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THEORY OF POWER CORRECTIONS TO DEEP-INELASTIC SCATTERING IN QUANTUM CHROMODYNAMICS II.

Q-4 EFFECTS, POLARIZED TARGET.

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THEORY OF POWER CORRECTIONS TO DEEP-INELASTIC SCATTERING IN QUANTUM CHROMODYNAMICS II. Q-4 EFFECTS, POLARIZED TARGET

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#### Abstract

The extension of our previous OPE calculations to  $Q^{-4}$  effects, connected with twist-6 operators is discussed. New type of interaction appears in this order between the struck quark and vacuum fluctuations caracterising by the nonzero vacuum average values of  $(G_{\mu\nu}^{\alpha})^2$  and  $(\overline{\psi}\psi)$ , the so called gluon and quark condensates. While the former also absent for Born diagrams, the latter is probably essential, especially for electromagnetic longitudinal structure function. We also discuss the case of the polarized target, in which twist-3 operators enter together with the leading twist-2 ones. We find OPE coefficients for them and determine power  $Q^{-2}$  corrections for polarized scattering.

## Introduction

This work is devoted to further studies of scaling violation in deep-inelastic scattering of leptons on nucleons. In our previous work [1] corrections of the order of Q<sup>-2</sup> was argued to be the main source of this phenomenon. The general consideration was made by the operator expansion method which expresses such corrections in terms of twist-4 operators averaged over the nucleon state. In this work we consider twist-6 effects in similar way, although some important modifications of our technical calculations are introduced.

But before we come to these calculations, let us discuss briefly the general features of twist-6 effects. In [1] we have shown that scattering on one and two quarks interferes, providing the main  $Q^{-2}$  corrections. Now among  $Q^{-4}$  effects there should be also diquark effects, and question may be asked whether they are numerically important. So, we start with simple order-of-magnitude estimates.

Twist-4 diquark operators of spin 2 considered in [1] were of the type

The increase in twist means two extra derivatives with one contraction of some pair of indices. They correspond to the contraction of quark momenta, of the same or of different quarks. Assuming nonrelativistic constituent quarks one get  $(\rho_1\rho_2)\sim 0.1$  Gev<sup>2</sup> which is reasonable guess. The next ingredient is the so called diquark density, much discussed

in [1]. This quantity n is the probability to find two quarks in the same point in the transverse plane in infinite momentum frame. It was estimated in [1] as  $n \sim 0.1 \text{ GeV}^2$ . With such numbers, corrections, say, to second moments should be of the order of

In our data analysis in [1] we have indeed found Q-4 correction of this order of magnitude. So, "ordinary" diquark effects can be important.

Below we discuss all y other effects which appear in Q-4 approximation, namely direct interaction of the struck quark with vacuum fluctuations, characterizing by the nonzero vacuum averages of the operators 6, 7, 4, called the gluon and quark condensates:

The role of these quantities in formation of low lying hadronic states was discussed in works [2,9]. Naive estimates of such effects like  $\langle 0/9^2 6^2/0 \rangle / Q^4$  give numbers essentially larger than diquark effects, so one may ask whether such effect is really present.

We start with the gluonic condensate, which is numerically larger. However we were able to prove that, neglected mixing of different operators under renormalization, no interaction with gluon condensate in deep- inelastic scattering is present. This proof needs particular formalism to

nal field. The most suitable (for this problem) gauge is X\_A\_n = =0, first proposed by Schwinger [3] but then rediscovered in important work [4]. We hope that this gauge will find more applications in different problems connected with hard processes. As an example of utility of the method developed we also calculate correction to polarization operator in external field. Although known from [2], this result is now obtained in much more simple way. This method also allows for simple determination of OPE coefficients in twist-6 case. As an an allows and example, we present such calculations for Gross-Llewellyn-Smith sum rules. Then we give the proof of the absence of interaction with gluon condensate for all moments.

Next topic is the interaction with quark condensate. In this case rather large coefficients are found. Moreover, they seem to be in intriguing correlation with MIT-SLAC data on  $6_{\rm L}/6_{\rm T}$  for the effect predicted is largest just for the interaction with longitudinally polarized photon, but not for W-boson, or transverse photon.

At the end of this work we turn to the scattering of the polarized leptons on the polarized target. In this case twist 2 and 3 effects come together in leading approximation in Q<sup>-2</sup>. There exist widespread mistake in the literature for twist-3 part: free quark states are used for the calculation of OPE coefficients, renormalization etc. However, these operators have zero averages over these states, so only introduction of gluons makes the calculation meaningful. We have found the corresponding coefficients in section 6. Power corrections of Q<sup>-2</sup> order are also briefly discussed, they are analogous to those discussed in [1] for F<sub>3</sub> structure function.

# 2. Green function at small distance

In the work [1] we have systematically used formal representation for the propagators in external field, which for massless quarks look as follows

he developed, see ely the dilectedition of properties in

Such representation for propagators has been used for construction of some amplitude (say, for Compton effect on quark), and then it was expanded in  $\mathbb{P}/J$ . Now we are going to make the same expansion for the propagator itself.

There is obvious difficulty on this way: unlike the amplitude, propagator is not gauge invariant quantity. The most convenient gauge is the following one

$$x_{\mu}A_{\mu}=0$$

which has several nice properties. First, there exists simple expression of the potential in terms of the field in this gauge [3,4]

$$= \frac{1}{\sum_{i=0}^{N} \frac{1}{K'(N+2)}} \times_{V} \times_{d_{\mathbf{1}}} \dots \times_{d_{\mathbf{K}}} \frac{1}{2} \cdot X_{d_{\mathbf{K}}} \cdot X$$

Note, that from (6) it is evident that  $A_{\mu}(0)$  O. Moreover, as it is pro-ved in details in the work [5], all ordinary derivatives in the expansion (6) can be turned into covariant ones. This statement can easily be checked for first terms

Gmv Quanticular gauge (5), looks as gauge covariant one. This fact becomes useful if we really calculate some invariant quantity, for then the result is obtained in its final form. Examples of the kind will be given later.

Now, let us calculate the propagator itself. For simplicity we start with scalar quark with propagator

$$\mathcal{D}(q) = \int d^4x \langle x | \frac{1}{(\rho + q)^2} | 0 \rangle$$
 (7)

Expanding in P/Q one has expressions of the type  $\langle x|P_{M_1}...$  ...  $P_{M_n}|0\rangle$ . Note that  $P_M=P_M+\frac{9}{2}t^AA^A$  and the action of  $P_M$  to the left and  $A_M$  to the right give zero

$$\int dx \langle x | \rho_{M} ... = 0 ; ... A_{M} | 0 \rangle = 0.$$
 (8)

The first fact follows from the definition

and the second one from  $A_{\mu}(\sigma) = 0$ . So, moving all  $p_{\mu}$  to the left and all  $A_{\mu}$  to the right we have finally only commutate ors  $[p_{\mu}A_{\nu}], [p_{\mu}[p_{\nu}A_{\lambda}]]$  which are derivatives of  $A_{\mu}$  at x=0. Such derivatives are easily found from (6). For example, starting with

$$\frac{1}{(p+q)^2} = \frac{1}{2^2} - \frac{2pq}{2^4} + \frac{4(pq)^2}{2^6} - \frac{p^2}{2^4} + \frac{1}{(10)}$$

$$+ \frac{g^{2}(2gq) + (2gq)g^{2}}{g^{6}} - \frac{g(gq)^{3}}{g^{2}} + O(g^{4}) (10)$$

one find that nonzero contribution appears only due to

$$\int dx (x | \mathcal{P}^{i} \mathcal{P}_{\alpha} | 0) = \frac{4}{6} \mathcal{Q}_{\beta} \mathcal{G}_{\beta \alpha}$$

$$\int dx (x | \mathcal{P}_{\alpha} \mathcal{P}^{i} | 0) = -\frac{4}{3} \mathcal{Q}_{\beta} \mathcal{G}_{\beta \alpha}$$

$$(11)$$

Further calculations along this line give

$$\mathcal{D}(q) = \frac{1}{q^2} - \frac{9}{396} (\mathcal{D}_{\sigma}(apqp) - \frac{9^2}{298})$$
(12)
$$[q_{\sigma}G_{\alpha p}G_{\rho \delta}Q_{\chi} + \frac{1}{4}G_{\gamma}^2] - \frac{i9}{98}(92)(\mathcal{D}_{\alpha}G_{\alpha p}Q_{\beta}) + \dots$$

In the case of real spin-1/2 quarks the result is more lengthy due to spin interaction with the external field

$$\frac{1}{2}(q) = \frac{1}{q} = \frac{2}{2}q_4 2\alpha G_{\alpha\beta}/\beta/5 + \frac{2}{3}q_6 \left[ q^2 D_{\alpha} G_{\alpha\beta}/\beta - \frac{1}{2} Q_{\alpha} G_{\alpha\beta}/\beta - \frac{1}{2}$$

where  $G_{\mu\nu} = \frac{1}{2} E_{\mu\nu} \gamma_5 G_{\nu} \gamma_5$  and  $G_{\mu\nu} = G_{\mu\nu}^{\alpha} t^{\alpha}$ . Similar calculations for gluons give  $D_{\mu\nu} = \int dx \langle x | \frac{1}{(q+p)^2} g_{\mu\nu} - 2g G_{\mu\nu} \rangle = \frac{g_{\mu\nu}}{q^2} + \frac{2g}{q^4} G_{\mu\nu} + \frac{4ig}{q^6} (q D) G_{\mu\nu} - \frac{2ig}{3g^6} g_{\mu\nu} D_{\alpha} G_{\alpha\beta} q_{\beta} + \frac{2g}{q^8} (q D) D_{\alpha} G_{\alpha\beta} q_{\beta} q_{\beta} + \frac{2g}{q^8} (q D) D_{\alpha} G_{\alpha\beta} q_{\beta} g_{\mu\nu} + \frac{2g}{q^8} (q^2 D^2 G_{\mu\nu} - 4(q D)^2 G_{\mu\nu}) + \frac{2g}{q^8} g_{\mu\nu} (q^2 G_{\alpha\beta} - 4(q D)^2 G_{\mu\nu}) + \frac{2g}{q^8} g_{\mu\nu} (q^2 G_{\alpha\beta} - 4(q D)^2 G_{\alpha\beta})^2 + \frac{4g^2}{q^8} G_{\alpha\beta} G_{\alpha\nu}$ (14)

where the field is also matrix in color space

$$G_{\alpha\beta}^{ab} = f^{acb}G_{\alpha\beta}^{c}$$
 (15)

Now, let us assume that the external field is that due to quantum fluctuations in QCD vacuum and try to average over them. Obviously, only scalars should have nonzero average value, with our accuracy only  $(G_{\mu\nu}^{\mathcal{A}})^2$ . Then it is interesting that both propagators for spin-0 and spin-1/2 particles receive no contribution of this type, while that for spin-1 gluon has the nonzero interaction with  $\langle G_{\mu\nu}^{\mathcal{A}} \rangle$  already at this level. The related statement has been proved in [4] for self-dual field:  $\log(x^2)$  singularity is absent for spin 0 and 1/2 cases but present for spin-1 case. We return to discussion of this point in the next section.

Now let us demonstrate the utility of these considerations by the calculation of the 62 correction to the polariza tion operator in external field;

$$\mathcal{T}_{\mu\nu}(x,0)=i\mathcal{T}_{\nu}\left[\gamma_{\mu}S(x,0)\gamma_{\nu}S(0,x)\right],$$

$$S(0,x)=\gamma_{0}S^{\prime}(-x,0)\gamma_{0}$$
The imaginary part of this quantity is connected with etc.

annihilation into hadrons.

It is sufficient to retain only two first term in the propagator S(x,c)= <x/1/0010>,

because next terms do not contain 62 effect under consideration. Substituting (17) into (16) one obtains

This result coinsides with the result found in the work  $\lceil 2 \rceil$ by more complicated diagrammatic method, Other examples of calculations with the propagators found above are given in next sections.

## 3. Q-4 corrections to first moments

Using the results obtained above we are able to find OPE coefficients for twist-6 operators in much more simple way than by the general method used in [1] . Let us start with the discussion of singularities of Compton amplitude in coordinate representation

The most singular part is connected with free propagator  $S^{(0)} = i \hat{\chi} / 2\pi^2 \times 4$ . The wave function  $\psi(x)$  can also be expanded in powers of Xx

Therefore, part of the amplitude with  $x^{-4}$  singularity is connected with leading twist operators. Note however that one should in fact subtract traces out of them, which produces extra terms with X -2 singularity. Such terms also come from the propagator. Separately they are not gauge invariant, but together they produce twist-4 effect. We have checked that the results coincide with those found in [1].

The important point is that next singularity of the amplitude, corresponding to twist-6 effects under consideration, is  $\log x^2$  . If one takes double trace of  $\psi(x)$  expansion he has x4 which cansels the singularity x4 but does not produce log x2. Therefore, it comes entirely from the propagator. This observation makes the use of the propagator expansion very simple for twist-6 terms.

Since we need only logarithmic terms we may use the following relations

and obtain the simplified logarithmic part of the propagator  $S(q)^{[log]} = 2 \mathcal{Q}_{\alpha} \mathcal{G}_{\alpha\beta} \mathcal{Y}_{\beta} + 2 \mathcal{G}_{\alpha\beta} \mathcal{Y}_{\beta} \mathcal{Y}_{\beta$ 

Hote that this expression vanishes both for vacuum averaging as well as for selfdual field  $G_{1,3} = \pm i G_{2,3}$ . The latter fact was proved in the work [4]. In this order two other structures  $[G_{2,p}, G_{p,3}]_{-}$ ,  $[G_{2,p}, G_{p,3}]_{+}$  are possible but do not enter (20), for they do not vanish for selfdual field.

Substituting (20) into the amplitude and extracting spin-1 operators ( see section 2.1 of  $\begin{bmatrix} 1 \end{bmatrix}$  ) we obtain the following twist-6 corrections to first moments

 $\Delta M_{2}^{V,NS} = -\frac{g^{2}}{360^{4}} < C \left[ 7i dt^{9}_{B}(t+y_{5})u^{9} \right] \\ \times D_{A} \left[ \overline{u}t^{9}_{B}(t+y_{5})d \right] + \left( \overline{u}i^{9}_{A}y^{9} u + \overline{d}t^{9}_{A}y^{9} \right) \cdot \overline{u}t^{9}_{A}(y_{p}i \overline{D}_{q}^{2} + y_{5})d \right] \\ + \left( \overline{u}i^{9}_{A}y^{9} + 2 \int_{a}^{a} \frac{g^{9}_{A}}{g^{9}_{A}} \cdot \overline{G}_{p}^{6} - \overline{G}_{p}^{a} \cdot \overline{G}_{p}^{8} \right) \overline{u}t^{9}_{A}u^{4}$   $+ 2 \left( G_{A}^{9} \cdot G_{p}^{8} - \frac{1}{4}g_{AB} \cdot G_{p}^{3} \cdot G_{p}^{8} \right) \overline{u}t^{9}_{A}t^{8}_{A}y^{9}_{A}u^{4}$   $- \left[ u \leftrightarrow d \right] > >$ 

 $\Delta M_{3}^{V,S} = \frac{g^{2}}{360^{4}} < < [at^{a}_{\beta}(1+\gamma_{5})u \cdot ut^{a}(13\gamma_{5}iD_{a}) - 5\gamma_{a}iD_{\beta})(1+\gamma_{5})d - (\bar{u}t^{a}_{\gamma}u + \bar{d}t^{a}_{\gamma}ad)\cdot \bar{u}t^{a}(\gamma_{5}iD_{a}) - 3\gamma_{a}iD_{\beta})(1+\gamma_{5})d - (\bar{u}t^{a}_{\gamma}u + \bar{d}t^{a}_{\gamma}ad)\cdot \bar{u}t^{a}(\gamma_{5}iD_{a}) - 3\gamma_{a}iD_{\beta})u + 2\rho^{abc}(G_{a\rho}G_{\rho\beta}$ 

Here << 0>> is reduced value of operator O, <N/ONN>=2p, <0>> and # t /2 D # = # t /2 (D #) (D #) t /2 # . The first term in r.h.s. of eqs. (21) is the contribution of diagrams where currents act on different quarks.

Note, that further calculations of twist-6 effects for next moments need next terms in the 1/Q expansion of the propagator.

charmonium theory. Its numerical value is rather large

The nonzero vacuum average of the operator (64) was introduced and estimated in the work [2] in the dispersion

<0192(6xx)210> = 0.56eV4 (22)

so one shuld first look for effects proportional to this quantity. In the corrections found above (21) we find no such contribution if the terms bilinear in G, are averaged in order to produce the scalar (22). This statement can be generalized to all moments.

Let us consider the operator of spin-n

for which the average over the nucleon state can be approxi-

$$\langle N | G_{\lambda_1} \dots \lambda_n | N \rangle \simeq$$

$$\simeq \langle o | g^2 (G_{\mu\nu}^2)^2 | o \times N | i^{n-1} \Psi Y_{\lambda_1} \dots \mathcal{D}_{\lambda_n} \Psi | N \rangle. \tag{24}$$

All other operators of relevant dimension can be determined in such way that no average of the type (24) be possible for them, say contain traceless combination  $G_{MA}G_{VA} - \frac{4}{4}g_{\mu\nu}G^2$ . We are going to prove that the operator  $G_{VA}$  has zero coefficient in the OPE.

The simplest way is based again on the property [4] that no logarithmic singularity of the propagator is present in selfdual field. Since (23) is the only operator in which two field strength are in state  $J^p=0^+$  and it is nonvanishing in such field, its coefficient should be zero. Another proof is more lengthy and it is the direct extension of the calculations made above for higher spins: the operator (23) does not appear automatically. We only note that in this case further terms in propagator are needed, but they produce only expressions such as

T(D. D[26 of 6 y By - = = 9 x 6 5 6 of ) y 4 (25)

which have no average values of the type (24). Note also that

eq. (24) implies low normalisation point  $\mu$  while our theorem refers to  $\mu \approx Q$ . The transition between these normalization points cause operator mixing due to renormalization. However, as we have seen for similar cases, such mixing is rather small numerically (see section 2.3 of [1]). Moreover, the coefficients for quark-gluon operators of twist-6 are also rather small.

Therefore we may conclude that the interaction with gluon condensate is absent on the level of Born diagrams and presumably small with all perturbative corrections.

#### 5. Interaction with quark condensate

The idea of spontaneously broken (in physical vacuum) chiral symmetry of strong interaction is well known [7] a Its consequence is the nonzero average value of the operator  $\mathcal{F}\varphi$ 

$$201\overline{u}\,u\,107 = 201\overline{d}\,d\,107 = -\frac{m_{\pi}^2 f_{\pi}^2}{2(m_{\pi} + m_{\theta})} \simeq (26)$$
  
=  $-(0.24 \,GeV)^3$ 

Note that in the exact massless limit this quantity remains fixed because  $m_q^2 = O(m_q^2)$ ,  $f_0 = O(m_q^2)$ . The following dimensional parameter then appears in the problem

$$\lim_{M_2 \to 0} \left( \frac{m_{\overline{1}}^2}{m_u + m_d} \right) \simeq 1.8 \text{ GeV} \qquad (27)$$

It is very large, although it is related to pion. So, effects proportional to (26) are in some sense large as well.

The interaction with the quark condensate has been shown to play very important role in the problem of hadronic masses

see [2, 9] and low energy e<sup>+</sup>e<sup>-</sup> annihilation [8], so it is natural to assume its importance in deep-inelastic scattering as well.

The physics of the effect we are going to discuss is similar to diquark effects cosidered in [1], the interference of scattering on one and two quarks in the nucleon, the latter being connected by hard gluon exchange. The novel feature now is that one of the quarks involved in fact from the vacuum condensate (26). Because vacuum is colorless, direct emission of gluon from the condensate is forbidden, and only exchange interaction between "physical" and "condensate" quarks is possible. It is easy to see that such effect is absent in operators of the type  $(T)_{\mu} \psi_{\nu}(T)_{\nu} \psi_{\nu}$  discussed in [1]: extracting  $(\psi)_{\nu}$  we obtain only such Lorentz structures as  $g_{\mu\nu}$  and  $g_{\mu\nu}$ , but they does not correspond to Lorentz spin 2. It is also easy to write down the operator Q which in simplest form contain the effect under consideration

$$Q_{\alpha_1 \dots \alpha_n} = i^n (\overline{\psi} \psi) \overline{\psi} \mathcal{D}_{\alpha_1 \dots} \mathcal{D}_{\alpha_n} \psi \qquad (28)$$

with the following approximate way for estimates of its average value:

The second factor is similar to the leadind twist operator, but in fact it has completely different chiral properties and at the moment we know no method to estimate reliably its

average value. It can also be shown that for free quarks it is zero and is present only due to color field. The spin-1 operator Follows

$$\overline{Y} \mathcal{D}_{\lambda} \mathcal{D}_{\beta} \Psi = \frac{1}{2} \overline{\Psi} \mathcal{J}_{\lambda} \hat{\mathcal{D}} \mathcal{D}_{\beta} \Psi =$$

$$= -\frac{i9}{4} \overline{\Psi} 6_{\lambda 6} G_{6\beta}^{6} E_{\Psi}^{6}$$
(30)

Now the problem is to calculate the coefficients for the operators (28). As for any diquark contributions, there are two classes of diagrams: those in which photons ( or W ) interact with the same or different quark lines. The latter one is simpler, so we begin with it.

As far as the effect we look for is proportional to gluon field, the natural technique is again formal discussion of deep-inelastic scattering in some arbitrary external field by Schwinger formalism. It give for four diagrams in question the following formal expression for the amplitude:

$$A_{\mu\nu} = \left(-\frac{q^{2}}{4}\right) \int d^{4}x \left(x\right) \left(\overline{\Psi} \mathcal{F}_{\nu} \frac{1}{\hat{\rho}+\hat{q}} \mathcal{F}_{\lambda} t^{a} \Psi\right).$$

$$\cdot \left(\frac{1}{(\mathcal{P}+\hat{q})^{2}-2G}\right)_{\lambda\lambda'}^{aq'} \left[\left(\overline{\Psi} \mathcal{F}_{\lambda'} t^{a'} \frac{1}{\hat{\rho}+\hat{q}} \mathcal{F}_{\nu} \Psi\right) + \left(\overline{\Psi} \mathcal{F}_{\lambda'} t^{a'} \frac{1}{\hat{\rho}-\hat{q}} \mathcal{F}_{\nu} \Psi\right)_{c}\right] |0\rangle$$

$$+ \left(\overline{\Psi} \mathcal{F}_{\lambda'} t^{a'} \frac{1}{\hat{\rho}-\hat{q}} \mathcal{F}_{\nu} \Psi\right)_{c} |0\rangle$$

Here "c" near brackets mean charge conjugation of all operators in it, corresponding to propagation of the antiparticle. Ve have used this relation in [1] only in zero order in operator  $\mathcal{P}_{\mathcal{M}}$ , so this complication was not needed there. Now we are going to calculate effects of the second order in  $\mathcal{P}_{\mathcal{M}}$ .

The simplest term is due to explicit in the dominator of gluon propagator (it correspond the interaction of external field with gluon magnetic moment). Expanding in this one find the following contribution to the amplitude

It is natural now to extract quark condensate on this stage by substitution

for all quark pairs. Two different ways of such pairing are present, and they cancel each other, as can be seen from the following calculation

This result does not contain spin-2 part we look for.

Calculations of other terms one may begin with pairing (33) in the original expression (31). However, this is wrong way, as can be seen from the following consideration. For example, one has the combination ...  $(D \forall \forall \forall \dots)$ , which is zero due to equations of motion  $(D \forall \forall \exists 0)$ , but after the

pairing (33) wrong nozero result can appear. The physics of this point is rather simple: external field polarizes the condensate, and it is not proper to consider as some x-independent number. Only after the explicite account for appropriate power of gluon field  $G_{\mu\nu}^{\alpha}$  such pairing becomes possible, as it was demonstrated above. This observation shows that in general no simplification of the type of (33) is possible.

However, there exists approximate way of doing this simplification using the number of colors N<sub>c</sub> as large parameter. Let our external field has nonzero elements only for definite pair of quark color indices. The condensate contains all colors and at N<sub>c</sub> going to infinity the condensate polarization becomes small. On more formal level, there are two types of color matrices entering (31), namely

$$t^{a}t^{a}t^{a}t^{b}=(2N_{c}-\frac{2}{N_{c}})t^{b}; t^{a}t^{b}t^{a}=-\frac{2}{N_{c}}t^{b}$$
 (35)

and only the latter case is connected with condensate polarization effect. At large N  $_{\rm c}$  we may drop it, with quite sufficient accuracy N  $_{\rm c}^{-2}$  .

The remaining step is the extraction of spin-2 part from reducible rank-4 tensor. Similar but much more complicated cases are discussed in [1] and we only give the result:

$$\frac{1}{4} \frac{1}{4} \frac{1$$

With this relation the final expression for the amplitude can be found to be

$$A_{\mu\nu} = \frac{4g^2}{g^6} (Q_{\mu\nu} Q^2 - Q_{\mu} Q_{\lambda} Q_{\mu\nu} - Q_{\nu} Q_{\lambda} Q_{\mu\mu} + (37)$$

$$+ g_{\mu\nu} Q_{\lambda} Q_{\mu} Q_{\lambda\beta}) - \frac{12g^2}{g^6} (g_{\mu\nu} - g_{\mu} Q_{\nu} / Q^2) Q_{\lambda} Q_{\mu} Q_{\lambda\beta}$$

The calculation of another type of diagrams, with both currents on the same quark line, can in principle be made with the method discussed in sections 2,3 above. In this case we have to calculate terms in propagator of the order of  $\mathcal{DGG}$  which is rather combersome. The main point is however not technical difficulties, but the absence of unique definition of the diquark operators. As discussed also in [1], they are introduced by the equation of motion

$$\mathcal{D}_{\mu} G^{\alpha}_{\mu\nu} = -\frac{9}{2} \overline{\Psi} \delta_{\nu} t^{\alpha} \Upsilon. \tag{38}$$

but in fact the left hand side can be rewritten in other ways as well. For example, consider the identity

$$\mathcal{D}^{2}G_{\alpha\beta} + \mathcal{D}_{\beta}\mathcal{D}_{\alpha}G_{\beta\beta} + \mathcal{D}_{\beta}\mathcal{D}_{\beta}G_{\beta\alpha} = 0$$
 (39)

between diquark and gluon operators. So, an attempt to separate diquark effects from quark-gluon ones becomes more and more hopeless with increasing twist.

The reader may now say that we can not define what we are looking for in this section. In principle, it is really so, but, very fortunately, the contribution calculated above turns out to have rather large coefficients while the troublemaking contributions of another type give effects several times smaller. It gives at least approximate sense to our calculati-

ons of the interaction with quark condensate. Similar phenomenon can be observed in section 3, where four-leg diagram contribution is larger than two-leg one.

Therefore we give our results in such form

$$\frac{18}{2} \Delta M_{L}(2, G^{2}) = \frac{(6\pm 1)}{Q^{4}} \ll G_{0,3} ; \frac{18}{5} \Delta M_{2}^{e, S} = \frac{(2\cdot 1)}{Q^{4}} \ll G_{0,3} ; \frac{18}{5} \Delta M_{2}^{e, S} = \frac{(40)}{Q^{4}} \ll G_{0,3} ;$$

where #1 means the uncertainty depending on the convention on the separation between diquark and quark-gluon operators.

Essential that such contribution of the four-leg diagram is absent for neutrino scattering because all quarks should be left-handed in this case and one can not compose condensate pair out of them. So, in contrast with Q<sup>-2</sup> effects, twist-6 operators contribute in qualitatively different way to electron and neutrino scattering. This conclusion deserves experimental test.

It is very interesting that the effect in question is maximal just for the longitudinal structure function, where large and so far unexplaned effect was observed in SLAC (see for more detailed discussion [1]), of the order of 1 Gev<sup>4</sup>/Q<sup>4</sup>. So large effect is evidenly absent in other cases. Our result (40) provides possible rational explanation for this, in contast to simple diquark effects considered in [1] and estimated by eq.(2).

Concluding this section we may say, that the interaction with quark condensate is good candidate to explain the long-standing  $\sigma_L/\sigma_T$  puzzle. Much better data are needed, and large difference between electron and neutrino scattering is predicted.

#### 6. Polarized target

In this section we discuss effects of higher twist operators for deep-inelastic scattering of pol<sup>8</sup>.rized electron ( or muon ) on the polarized nucleon. The extension of the results to the case of neutrino scattering is straighforward.

In this case the operators of leading twist 2 enter the structure function  $g_2(x,0)$  (see below) together with operators of mixed symmetry or twist-3 even in leading power of  $Q^{-2}$ . This question has been discussed in works [10], but to our understanding their tratment of twist-3 operators is incorrect. So, we start in this section from the derivation of the OPE and 3 coefficients for twist 2 operators of arbitrary spin.

Then we discuss briefly the renormalization of twist-3 operators. The discussion of this question in [10] is incorrect as well, for even not all mixing operators are discussed. We present also result for the case, in which only one operator is present.

Finally, we turn to power corrections to Bjorken sum rule, which is similar to discussion of section 2,1 of the first part of our work [1].

As it is well known [6], two additional structure functions  $g_1(x,Q^2)$ ,  $g_2(x,Q^2)$  arise for polarized scattering. The spin dependence of the imaginary part of Compton amplitude

$$\frac{A_{\mu\nu} \text{ can be represented as}}{\frac{1}{2\pi} \frac{1}{2\pi} \frac{1}{m} \frac{A_{\mu\nu}}{\mu\nu} \left| \frac{spin}{dependent} - \frac{i}{2pq} \frac{\epsilon_{\mu\nu\rho\epsilon} q_{\rho}}{e^{\mu\nu\rho\epsilon} q_{\rho}} \right|^{\kappa} \\
\times N \left[ \chi_{\epsilon} \chi_{5} g_{2}(x,Q^{2}) + \frac{m_{\nu}}{pq} q_{5} \delta_{5} \delta_{5} g_{2}(x,Q^{2}) \right] N (41)$$

where N is the nucleon spinor normalized by  $N^{\dagger}N=2E$ ,  $\rho$  and q are momenta of the nucleon and  $p^{\dagger}$  quantum,  $m_N$  is the nucleon mass. This form of spin dependence emphasizes the kinematical supression of the second structure function in the scaling limit.

As in the rest of the present work, we start from the quantity ( its antisymmetric part )

where some external field  $\mathcal{A}_{\mu}$  is implied. Using algebra of commutators and the equations of motion for quark field  $\psi$ , one gets the following form of  $\mathcal{T}_{\mu\nu}$ 

$$T_{\mu\nu} = i \xi_{\mu\nu\rho\delta} q_{\rho} \int dx \langle x| \overline{\psi} \left[ \chi_{\delta} \chi_{\delta} \frac{1}{(q+p)^{2}} + \frac{2}{2} \frac{1}{(q+p)^{2}} \widetilde{G}_{\delta} \delta \chi_{\delta} \frac{1}{(q+p)^{2}} \right] \psi |0\rangle$$

$$+ \frac{q}{2} \frac{1}{(q+p)^{2}} \widetilde{G}_{\delta} \delta \chi_{\delta} \frac{1}{(q+p)^{2}} \left[ \psi |0\rangle \right]$$

where terms with twist > 5 are omitted. For twist-2 operators

(43)

(43)

(43)

(43)

(44)

$$T_{MV} = i \xi_{MV} p_{\delta} q_{p} \cdot \frac{1}{q^{2}} \sum_{n=0}^{\infty} \left(\frac{2}{q^{2}}\right) q_{M_{1}} ... q_{M_{n}} \times \left[U_{\delta M_{1} ... M_{n}} + \frac{2n}{n+1} V_{\delta M_{2} ... M_{n}}\right]$$
(44)

with the following definition of operators

Here, S, A means symmetrisation and antisymmetrisation of given indices.

Note the important point here: the operators V vanish for free quarks and are present only due to nonzero field  $G_{\mu\nu}$  in the scattering point. This fact becomes evident if one uses the substitution  $D_{\mu\nu} = (1/2)(y_{\mu\nu}\hat{D} - \hat{D}_{\mu\nu})$  and equations of motion  $\hat{D}_{\nu\nu} = 0$ , similar as it was shown for the operator  $\hat{Q}_{\mu\nu}$  of the preceding section. This fact was overlooked by the authors of ref.  $\begin{bmatrix} 10 \end{bmatrix}$ , who have tried to find coefficients of V using free quark states, which is not possible. The simplest process to be considered is the transition of quark state into quark plus gluon one. Another form of operator V used for this aim can be written as

Using eq.(44) with account of flavor structure of electromagnetic current and the definition (41) of structure functions we obtain the following expressions for moments in leading approximation ( in powers of  $Q^{-2}$  )

$$\int_{0}^{4} dx \, x^{n} (g_{1}^{p} - g_{1}^{n}) = \frac{1}{6} \, \angle \mathcal{U}_{n}^{NS} >> , \quad (n = 0, 2, ...)$$

$$\int_{0}^{4} dx \, x^{n} (g_{2}^{p} - g_{2}^{n}) = \frac{1}{6} \, \frac{n}{n+1} \, (\angle \mathcal{U}_{n}^{NS} >> - \langle V_{n}^{NS} \rangle), \quad (n = 2, 4, ...)$$

$$\int_{0}^{4} dx \, x^{n} (g_{1}^{p} + g_{1}^{n}) = \frac{5}{18} \, \angle \mathcal{U}_{n}^{S} >> , \quad (n = 0, 2, ...)$$

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$$\int_{0}^{4} dx \, x^{n} (g_{2}^{p} + g_{2}^{n}) = \frac{5}{18} \, \frac{n}{n+1} \, (\angle \mathcal{U}_{n}^{S} >> - \angle V_{n}^{S} >>), \quad (n = 2, 4...)$$

where the reduced matrix elements are defined as follows

$$\langle N|U_{\delta M_1...M_n}|N\rangle = \langle \langle U_n \rangle \rangle S \overline{N}_{\chi_{\delta} \chi_{\delta} N} P_{M_1}...P_{M_n} - T_{naces},$$

$$-T_{naces},$$

$$\langle N|V_{\delta M_1...M_n}|N\rangle = \langle \langle V_n \rangle \rangle S' A S' \times (48)$$

$$\langle M_1...M_n \rangle \langle M_2...M_n \rangle \langle M_2...M_n \rangle \langle M_1...M_n \rangle$$

and singlet and nonsinglet operators are (0.=U,V)

$$O^{NS} = O(\psi \to u) - O(\psi \to d)$$

$$O^{S'} = O(\psi \to u) + O(\psi \to d) + (49)$$

$$+ \frac{18}{5} \sum_{q \neq u, d} Q_q^2 \cdot O(\psi \to q)$$

Now we consider the renormalization of the operators

Its discussion in ref. [10] is also incorrect, for as it was noted by L.N.Lipatov instead of one operator 
one should now consider the mixing of (n-1) operators entering the r.h.s. of the equation (46). Again, the discussion of free quark states was misleading.

We have done calculations for the simplest case n=2 where only one twist-3 operator  $V_{6n_1n_2} = S(98)\Psi \tilde{G}_{n_1} + V_{n_2}\Psi$  exists, which are similar to calculations of the spin-1 twist-4 operator discussed in our previous work. The results are as follows:

Tows:  $V_{6}^{NS}(Q) \rightarrow \left[\frac{2}{\sqrt{s(Q)}}\right]^{-\frac{3}{8}\left[3N_{c}-\frac{1}{6}\left(N_{c}-\frac{1}{N_{c}}\right)\right]} V_{6}^{NS}(Q)$   $V_{6}^{NS}(Q) \rightarrow \left[\frac{2}{\sqrt{s(Q)}}\right]^{-\frac{3}{8}\left[3N_{c}-\frac{1}{6}\left(N_{c}-\frac{1}{N_{c}}\right)\right]} V_{6}^{NS}(Q)$ 

 $V_{\beta_{M_{1}N_{2}}}^{S}(Q) \rightarrow \begin{bmatrix} \frac{1}{3N_{c}} - \frac{1}{6}(N_{c} - \frac{1}{N_{c}}) + \frac{2}{3}N_{f} \end{bmatrix} \times V_{\beta_{M_{2}N_{2}}}^{S}(A)$   $V_{\beta_{M_{1}N_{2}}}^{S}(Q) \rightarrow \begin{bmatrix} \frac{1}{3N_{c}} - \frac{1}{6}(N_{c} - \frac{1}{N_{c}}) + \frac{2}{3}N_{f} \end{bmatrix} \times V_{\beta_{M_{2}N_{2}}}^{S}(A)$ 

Here arguments Q and  $\mu$  show the normalization points of operators , N<sub>f</sub> and N<sub>c</sub> are numbers of colours and flavours,  $\delta = \frac{42}{3}N_c - \frac{2}{3}N_{\ell}$ 

This result also disagrees with [10]. Note that in our case both Casimir invariants  $C_2(R) = \frac{2}{2}(N_c - \frac{1}{N_c})$ ,  $C_2(G) = N_c$  are present, while in [10] all the results contain only  $C_2(R)$ .

The last topic of this section is nonleading power corrections connected with operators of twist 4. As an example we

discuss Bjorken sum rule for isovector axial current. The calculation is also similar to corrections to sum rules discussed in our work [1]. The result is as follows

$$\int_{0}^{3} dx \left\{ \left[ g_{1}^{p} - \frac{8}{3} \frac{m_{N}^{2}}{q^{2}} x^{2} \left( g_{2}^{p} + \frac{5}{6} g_{1}^{p} \right) \right] - \left[ p \to n \right] \right\} =$$

$$= \frac{1}{6} \left\{ g_{A} \left( 1 - \frac{\lambda_{S}(Q)}{\pi} \right) - \frac{4}{9} \frac{1}{Q^{2}} \left[ \frac{\lambda_{S}(\omega)}{\lambda_{S}(Q)} \right]^{\frac{32}{96}} \left( \frac{\gamma_{NS}}{Q^{2}} \right) \right\} (51)$$

$$\int_{0}^{3} dx \left\{ \left[ g_{1}^{p} - \frac{8}{3} \frac{m_{N}^{2}}{Q^{2}} x^{2} \left( g_{2}^{p} + \frac{5}{6} g_{1}^{p} \right) \right] + \left[ p \to n \right] \right\} =$$

$$= \frac{5}{18} \left\{ g_{A}^{S} \left( 1 - \frac{\lambda_{S}(Q)}{\pi} \right) - \frac{4}{9} \frac{1}{Q^{2}} \left[ \frac{\lambda_{S}(\omega)}{\lambda_{S}(Q)} \right]^{\frac{32}{6} \left( \frac{32}{9} + \frac{2}{3} N_{A} \right)} \left( \frac{\gamma_{S}^{S}}{Q^{2}} \right) \right\}$$

Here  $g_A$  is the known  $\beta$  decay constant and  $g^S$  corresponds to the isocinglet axial current of the form (49). The twist dependence is defined as

$$\widetilde{O}_{\delta}^{NS} = y \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, u - (u \rightarrow d),$$

$$\widetilde{O}_{\delta}^{S'} = g \, \widetilde{u} \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, u + (u \rightarrow d) +$$

$$+ \frac{18}{5} \, \widetilde{z}^{\gamma} \, Q_{2}^{\gamma} \, g \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, q$$

$$= \frac{18}{5} \, \widetilde{z}^{\gamma} \, Q_{2}^{\gamma} \, g \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, q$$

$$= \frac{18}{5} \, \widetilde{z}^{\gamma} \, Q_{2}^{\gamma} \, g \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, q$$

$$= \frac{18}{5} \, \widetilde{z}^{\gamma} \, Q_{2}^{\gamma} \, g \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, q$$

$$= \frac{18}{5} \, \widetilde{z}^{\gamma} \, Q_{2}^{\gamma} \, g \, \widetilde{G}_{\delta\delta}^{\alpha} t^{\alpha} y_{\delta} \, q$$

Note the similarity of eqs.(51) with the corrections to sum rules for unpolarized target.

The first order in  $\propto_S$  correction is taken from [10], and anomalous dimension of  $\sim_S$  from our work [1].

Note also, that we do not use complete Nachtmann moments but only their expansion up to first order in  $Q^{-2}$ .

# 7. Conclusions

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The consideration of the propagator in X = 0 gauge provide useful method to calculate OPE coefficients for twist-6 operators. We have found corrections to first moments. The general features of these result is rather small coefficients for quark-gluon operators, while for quark operators they are of the order of unity due to diquark diagram in which current act on different quarks. Similar trend, but somehow weaker, was observed in [1] for Q-2 effects. This imply that most probably the diquark effect dominates over quark-gluon ones, whatever is the convention separating these sets of operators. With further progress in experimental accuracy, average values of twist-6 operators can also be found from our results.

Next, in contrast to naive estimates of the effect of the gluon condensate presented in the introduction, this effect is absent completely in Born approximation, and negligibly small due to renormalization mixing of operators.

On the contrary, the interaction with quark condensate is connected with rather large OPE coefficients. Moreover, they are such that clearly separate neutrino and electron scattering and longitudinal and transverse photons. Therefore, they are very promizing candidate to explain the long-standing puzzle of large 64/67 ratio observed in SLAC.

The theory of the scattering on the polarized target can be developed in the same formalism. The interesting point here is that quark-gluon effects in form of twist-3 operators appear here even in the main approximation in structure function  $g_2$ . Most of the previous results reported in literature on their coefficients and renormalization was incorrect.

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