the α_s is indeed frozen dynamically in the region of distances, $(2\,GeV)^{-1}<r<(1\,GeV)^{-1}$, has been shown from the under-threshold sum rules for the Υ-family (see Ref./5/). Note that the determining role in this phenomena is played by small size instantons.

Fig.2 demonstrates the curve corresponding to the Coulomb potential $V_{fr}(r)$ with $\alpha_s=\alpha_s^{eff}=$ const ($\alpha_s^{eff}=0.27$)

$$V_{fr}(r)=-\frac{N_c^2-1}{2N_c}\cdot\frac{\alpha_s^{eff}}{r}+\Delta m \qquad (12)$$

($\Delta m=-0.17\,GeV$). At $(0.9\,GeV)^{-1}>r>(2.2\,GeV)^{-1}$ there is a good agreement of $V_{fr}(r)$ and $V_{th}(r)(V_{Rich}(r))$.

Let us introduce now the "frozen" coupling constant $\alpha_s^{fr}(r)$ ($\alpha_s^{fr}(r)\approx\alpha_s^{eff}$ at $(0.9\,GeV)^{-1}>r>(2.2\,GeV)^{-1}$) so that

$$V_{th}(r)=-\frac{N_c^2-1}{2N_c}\cdot\frac{\alpha_s^{fr}(r)}{r}+\Delta m \qquad (13)$$

The corresponding curve of $\alpha_s^{fr}(r)$ is depicted in Fig.3. The physical reason of the phenomenon of "freezing" of coupling constant α_s consists, likely, in that the small size instantons lead to a strong modification of the Green's function of gluon already at fairly short distances $r\sim(2\ GeV)^{-1}$. And this modification manifests itself as the appearance of the effective gluon mass at such virtualities (in case of very large virtualities "the gluon mass" decreases in the power manner (Ref./4/): $\mu_G^2(q^2) \sim \langle \frac{\alpha_s}{\pi} G^2 \rangle / q^2$; recall that the phenomenological gluon mass in Ref./9/ is $\mu_G \approx 0.8$ GeV) .

It should be emphasized that in case of a longwave VF of Ref./2/ the interaction operator is not local. Therefore, its interpretation in the form of a simple redefinition of the local coupling constant α_s (Ref./13/) has little sense.

Moreover, one can say that the empirically observed freezing of α_s in the infrared region also indicates in favour of a small characteristic size ρ_c of the VF. The very concept of a coupling constant in the infrared region may be reasonable if only ρ_c is small enough.

5. It is very interesting that Fig.2 clearly demonstates also the reason for which the small value of the gluon vacuum condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.018$ GeV^{-4} was obtained in Ref./7/. It has been supposed in this paper that α_s = const, what means the unjustified neglection of the corrections $\alpha_s^m \ln^m r$. The curve for $V_{SR}(r)$ in Fig.2 is close to $V_{th}(r)$ and $V_{Rich}(r)$ in the region of "freezing". Its deviation from $V_{Rich}(r)$ in the range of small r corresponds to not taking into account of the other levels, except for the Υ-resonance, in the fitting of Ref./7/. Moreover, deviations in the region $r \gtrsim (1 \text{ GeV})^{-1}$ were erroneously connected there with the contribution of the operator E^2 ; these are approximately one order of magnitude less than the deviations of $V_{Rich}(r)$ (and $V_{th}(r)$ as well) from $V_{Coul}(r)$ (with the corrections $\alpha_s^m \ln^m r$ taken into account). All this is in full accordance with the argumentation in Ref./5/ in the analysis of the sum rules for the Υ-family. The nontrivial test of these assertions is to fit the curve $V_{th}(r)$ by means of the potential $V_{SR}(r)$ of the form (12) in the region $r \sim (1 \text{ GeV})^{-1}$ with $\alpha_s = 0.30 \pm 0.03$ from /7/, and, with the calculated constant Δm, to determine the "b-quark mass" m_b' . In this case, the mass turns out to be equal to $m_b' = 4.78$ GeV (if $\alpha_s = 0.27$); note that $m_b' = 4.80 \pm 0.05$ GeV in Ref./7/.

It is worth noting that in Ref./2/ both the variation of α_s according to the renormalization group in the perturbative corrections and the very Coulomb "corrections" have not been taken into account, despite the fact that in the sum rules in the moments representation considered there for the ψ-family the Coulomb parameter

$$\frac{\alpha_s(v)}{v} \sim \sqrt{n} \cdot \alpha_s\left(\frac{2m_c}{\sqrt{n}}\right) \sim 1$$

at the same values of n at which the value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ was extracted.

6. It is possible to show that the potential $V_{th}(r)$ gives rise to the formation, at least, one bound state unlike $V_{Coul}(r)$ with running α_s (for the ψ- and Υ-families), i.e. small size instantons can form the observed resonances. In conclusion, we would like to note that in case of the shortwave mechanism of the asymptotic freedom breaking it has turned out to be possible to introduce, for the ψ- and Υ-families, the approximately local interaction potential. This potential is in accordance with the phenomenological potentials at the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, ρ_c of the nonperturbative QCD vacuum, close to those obtained in Ref. /5/. In addition, this potential clearly demonstrates the dynamical freezing of α_s in QCD.

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FIGURE CAPTIONS

Fig.1 Curve 1 - $V_{Rich}(r)$, curve 2 - $V_{th}(r)$, curve 3 - $V_{Coul}(r)$, curve 4 - $V_{Dip}(r)$. The corresponding parameters are indicated in Eq. (10).

Fig.2 Curve 1 - $V_{th}(r)$, curve 2 - $V_{Coul}(r)$, curve 3 - $V_{fr}(r)$, $\Lambda_{Q\bar{Q}} = 0.25\ GeV$.

Fig.3 The coupling constant α_s as a function of the distance r : curve 1 - $\alpha_s^{fr}(r)$, curve 2 is the dependence of $\alpha_s(r)$ according to the renormalization group ($\Lambda_{Q\bar{Q}} = 0.24 GeV$), the straight line $\alpha_s = \alpha_s^{eff} = 0.27$ is denoted by the number 3 .

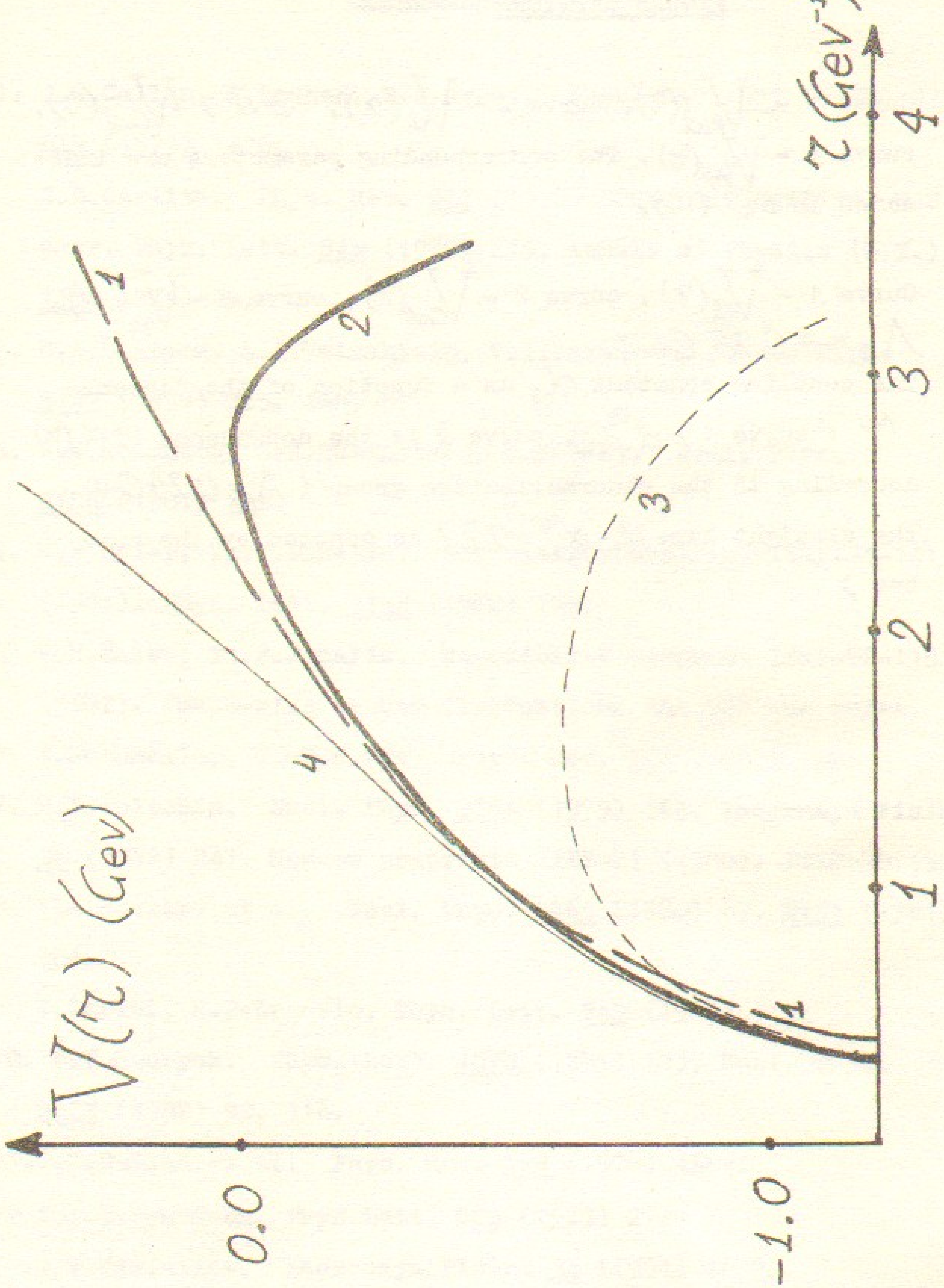

FIG.1

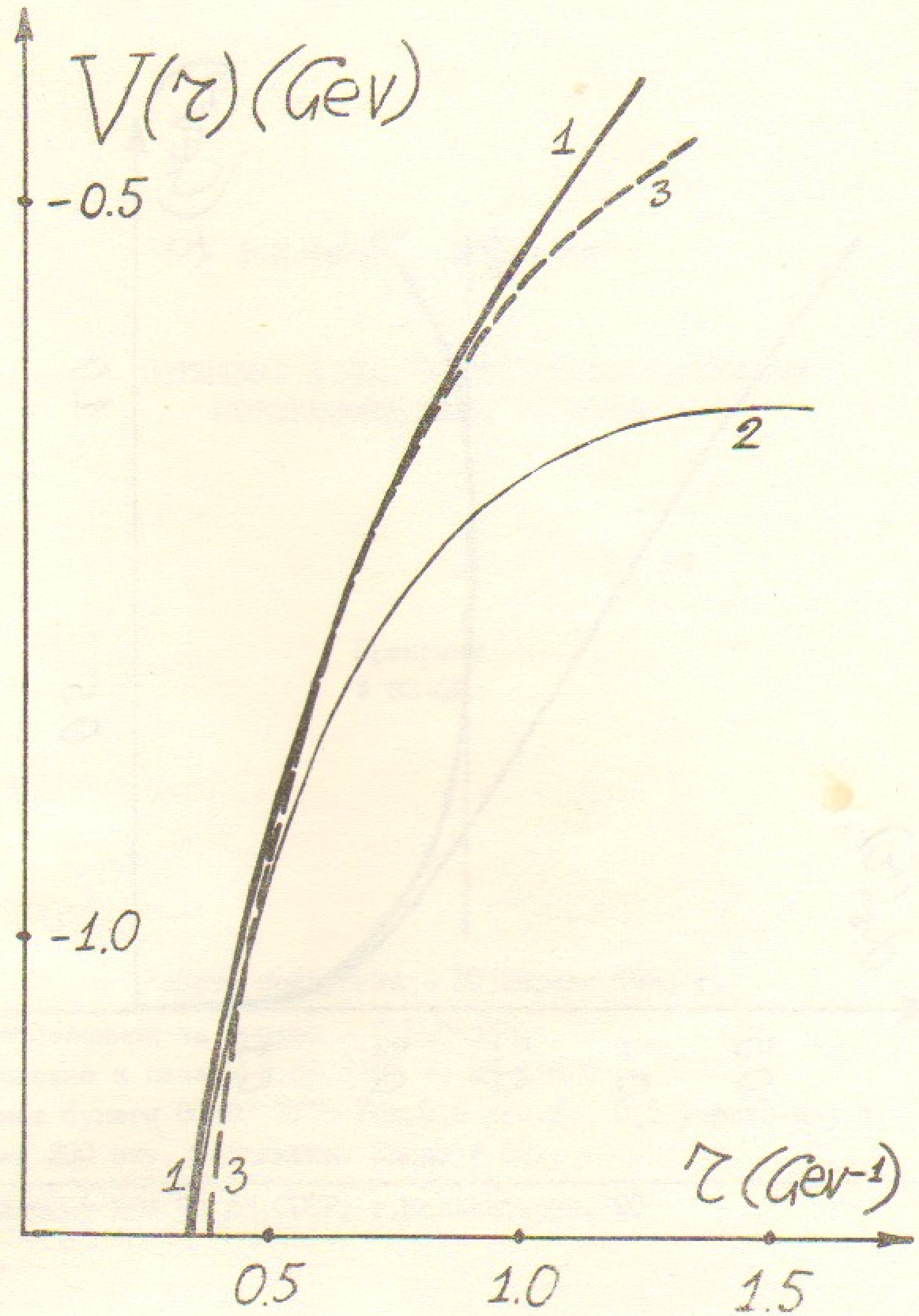

FIG.2

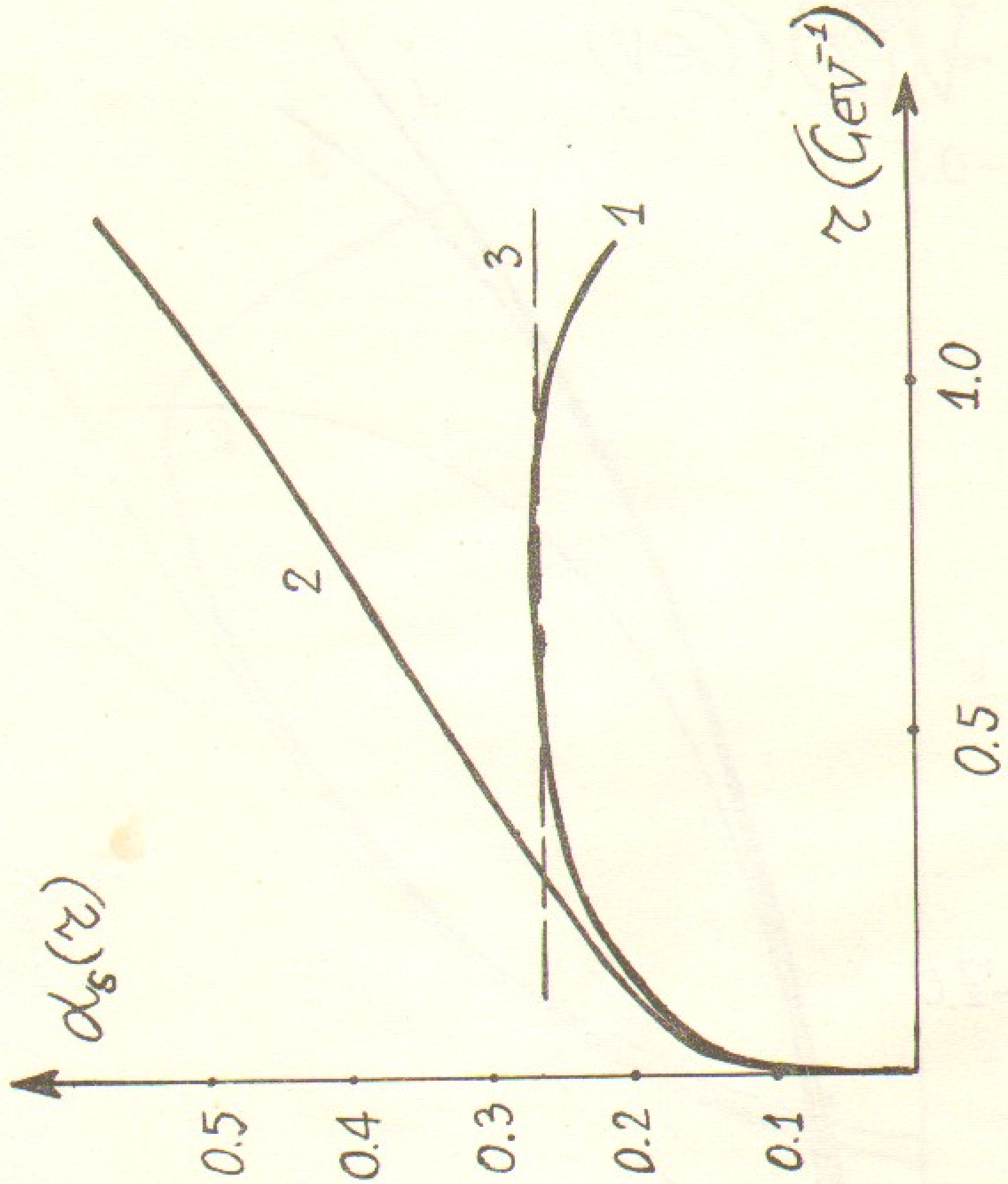

FIG.3

В.Н.Байер, Ю.Ф.Пинелис

ПОТЕНЦИАЛ В КХД, ОБУСЛОВЛЕННЫЙ ВАКУУМНЫМИ ФЛУКТУАЦИЯМИ МАЛЫХ РАЗМЕРОВ

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