38

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DOES LIGHT PROPAGATE IN GRAVITATIONAL FIELD WITH THE VELOCITY OF LIGHT ?

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A b s t r a c t

Since the velocity of wave front propagation is determined by the high frequency asymptotics of refraction index, superluminal velocity found in nonrenormalizable theories is in fact only the result of low order perturbative calculations.

It was claimed recently[1] that QED radiative corrections to the Maxwell equations in gravitational field can result in local velocity of light exceeding unity. To show this the authors of Ref.[1] use the following effective Lagrangian:

$$L_{eff} = \frac{\alpha}{m^2}\left(aRF^{\mu\nu}F_{\mu\nu} + bR_{\mu\nu}F^{\mu\sigma}F^{\nu}_{\ \sigma} + cR_{\mu\nu\sigma\tau}F^{\mu\nu}F^{\sigma\tau} + dD_{\mu}F^{\mu\nu}D_{\sigma}F^{\sigma}_{\ \nu}\right) \qquad (1)$$

where $\alpha = 1/137$, m is the electron mass, $F_{\mu\nu}$ is the electromagnetic field strength, $R_{\mu\nu\sigma\tau}$ is the Riemann tensor, $R_{\mu\nu} = R_{\mu\sigma\nu}^{\ \ \ \ \sigma}$, $R = R_{\mu}^{\ \mu}$; a, b, c and d are numerical constants. This Lagrangian was obtained previously[2] in the lowest order in electromagnetic interaction. Note that the last term in Eq.(1) influences the equation of motion in the order of α^2 only.

One can easily see that being of the same order in the photon frequency ω as the free field Lagrangian, L_{eff} (1) gives the contribution into the refraction index n independent of ω and different generally speaking for different polarizations. In particular, in the Schwarzschild background[4] n proves to be smaller than unity for one of the polarization states and so, according to Ref.[1], this state propagates with a superluminal velocity. We think however that this conclusion is unfounded because of the following reasons.

It is well-known[3] that the velocity of wave front propagation in a dispersive medium is determined by the asymptotics of the refraction index $n(\omega)$ at $\omega \to \infty$. Indeed, consider a wave packet

$$A(x,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-nx)} f(\omega) \qquad (2)$$

where $n(\omega)$ is known to be analytic in the upper half-plane ω. It can be easily seen that the function $f(\omega)$ should possess the same property to garantee the vanishing of the signal for $x>t$ in the region where the external field is absent. So for $t<nx$ one can shift up the integration contour in the complex ω-plane in Eq.(2) and find consequently that the wave front velocity is

$$v = \lim_{\omega\to\infty} \frac{1}{n(\omega)} \qquad (3)$$

Since $n(\omega)$ is proportional to the amplitude of the forward scattering in external field, the wave front velocity is determined by the high energy asymptotics of this amplitude. Meanwhile, effective Lagrangian (1) used in Ref.[1] describes in fact low energy limit of light scattering in gravitational field. Although to the lowest order in α the refraction index calculated in Ref.[1] is exact as a function of ω [4], the higher order corrections rise with ω faster and faster. In particular, the last term in Eq.(1) omitted in the calculations made in Ref.[1] leads to the extra contribution to the refraction index which increase quadratically with frequency:

$$\delta n \sim \frac{\alpha^3 \omega^2}{m^6} R^2 \qquad (4)$$

(We suppress here the indices of R describing the tensor structure of δn.)

So the consideration of the lowest order (or any finite order) in perturbative expansion in a nonrenormalizable theory is insufficient for the conclusion about the velocity of wave front propagation. One can reverse the arguments and say that the causality condition implies a restrictive bound on the rise of a scattering amplitude.

Note that in a number of papers[5] it was claimed that higher spin ($s \geqslant 1$) particles can propagate in external fields with superluminal velocity. The presented arguments show that this assertion is unfounded by the same reason - all those theories are nonrenormalizable.

A c k n o w l e d g e m e n t s

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R e f e r e n c e s

1. I.T.Drummond and S.J.Hathrell, Phys.Rev. D 22, 343 (1980).
2. F.A.Berends and R.Gastmans, Ann.Phys.(N.Y.) 98, 225 (1976).
3. M.A.Leontovich, in L.I.Mandelstam, Lectures on Optics, Relativity and Quantum Mechanics (Nauka, Moscow, 1972, p.308).
4. Since the invariant amplitudes determined by the Feynman diagrams with three external lines depend only on the four-momenta squared of the external particles, then in the case of the forward scattering of light in gravitational field these amplitudes are constant for any ω.
5. List of references on this subject is too lengthy for this short note. One can find it in the bibliographical review by V.F.Perepelitsa, ITEP-100,165 (1980).